Strong-Field Scattering of Two Black Holes: Numerics Versus Analytics

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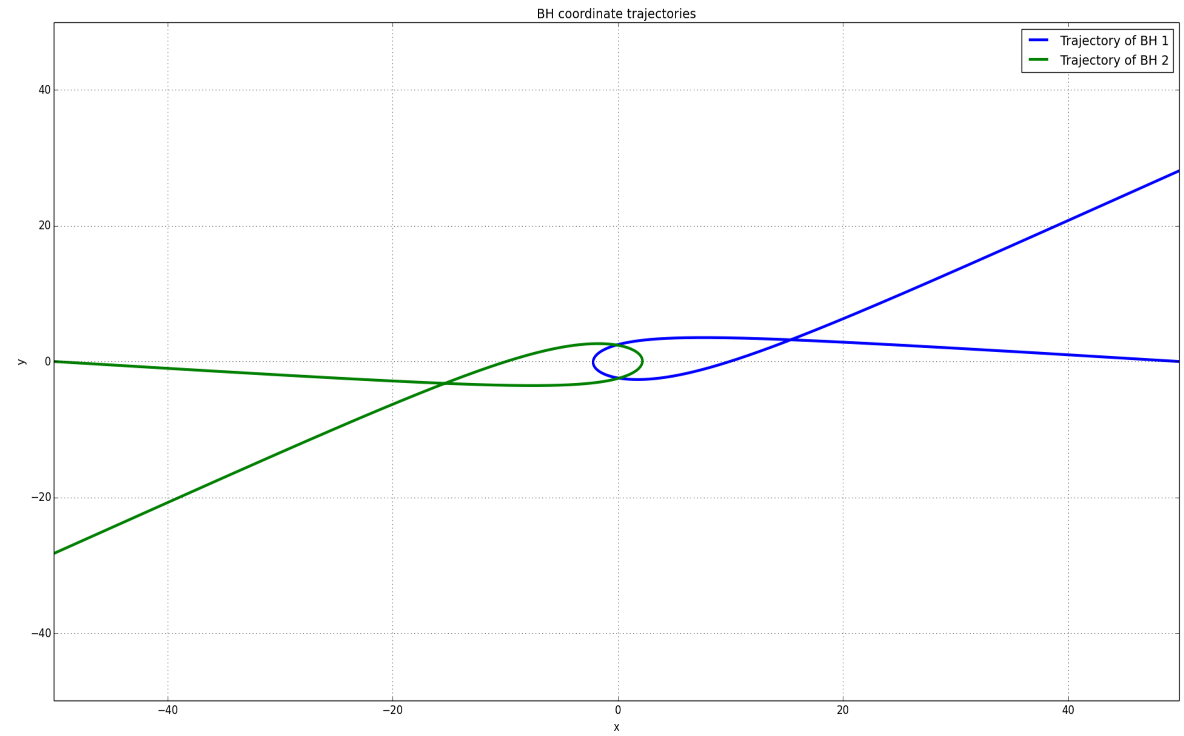
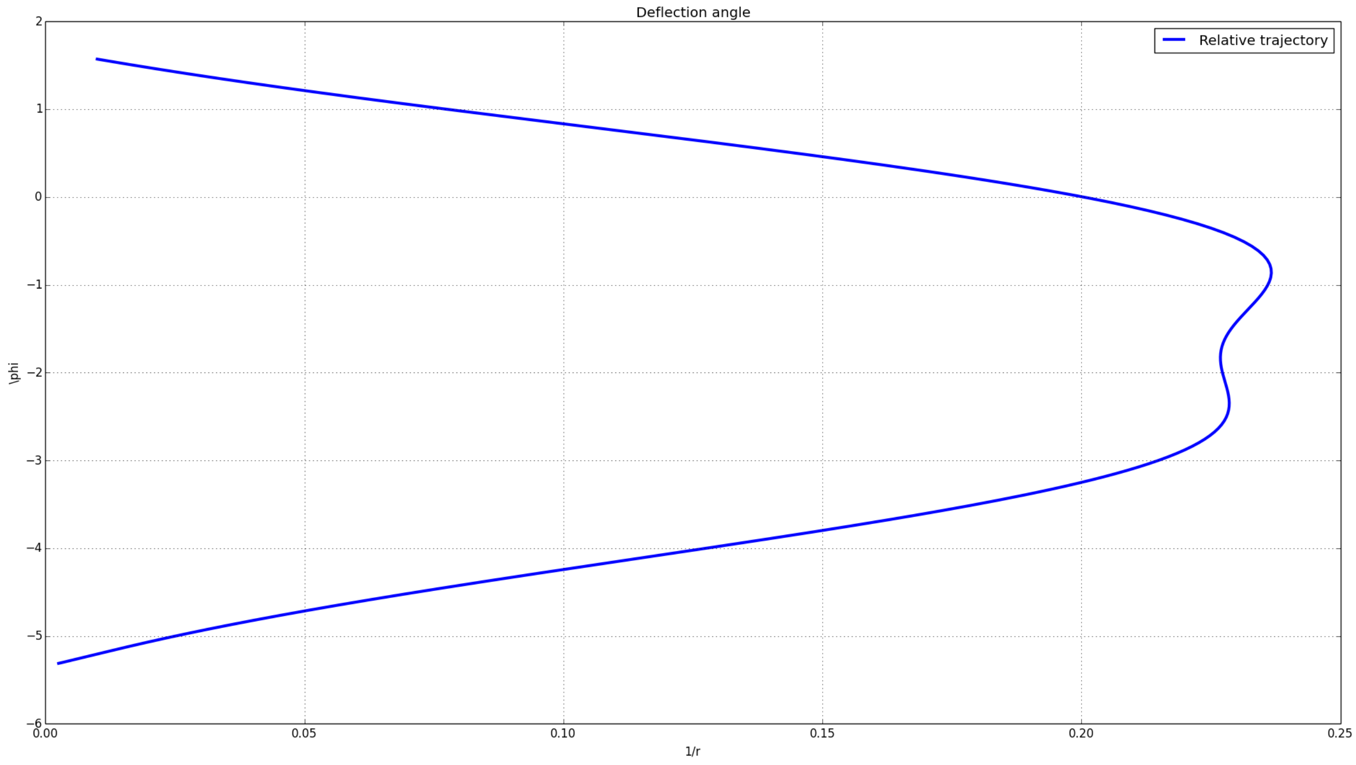
Binary BHs on hyperbolic orbits



Deflection angle



Effective One Body model

- from 2-body to 1-body problem
- Geodesic motion in Schwarzschild-like spacetime

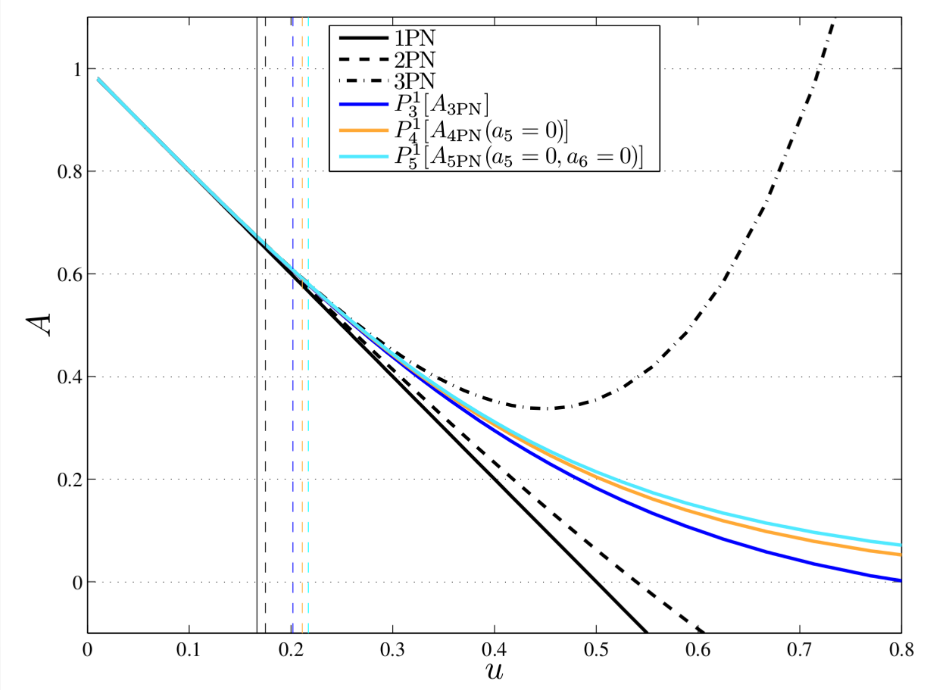
$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$u = 1/r \qquad\qquad A(u)_{3PN} = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

- Resummation as Padè approximant:

$$A(u) = \frac{a_0 + a_1 u^1 + a_2 u^2 + \ldots + a_n u^n}{1 + b_1 u^1 + b_2 u^2 + \ldots + b_m u^m}$$

„Padèing“



$$A(u)=\frac{1+n_1 u}{1+d_1 u^1+d_2 u^2+d_3 u^3}$$

Effective One Body model

- A Hamiltonian of the system (describes conservative dynamics)
- Radiation reaction force terms to be added to the equation of motion
- A description of the asymptotic gravitational waveforms

Effective One Body Hamiltonian

$$H_{EOB}(r, p_\varphi, p_r) = M\sqrt{1+2\nu\left(H_{eff}/\mu-1\right)}$$

$$H_{eff} = \mu\sqrt{A(r)\left(1+J^2u^2 2\nu(4-3\nu)u^2 p_{\bar{r}}^4\right)+p_{\bar{r}}^2}$$

$$p_{\bar{r}} = p_r\sqrt{\frac{A}{B}}$$

EOB deflection angle

$$\frac{\chi}{2}=\int_0^{u_{max}(E,J)} U(u,J,H_{eff})\,du-\frac{\pi}{2}$$

$$U(u,J,H_{eff})=J\frac{\sqrt{A(u)B(b)}}{\sqrt{H_{eff}^2-A(u)(1+J^2u^2)}}$$

Radiation reaction terms

$$\frac{\partial x^i}{dt} = \frac{\partial H}{\partial p_i}$$

$$\frac{\partial p^i}{dt} = \frac{-\partial H}{\partial x_i} + F_i$$

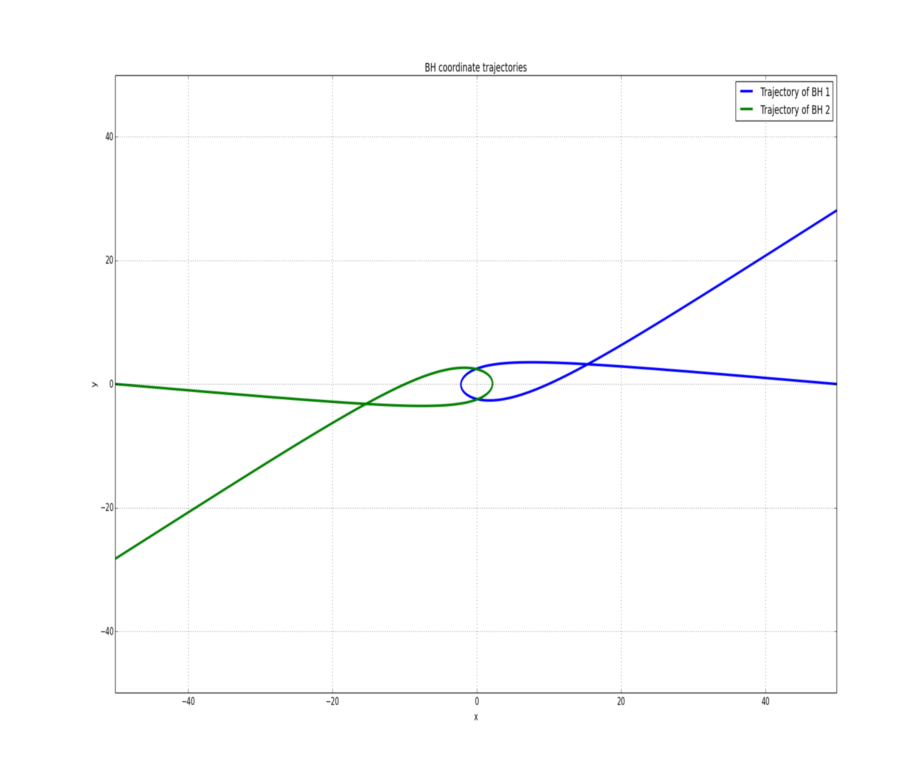
Radiation reaction in the deflection angle

- When neglecting terms qudratic in F_i (of order $(v/c)^{10}$) :

$$\chi^{(non-conservative)} = \chi^{(conservative)}(\bar{E}, \bar{J})$$

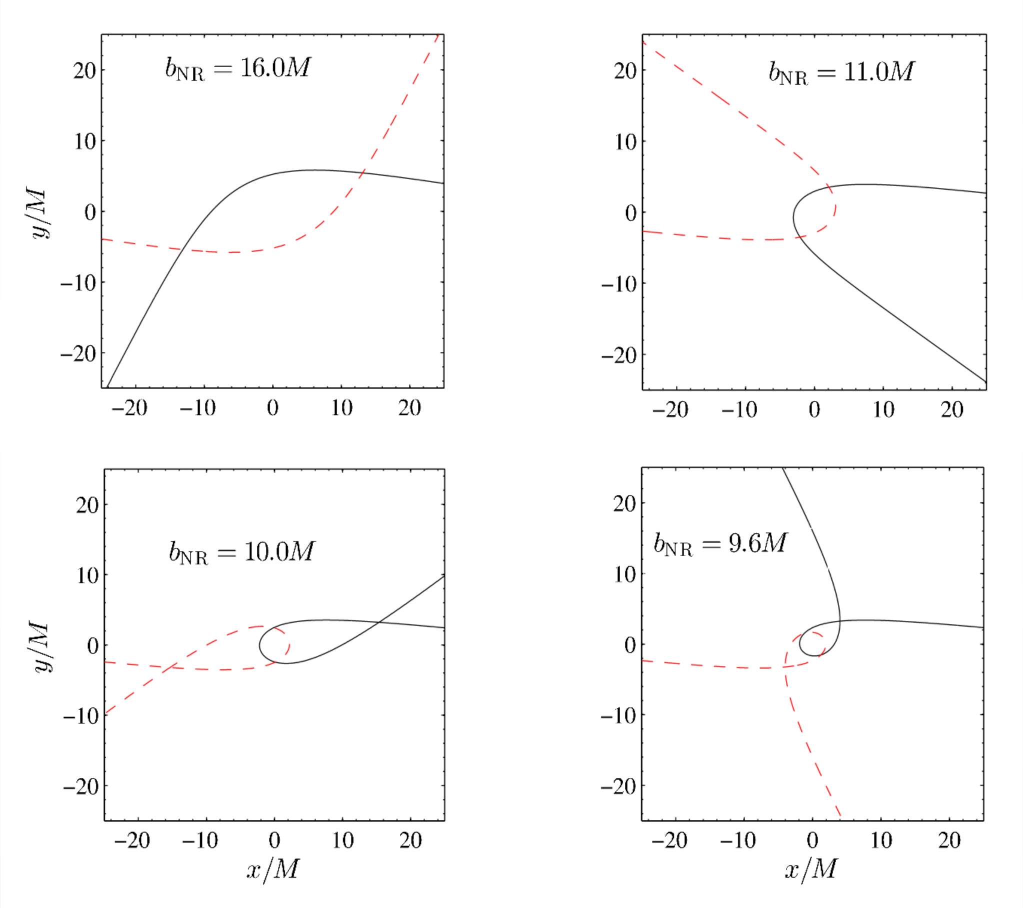
$$E = \frac{1}{2}(E_{incoming} + E_{outgoing}) \qquad J = \frac{1}{2}(J_{incoming} + J_{outgoing})$$

Initial data



- Equal mass BHs (m=0.5 M)
- Non spinning
- Equal anti-parallel initial momenta (|p|=0.12 M)
- Initial separation: 100 M
- Varying impact parameter b
- TwoPunctures code (spectral method)

Initial data: possible configurations



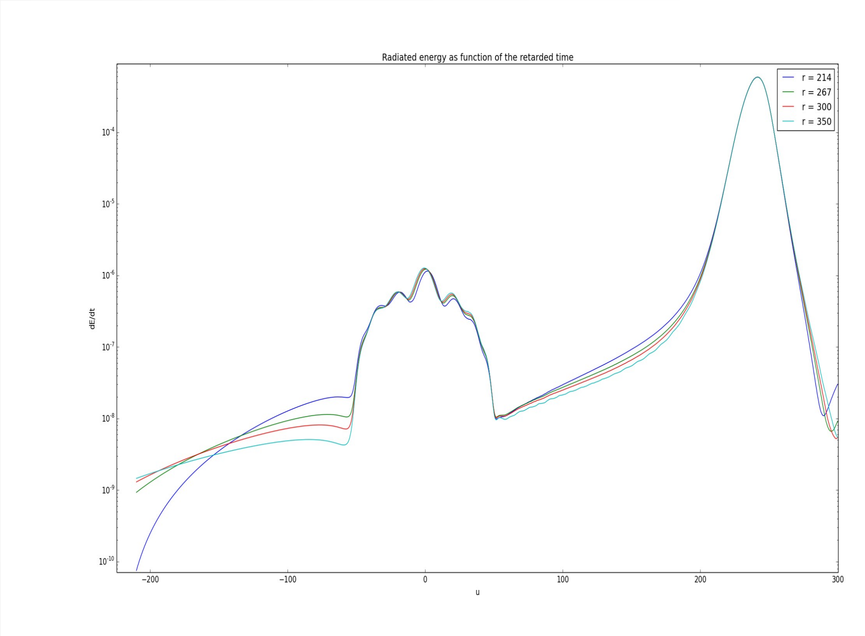
Initial energy and angular momentum

b	E/M	J/M^2
9.6	1.0225555	1.099652
9.8	1.0225722	1.122598
10.0	1.0225791	1.145523
10.6	1.0225870	1.214273
11.0	1.0225884	1.260098
12.0	1.0255907	1.374658
13.0	1.0225924	1.489217
14.0	1.0225931	1.603774
15.0	1.0225938	1.718331
16.0	1.0225932	1.832883

Evolution

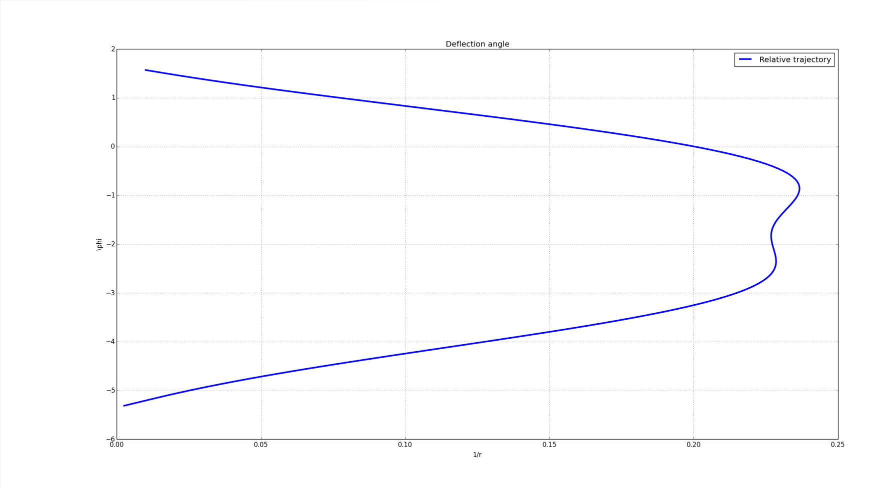
- BSSN formulation of Einstein equations
- Spatial derivatives: 8th-order finite-difference (McLachlan code)
- 7 box-in-box mesh refinement levels for each BH
- Cartesian grid (no multipatch)
- Time evolution: Method of lines 4th-order Runge-Kutta time integrator

Radiated energy and angular momentum



- Weyl scalar psi4
- Multipole decomposition up to l=8
- 4 extraction radii
- Extrapolation to null infinity
- Error sources: finite resolution, extrapolation, junk radiation

Deflection angle



- Polynomial fit of theta as function of 1/r
- Extrapolation to 1/r=0
- Choice of degree of the polynomial

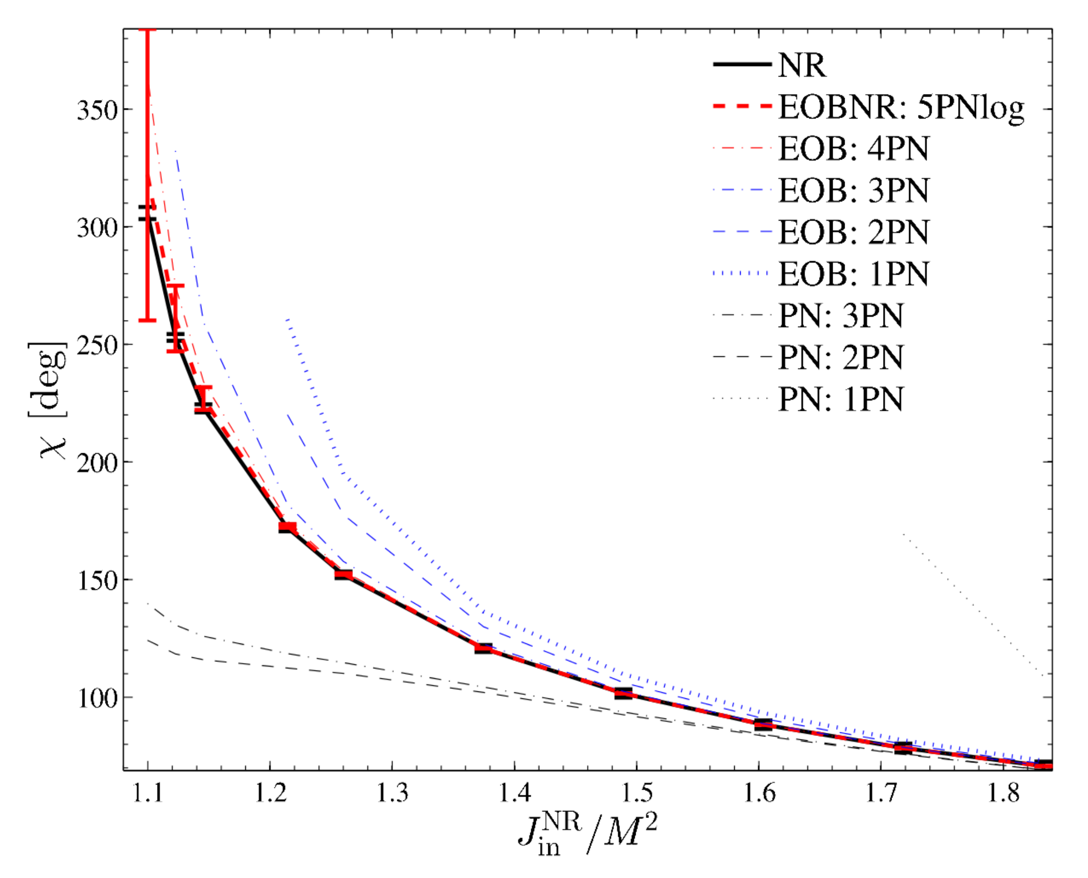
Singular value decomposition

- Linear least squares problem: **A x = b**
- SVD decomposition: $\mathbf{A} = \mathbf{M}\ \mathbf{W}\ \mathbf{V}^T$
- $\mathbf{W} = \mathrm{diag}\{w_1, w_2, .., w_i\}$ and **x** depends linearly on the reciprocals $1/w_i$
- Treshold T: if $w_n < T^*\max(w_i)$, then $1/w_n = 0$
- Coefficients and the extrapolant do not vary for polynomials of degree $n > N$

Results

b	chi_{NR}	chi_{5PN}^{EOB}	chi_{4PN}^{EOB}	chi_{3PN}^{EOB}	chi_{2PN}^{EOB}	chi_{1PN}^{EOB}	chi_{3PN}^{PN}	chi_{2PN}^{PN}	chi_{1PN}^{PN}
9.6	**305.8(2.6)**	**322(62)**	364.29	…	…	…	139.9	124.2	…
9.8	**253.0(1.4)**	**261(14)**	274.92	332.24	…	…	131(2)	118.46	…
10.0	**222.9(1.7)**	**227(5)**	234.26	259.46	…	…	126(1)	115.89	…
10.6	**172.0(1.4)**	**172.8(7)**	174.98	182.09	220.11	260.53	118.5(3)	112.43	…
11.0	**152.0(1.3)**	**152.4(3)**	153.59	157.68	177.60	194.90	114.7(2)	110.14	…
12.0	**120.7(1.5)**	**120.77(6)**	121.17	122.63	129.98	136.42	104.34(4)	102.06	…
13.0	**101.6(1.7)**	**101.63(2)**	101.80	102.48	106.20	109.80	93.69(2)	92.54	…
14.0	**88.3(1.8)**	**88.348(8)**	88.43	88.80	90.95	93.30	84.111(7)	83.55	…
15.0	**78.4(1.8)**	**78.427(4)**	78.47	78.69	80.03	81.699	75.962(3)	75.71	169.298

Results


NR
EOBNR: 5PNlog
EOB: 4PN
EOB: 3PN
EOB: 2PN
EOB: 1PN
PN: 3PN
PN: 2PN
PN: 1PN
χ [deg]
$J_{\rm in}^{\rm NR}/M^2$
350
300
250
200
150
100
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8

Conclusions

- Compared full GR simulations of BHs on hyperbolic orbits with PN-EOB predictions
- Found agreement for the 5PN NR-calibrated EOB case for every b
- Even for non circular orbits
- Possibility of extracting information from scattering experiments to complete the EOB model