Numerical relativity simulations of thick accretion disks around tilted Kerr black holes

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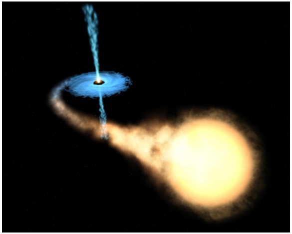
Outline

Thick accretion disks in the universe

- Thick accretion disks: believed to be formed in NS-NS and NS-BH mergers (a mechanism for *sGRB*), as well as in the CC of massive stars (a mechanism for *lGRB*).
- *Rezzolla et al (2010)* have shown that massive, thick disks form in simulations of unequal mass NS-NS mergers.
- to explain *sGRB* as energy released from accreted material coming from a thick disk, it must survive long enough > stability.



Instabilities of disks

- Papalouizou-Pringle Instability (PPI) (Papaloizou and Pringle (1984)): axisymmetry in the disk is broken and *m* planetary structures emerge, where *m* is the dominant mode.
- Runaway Instability (RI) (Abramowicz et al (1983)): initially stable disk is being accreted almost completely in a few dynamical time-scales onto the central object.
- Aim to understand under which conditions (and if) these instabilities develop, and investigate the effect the BH tilt has on these instabilities.

Tilted Disks: Motivation and previous work

- Pioneering work in this field by *Fragile et al (2005,2006)* who have analysed tilted disks in the Cowling (fixed background spacetime) approximation.
- No reason to expect that $\mathbf{S_{BH}}$ is aligned with the orbital plane of the NS-BH merger.
- Perform simulations with spacetime evolution to investigate effects of BH tilt to BH+torus evolution.
- Computationally cheaper (due to symmetries) to consider $\mathbf{S_{BH}}$ and $\mathbf{L_{disk}}$ aligned.
- Test effect of Spin magnitude and Spin direction on evolution of disk and search for imprint on GW.

Model and initial data

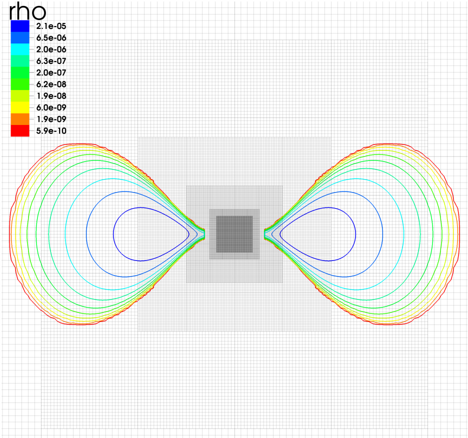
- Self-gravitating, massive tori around non-rotating stellar mass BH, *Stergioulas (2011)*.
- Starting from an AJS disk (Polish doughnut), the field equations of the QI spacetime and the hydrostatic equilibrium equations are solved iteratively until an equilibrium solution is found.
- Tilted simulations: Kerr BH in improved QI coordinates (*Liu et al 2009*)
- Tilt BH by rotating the coordinate system by an angle θ about the x-axis

Simulation software

- Simulations were performed using the publicly available *Einstein Toolkit* (`www.einsteintoolkit.org`).
- We solve the 3D Einstein equations: $G^{\mu\nu} = 8\,\pi\,T^{\mu\nu}$ in the so-called BSSN formulation (`MacLachlan` thorn).
- Solve the relativistic hydrodynamic equations in conservative form (*Valencia formulation*) for a perfect fluid, using High Resolution Shock Capturing schemes, coupling the hydro evolution to the spacetime via the stress-energy tensor (`GRHydro` thorn).
- Mesh: `Carpet Code`, providing AMR.

Disk Models

Model	ρ_{max} $[G{=}c{=}M_\odot{=}1]$	l	M_{torus}/M_{BH}	f_{orb} $[Hz]$
D2	1.05e-05	3.75, const.	4.4e-02	1360
C1B	5.91e-05	3.67, const.	1.6e-01	1300
NC1	1.69e-05	3.04, non-const.	1.1e-01	843



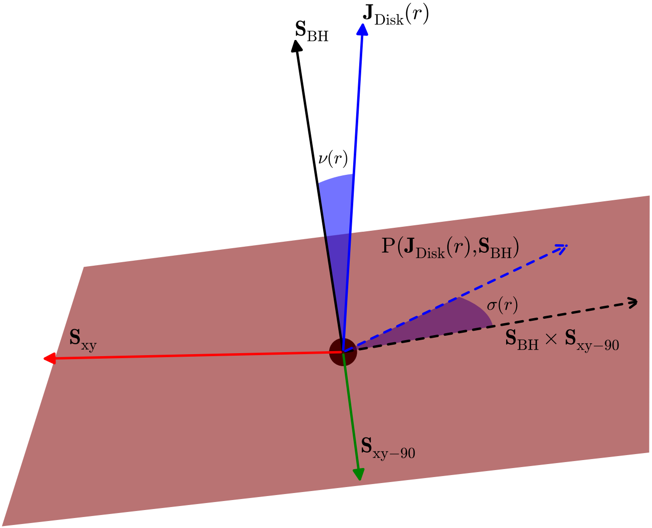
Analysis of Twist and Tilt

- We analyse the response of the disk to the tilted BH by two quantities:
- The twist: $\sigma(r) = \angle(\mathbf{S}_{\mathrm{BH}} \times \mathbf{S}_{\mathrm{xy}-90}, \mathrm{P}(\mathbf{J}_{\mathrm{Disk}}(r), \mathbf{S}_{\mathrm{BH}}))$, where

$$\mathrm{P}(\mathbf{a}, \mathbf{b}) = \mathbf{a} - \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\,\mathbf{b}\,, \qquad (1)$$

  is the projection of vector **a** onto the plane with normal **b**.
- and tilt: $\nu(r) = \angle(\mathbf{S}_{\mathrm{BH}}, \mathbf{J}_{\mathrm{Disk}}(r))$
- The disk is said to become twisted (warped), if $\sigma(r)$ ($\nu(r)$) vary with r

Analysis of Twist and Tilt II



Measuring BH spin direction in 3D Cartesian grids

- One of the ways to measure spin direction (Campanelli et al, 2006) is by using flat-space rotational Killing vectors

$$\begin{aligned} \xi_x &= (0, -z, y) \\ \xi_y &= (z, 0, -x) \\ \xi_z &= (-y, x, 0) \end{aligned} \tag{2}$$

in the angular momentum integral of the isolated and dynamical horizon formalism (Ashtekar and Krishnan):

$$S_i = \frac{1}{8\pi} \int_S \left(\xi_i^a R^b K_{ab} \right) dS, \tag{3}$$

Measuring BH spin direction in 3D Cartesian grids II

- One can derive the angular momentum integral (3) from the Weinberg pseudotensor (Weinberg, 1972) when the pseudotensor is expressed in Gaussian coordinates ($\alpha = 1$, $\beta^i = 0$), (Mewes et al, submitted to PRD).
- Both integrals are equal to the Komar angular momentum integral, when the Komar integral is written in a foliation adapted to the axisymmetry of the problem.
- Pseudotensors are problematic, because they are not coordinate independent quantities, however, by using Gaussian coordinates, we restore coordinate freedom in the Weinberg pseudotensor.

Results

- PPI universality and BH movement
- Bardeen-Petterson effect and tilt evolution

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PPI universality and BH movement

- The models that develop the PPI do so irrespective of initial tilt angle and BH spin magnitude.
- The over-density lump (planet) that develops causes the BH to start moving in a spiral.
- For tilted models, the spiral plane is tilted and causes a mild kick in the vertical direction when the PPI saturates.

Model	ρ_{max} [$G=c=M_{\odot}=1$]	l	M_{torus}/M_{BH}	f_{orb} [Hz]
D2	1.05e-05	3.75, const.	4.4e-02	1360
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PPI development in untilted C1B, xy-plane

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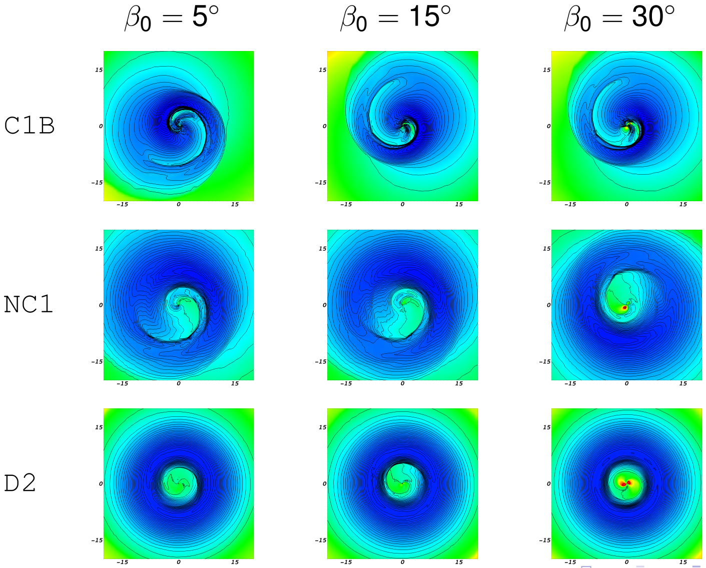
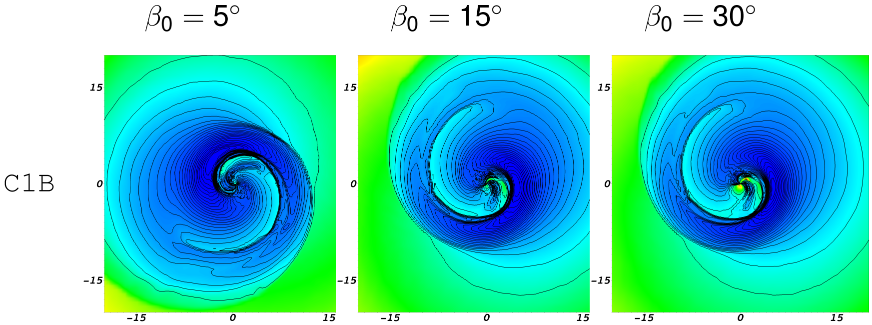
No PPI development in untilted model D2

Tilted disk D2 isovolume animation

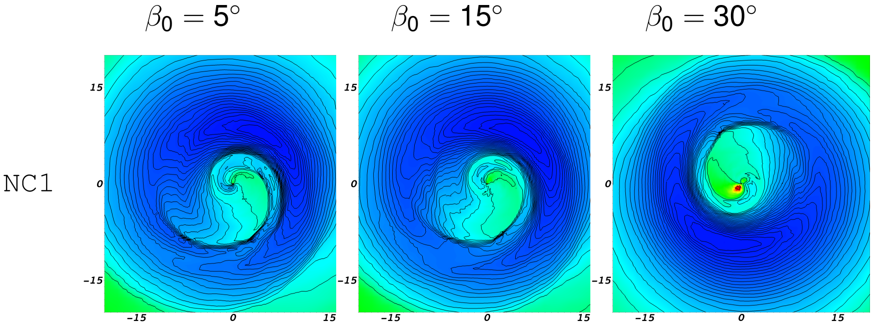
PPI in tilted disks, a=0.1



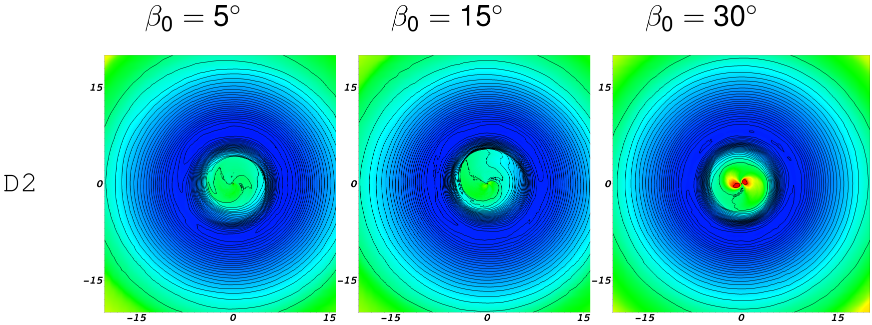
PPI in tilted disks, a=0.1



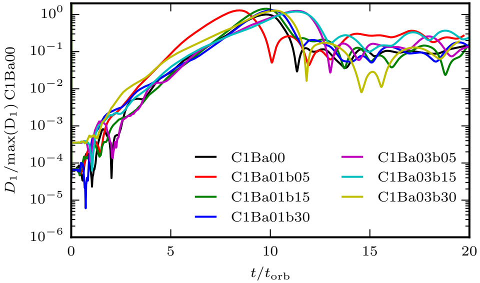
PPI in tilted disks, a=0.1

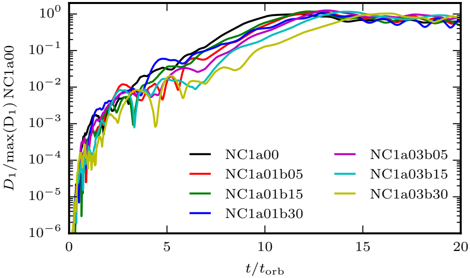


PPI in tilted disks, a=0.1



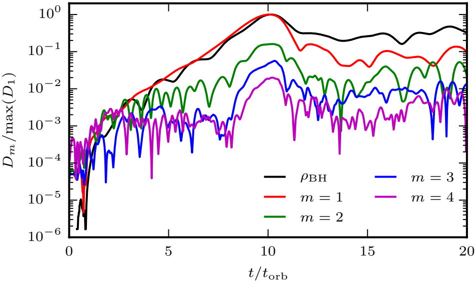
M=1 mode evolution in `C1B` and `NC1`

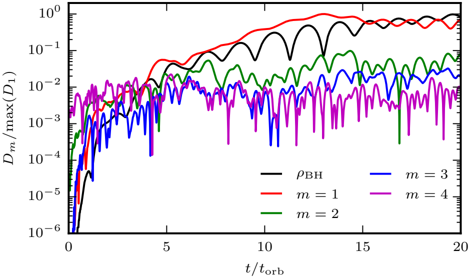




$$D_m = \int \rho\, e^{-im\phi}\, d^3x$$

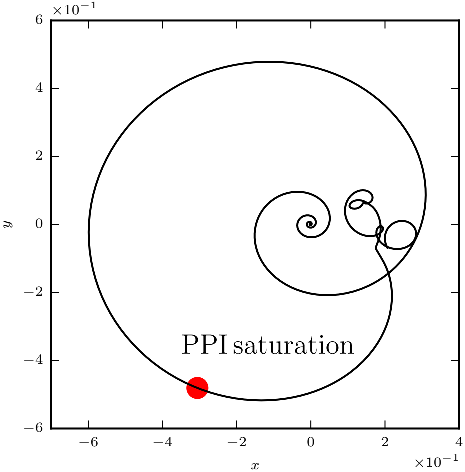
Mode evolution in `C1Ba01b30` and `NC1a01b30`

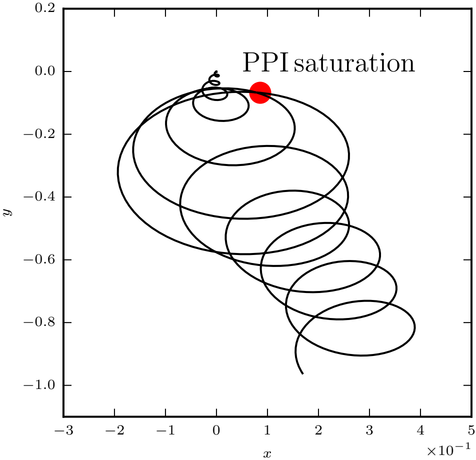




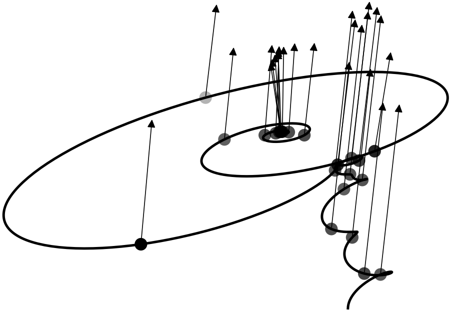
$$D_m = \int \rho \, e^{-im\phi} \, d^3x$$

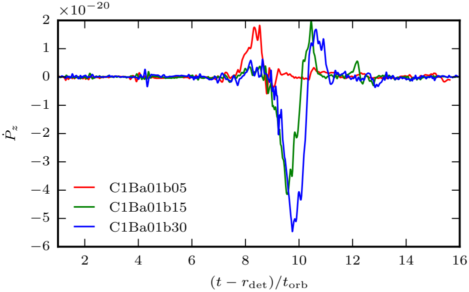
BH xy-movement in `C1Ba01b30` and `NC1a01b30`





Full BH trajectory and radiated linear momentum in z-direction for `C1Ba01b30`





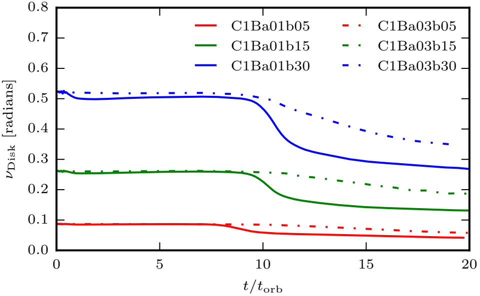
Results

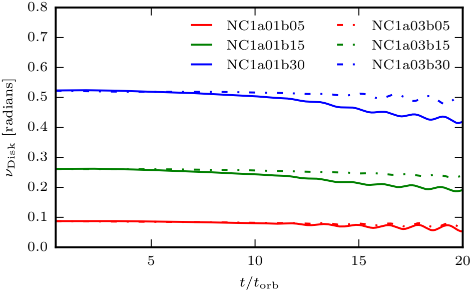
- universality and BH movement
- Bardeen-Petterson effect and tilt evolution

Bardeen-Petterson effect

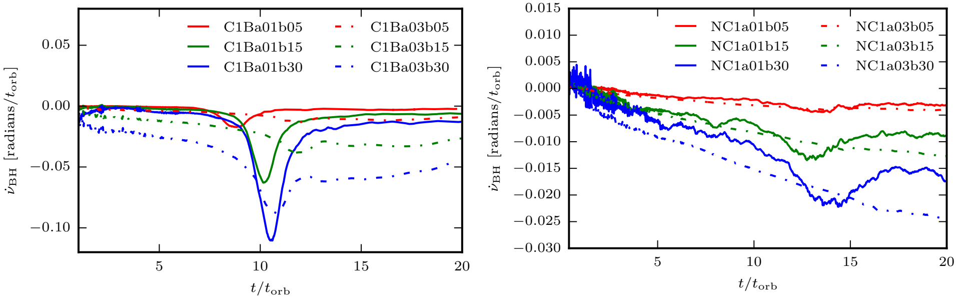
- BP-effect causes alignment of BH spin and disk angular momentum ($\nu(r) = 0$) in the inner regions of the disk.
- Generally only expected for thin disks.
- We observe that during the growth of the PPI the BH spin and total disk angular momentum vector tend to align.
- There are strong oscillations in the tilt for models `NC1` for the inner disk region due to the persistent $m = 1$ structure.

Global disk tilt evolution for C1B and NC1

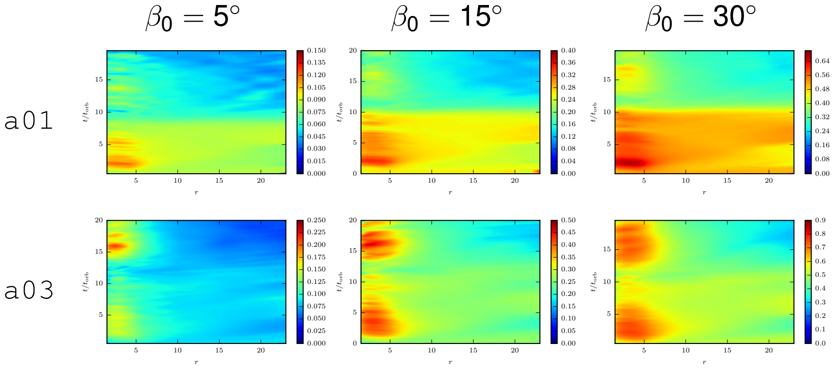




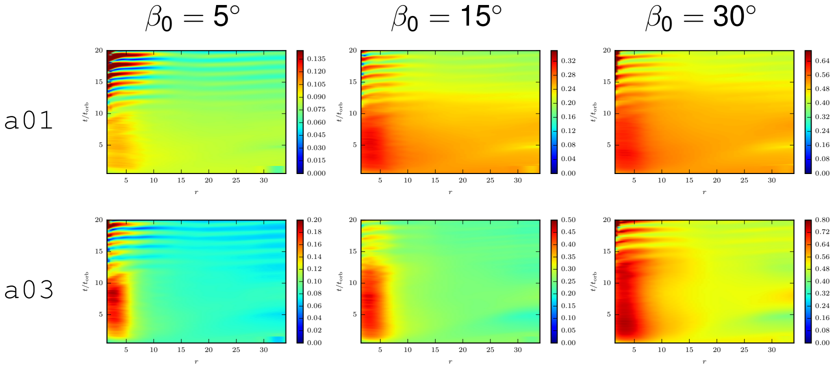
BH nutation rate about the z-axis for `C1B` and `NC1`



T-r diagrams of complete tilt evolution `C1B`



T-r diagrams of complete tilt evolution `NC1`

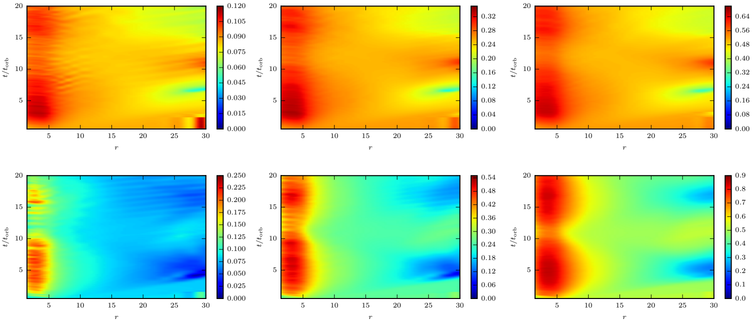


T-r diagrams of complete tilt evolution D2

$\beta_0 = 5°$ $\beta_0 = 15°$ $\beta_0 = 30°$

a01

a03



Conclusions

- The PPI seems to be a universal feature of the models that develop it: we have seen that it grows for all initial tilt angles and spin magnitudes investigated.

- The PPI causes a realignment between BH and global disk angular momentum during it's growth. For models `NC1`, the persistent $m = 1$ structure in the disk causes a rapid oscillation of the tilt angle $\nu(r)$ in the inner region of the disk.

The End

Thank you for your attention!

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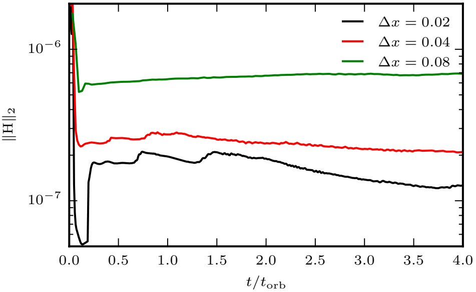
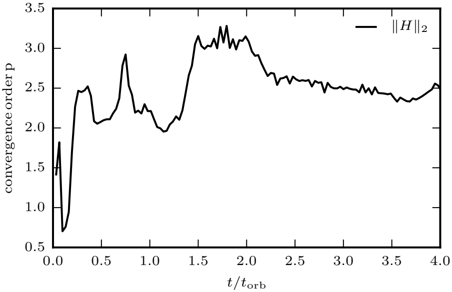
Other codes and their results

- Until very recently, full 3D GR simulations of self-gravitating tori around BH were not possible.
- *Kiuchi et al (2011)*, *Montero et al (2010)* (in 2D) and *Korobkin et al (2011,2012)* performed simulations of self-gravitating thick tori with different codes.
- We use a fourth code and different setup in our current work to redo some of the simulations of *Korobkin et al (2011/2012)* with similar initial data.

Convergence



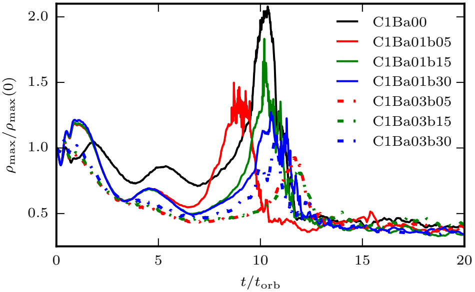


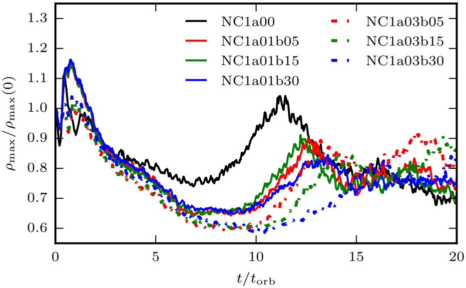
$$p = \frac{1}{\log(f)} \log\left(\frac{\left|\|H\|_2^{\rm low} - \|H\|_2^{\rm med}\right|}{\left|\|H\|_2^{\rm med} - \|H\|_2^{\rm high}\right|}\right)$$

Where does the non-axisymmetric perturbation in the torus causing the PPI come from?

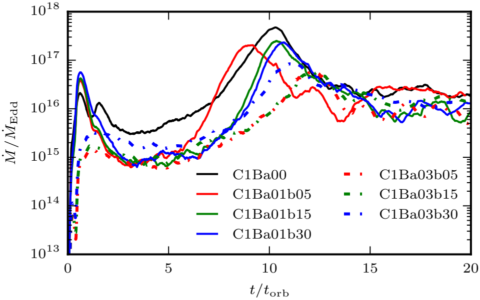
- Spherically symmetric (Schwarzschild BH) or axisymmetric (Kerr BH) gauge pulse travelling through the torus initially.
- Cartesian grid causes small delays or advances in the perturbation > it is not axisymmetric any more.
- In Spherical codes, PPI has to be seeded manually.

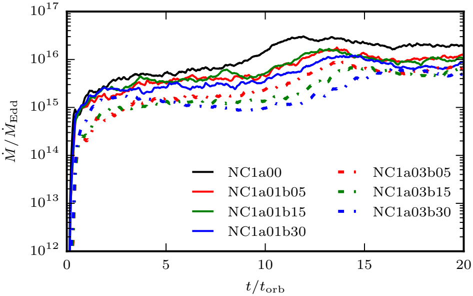
evolution of $\rho_{\max}$ for C1B and NC1



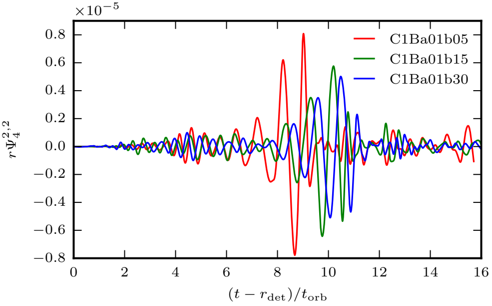


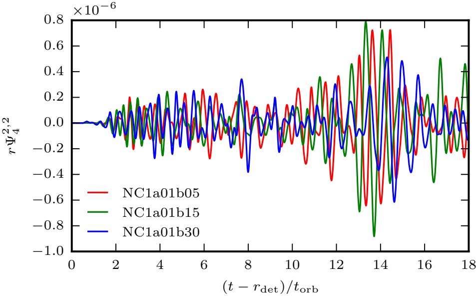
Accretion rate for C1B and NC1





$l = m = 2$ mode of the real part of the Weyl scalar Ψ_4 for C1B and NC1





Angular momentum transport for `C1Ba00` and `NC1a00`