SAPIENZA
UNIVERSITÀ DI ROMA

FACULTY OF MATHEMATICAL, PHYSICAL AND NATURAL SCIENCES

Influence of neutron star's equation of state on its deformability

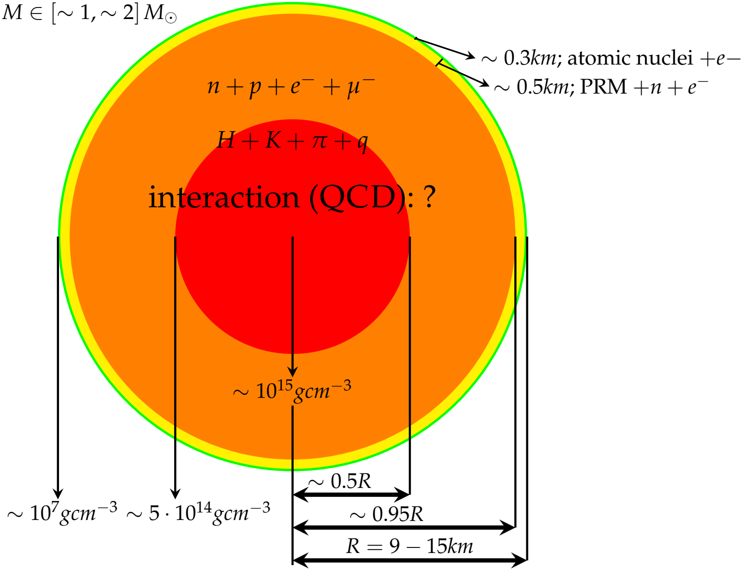
MASTER'S DEGREE THESIS

AUTHOR – Francesco Torsello

SUPERVISOR – Prof. Valeria Ferrari

Internal structure of a neutron star

$M \in [\sim 1, \sim 2]\, M_\odot$



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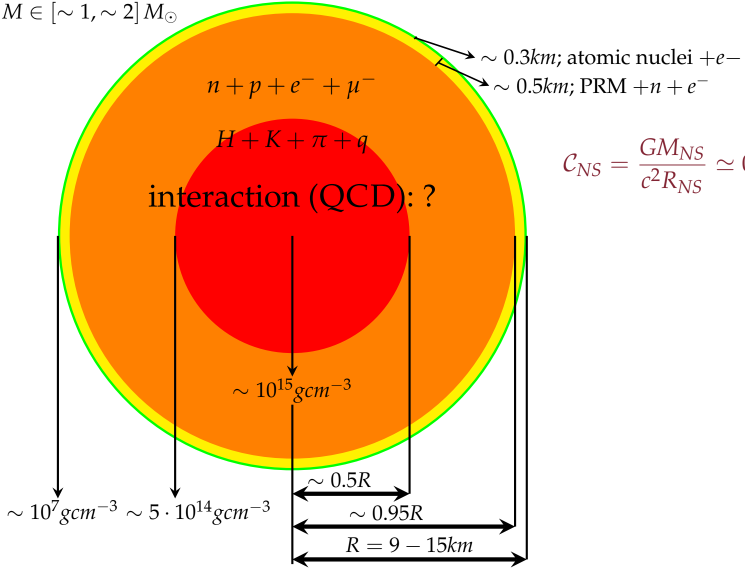

$$\mathcal{C}_{NS} = \frac{GM_{NS}}{c^2 R_{NS}} \simeq 0.1$$

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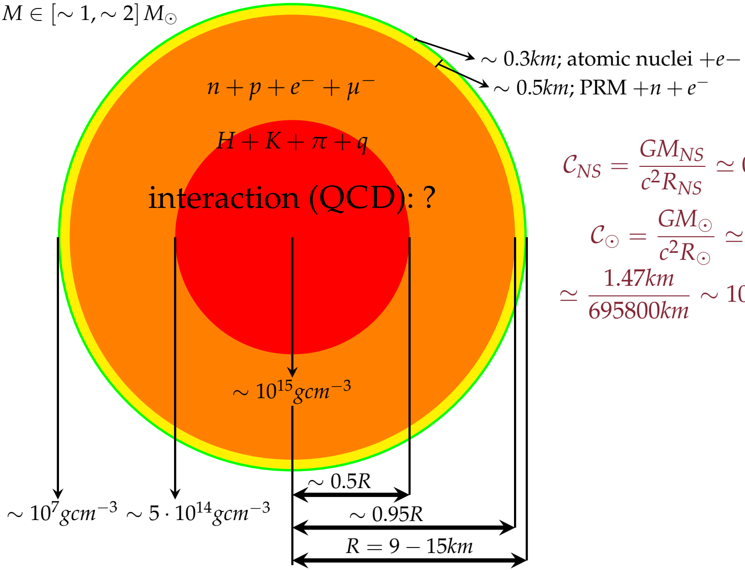

$$\mathcal{C}_{NS} = \frac{GM_{NS}}{c^2 R_{NS}} \simeq 0.1$$

$$\mathcal{C}_\odot = \frac{GM_\odot}{c^2 R_\odot} \simeq$$

$$\simeq \frac{1.47km}{695800km} \sim 10^{-6}$$

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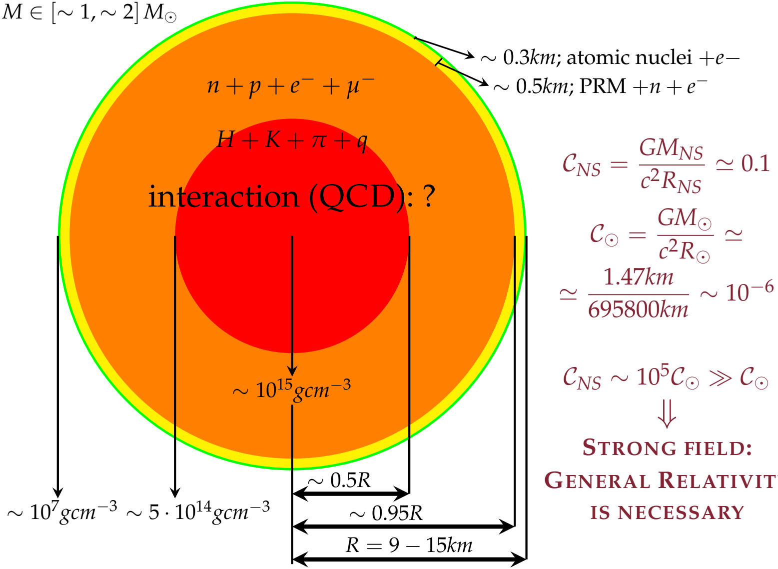


$$\mathcal{C}_{NS} = \frac{GM_{NS}}{c^2 R_{NS}} \simeq 0.1$$

$$\mathcal{C}_{\odot} = \frac{GM_{\odot}}{c^2 R_{\odot}} \simeq$$

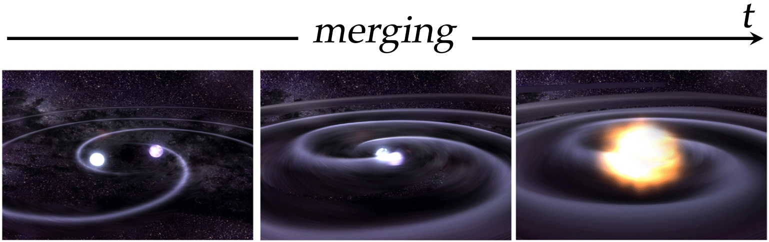
$$\simeq \frac{1.47km}{695800km} \sim 10^{-6}$$

$$\mathcal{C}_{NS} \sim 10^5 \mathcal{C}_{\odot} \gg \mathcal{C}_{\odot}$$

$$\Downarrow$$

STRONG FIELD: GENERAL RELATIVITY IS NECESSARY

Binary systems NS-NS or NS-BH



GRAVITATIONAL WAVES EMISSION

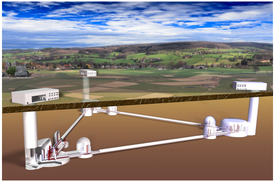
- Initial phase: point-like approximation, the gravitational wave does not depend on the internal structure of neutron stars
- Final phase: **neutron stars are deformed due to mutual tidal interaction and the gravitational wave depends on their internal structures**

Gravitational waves revelation

II generation – AdvancedLIGO, AdvancedVIRGO
III generation – Einstein Telescope

AdvancedLIGO – Caltech, MIT

AdvancedVIRGO – Centre National de la Recherche Scientifique (CNRS), INFN



Einstein Telescope – EGO, INFN, Max-Planck-Institut für Gravitationsphysik, CNRS, University of Birmingham, University of Glasgow, NIKHEF, Cardiff University

Purpose of this thesis

UNDESTANDING OF WHAT IS THE INTERNAL REGION OF A NEUTRON STAR THAT MAINLY DETERMINES ITS DEFORMABILITY,

in order to establish what is the internal region of a neutron star about which we can obtain information by measuring gravitational waves emitted by binary systems shortly before the merging

Static and spherically symmetric neutron star with mass M and radius R (non rotating star)

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\vartheta^2 + \sin^2(\vartheta)d\varphi^2\right),$$

with $G = c = 1$

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$r \leq R$

TOV equations (Tolman-Oppenheimer-Volkoff)

$$\begin{cases} \dfrac{dm(r)}{dr} = 4\pi r^2 \epsilon(r) \\ \dfrac{dp(r)}{dr} = -\dfrac{[\epsilon(r) + p(r)]\left[m(r) + 4\pi r^3 p(r)\right]}{r\left(r - 2m(r)\right)} \end{cases}$$

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3 unknown functions e 2 ODE:

1. $m(r) \equiv \dfrac{r}{2}\left(1 - e^{-2\lambda(r)}\right)$
2. $\epsilon(r) =$ mass-energy density
3. $p(r) =$ pressure

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THIRD EQUATION

Equation of State (EoS)

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THIRD EQUATION

Equation of State (EoS)

$r \geq R$

Schwarzschild metric

$$e^{2\nu(r)} = 1 - \frac{2M}{r} \qquad e^{2\lambda(r)} = \left(1 - \frac{2M}{r}\right)^{-1}$$

Piecewise polytropics

Influence of neutron star's equation of state on its deformability

MASTER'S DEGREE THESIS

Torsello

Neutron stars and tidal deformability

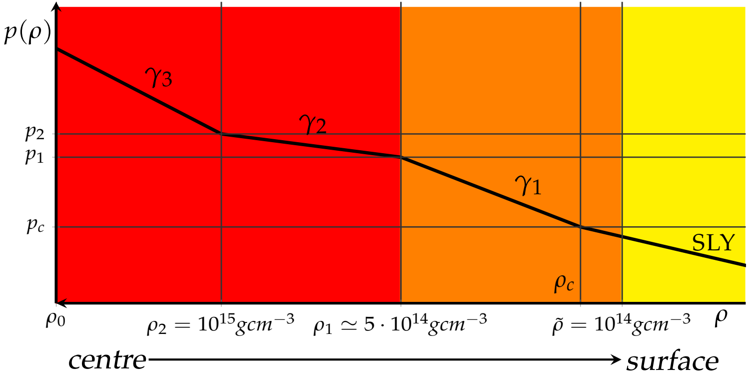
Work done

Results

Read, Lackey, Owen, Friedman, *Physical Review* D79, 124032, 2009

$[\rho_{i-1}, \rho_i]$, with $i = 1, ..., n$ and n number of intervals

$$\rho \in [\rho_{i-1}, \rho_i] \Longrightarrow \begin{cases} p(\rho) = K_i \rho^{\gamma_i} & \text{: EoS is polytropic} \\ \mathrm{d}\left(\frac{\epsilon}{\rho}\right) = -p\,\mathrm{d}\left(\frac{1}{\rho}\right) & \text{: the first principle of Thermodynamics is valid} \end{cases}$$



Tidal deformability λ: definition

- TIDAL QUADRUPOLAR FIELD $\Longrightarrow \ell = 2$

Tidal Quadrupolar Tensor

$$\mathcal{E}_{ij} = R^{\text{tid}}_{\mu\nu\alpha\beta}\left(e^{\mu}_{(i)}e^{\nu}_{(0)}e^{\alpha}_{(j)}e^{\beta}_{(0)}\right) = \sum_{m=-2}^{2}\mathcal{E}_m\mathcal{Y}^{2m}_{ij}$$

Mass Quadrupole Moment

$$Q_{ij} = \int \mathrm{d}^3x\,\delta\rho(\vec{x})\left(x_ix_j - \frac{1}{3}r^2\delta_{ij}\right) = \sum_{m=-2}^{2}Q_m\mathcal{Y}^{2m}_{ij}$$

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- WEAK TIDAL FIELD:

FIRST PERTURBATIVE ORDER, $\mathcal{O}(\mathcal{E}_{ij})$

$$Q_{ij} = -\left[\frac{2R^5}{3G}k_2\right]\mathcal{E}_{ij} \Longrightarrow \lambda \equiv \frac{2R^5}{3G}k_2$$

Tidal deformability λ of a static and spherically symmetric neutron star in an external tidal quadrupolar gravitational field

$$\delta G_{\mu}{}^{\nu} = 8\pi \delta T_{\mu}{}^{\nu}$$

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Thorne, *Physical Review* D58, 124031, 1998

$$g_{00} = -1 + \frac{2M}{r} + \left(-\frac{3\lambda}{r^3} - r^2\right) \mathcal{E}_0 Y_{20}(\vartheta)$$

Tidal deformability λ of a static and spherically symmetric neutron star in an external tidal quadrupolar gravitational field

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$$\searrow \qquad \swarrow$$

$$\lambda = \left(\frac{2R^5}{3G}\right)\frac{8\mathcal{C}^5}{5}(1-2\mathcal{C})^2\left(2+2\mathcal{C}(y-1)-y\right)\cdot$$
$$\cdot\left[2\mathcal{C}\left(6-3y+3\mathcal{C}(5y-8)\right)+4\mathcal{C}^3\left(13-11y+\mathcal{C}(3y-2)+\right.\right.$$
$$\left.\left.+2\mathcal{C}^2(1+y)\right)+3(1-2\mathcal{C})^2\left(2-y+2\mathcal{C}(y-1)\right)\ln(1-2\mathcal{C})\right]^{-1},$$

$$\mathcal{C} = \frac{M}{R}, \quad y = \frac{RH'(R)}{H(R)}$$

Hinderer, *Astrophysical Journal* 677: 1216-1220, 2008

$H(r)$ is the radial perturbation of the Schwarzschild metric

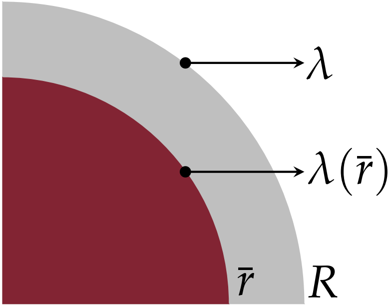
Estimate of λ in the interior of the star

APPROXIMATION

We evaluate the tidal deformability λ in $\bar{r} < R$ supposing that the internal region $\bar{r} < r \leq R$ does not exist.

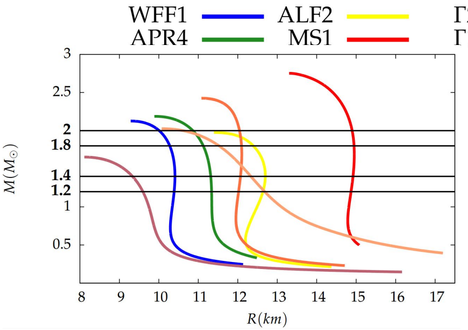
$\Downarrow$

To calculate $\lambda(\bar{r})$ it suffices to substitute $R \to \bar{r}$ in the previous formula of λ



$\bar{r} \leq R$

Choice of EOS and of stellar configurations used in our study



REASONS

- "Extreme" EOS: most and least compact neutron stars
- Different composition of matter in the interior of the stars:
  - MS1: n, p, e^-, μ^-, relativistic mean field theory
  - ALF2: n, p, e^-, μ^-, q, variational method
  - ecc...

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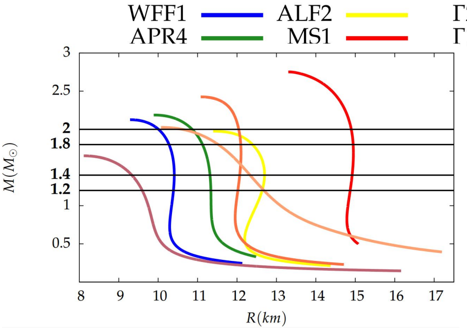

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EOS in decreasing order of produced neutron stars' compressibility $\chi = (\gamma_i p)^{-1}$:

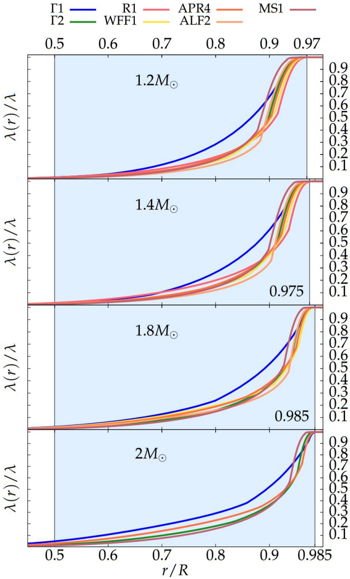
R1, WFF1, APR4, Γ1, Γ2, ALF2, MS1

⟶ *stiff*, minor $\mathcal{C} = \frac{M}{R}$

greater $\mathcal{C} = \frac{M}{R}$, *soft* ⟵

R1, Γ1, Γ2 are defined in Yagi, Stein, Pappas, Yunes, Apostolatos, *Physical Review* D90, 063010, 2014

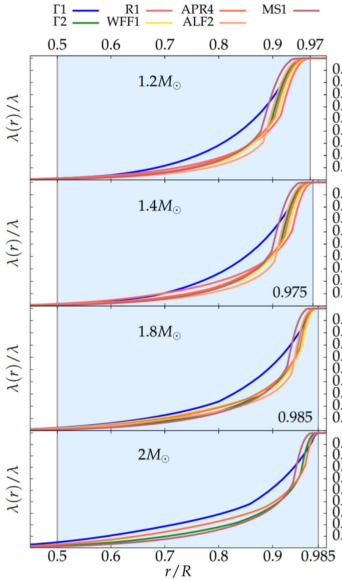
Radial profiles of tidal deformability



$$\left(\frac{\lambda(r)}{\lambda}\right) \text{ vs } \left(\frac{r}{R}\right)$$

$$\mathcal{D} = [0.5R, \sim 0.98R]$$

Radial profiles of tidal deformability

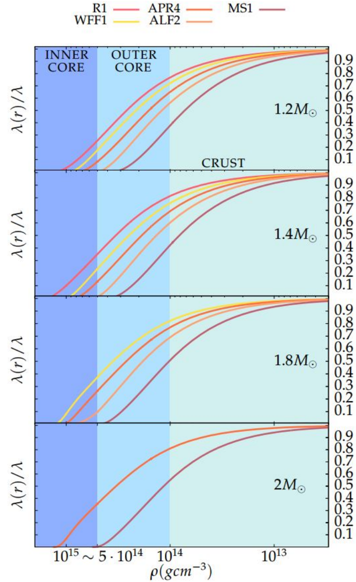


$$\left(\frac{\lambda(r)}{\lambda}\right) \text{ vs } \left(\frac{r}{R}\right)$$

$$\mathcal{D} = [0.5R, \sim 0.98R]$$

What is the physical internal region (inner core, outer core, inner crust, outer crust) that mainly contributes to determine λ?

Tidal deformability profiles depending on mass density



$$\left(\frac{\lambda(r)}{\lambda}\right) \text{ vs } \rho(r)$$

CONTRIBUTIONS TO $\lambda\,(1.4M_\odot)$

PHYSICAL REGION	EOS *stiff*	EOS *soft*
CORE	45%	80%
CRUST	55%	20%
INNER CORE	$\sim 0\%$	25%
OUTER CORE	45%	55%

Binary system NS-NS: gravitational wave

$$h(f) = \mathcal{A}(f)e^{i\psi(f)}$$, with f wave frequency

$$\psi(f) = \psi_{\text{pp}}(f) + \psi_{\bar{Q}}(f) + \psi_{\bar{\lambda}}(f)$$

$\psi_{\text{pp}}(f)$ $\downarrow$ "point particle" model's contribution

$+\psi_{\bar{Q}}(f)$ $\downarrow$ rotational contribution: $\bar{Q} = -\frac{MQ^{\text{rot}}}{J^2}$

$+\psi_{\bar{\lambda}}(f)$ $\downarrow$ tidal contribution: $\bar{\lambda} = \frac{\lambda}{M^5}$

Binary system NS-NS: gravitational wave

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- $\psi_{\rm pp}(f)$ → "point particle" model's contribution
- $\psi_{\bar{Q}}(f)$ → rotational contribution: $\bar{Q} = -\frac{MQ^{\rm rot}}{J^2}$
- $\psi_{\bar{\lambda}}(f)$ → tidal contribution: $\bar{\lambda} = \frac{\lambda}{M^5}$

$$\psi_{\bar{\lambda}}(f) = -\frac{3}{128} (\mathcal{M}\pi f)^{-5/3} \left\{24 \left[\left(1 + 7\eta - 31\eta^2\right) \lambda_S + \right.\right.$$
$$\left.\left. + \left(1 + 9\eta + 11\eta^2\right) \lambda_a \delta m\right] x^5\right\} + \mathcal{O}(x^6)$$

$\mathcal{M} = \left(\mu^3 (M_1 + M_2)^2\right)^{1/5}$ ="chirp mass", $\mu = \frac{M_1 M_2}{M_1 + M_2}$, $\eta = \frac{M_1 M_2}{(M_1 + M_2)^2}$,

$\delta m = \frac{M_1 - M_2}{M_1 + M_2}$, $x = ((M_1 + M_2)\pi f)^{5/3}$, $\lambda_S = \frac{\bar{\lambda}_1 + \bar{\lambda}_2}{2}$, $\lambda_a = \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2}$

Conclusions

Future measure of λ by gravitational waves revelation

Conclusions

Future measure of λ by gravitational waves revelation

$\Swarrow$

λ close to the value related to **soft** EOS

Conclusions

Future measure of λ by gravitational waves revelation

$\swarrow$

λ close to the value related to soft EoS

$\Downarrow$

λ is mainly determined by the star's core

Conclusions

Future measure of λ by gravitational waves revelation

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EOS in the innermost region of the star is soft

Conclusions

Future measure of λ by gravitational waves revelation

$\swarrow$ $\searrow$

λ close to the value related to soft EoS

λ close to the value related to stiff EoS

$\Downarrow$

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Conclusions

Future measure of λ by gravitational waves revelation

$\swarrow$ $\searrow$

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$\Downarrow$

λ is mainly determined by the star's core

$\Downarrow$

EOS in the innermost region of the star is soft

λ close to the value related to stiff EOS

$\Downarrow$

λ is mainly determined by the star's crust

Conclusions

Future measure of λ by gravitational waves revelation

$\swarrow$ $\searrow$

λ close to the value related to soft EOS	λ close to the value related to stiff EOS
$\Downarrow$	$\Downarrow$
λ is mainly determined by the star's core	λ is mainly determined by the star's crust
$\Downarrow$	$\Downarrow$
EOS in the innermost region of the star is soft	Little information about EOS in the innermost region