The Shadow of Black Holes

Arne Grenzebach • Volker Perlick, Claus Lämmerzahl

25th November, 2014
FRANKFURT AM MAIN

Astro Coffee

Universität Bremen

RESEARCH TRAINING GROUP
Models of Gravity

CENTER OF
APPLIED SPACE TECHNOLOGY
AND MICROGRAVITY
ZARM

Contents

- Kerr–Newman–NUT–(anti-)de Sitter space-time
  - Properties
- Effects of Gravitational Lensing
  - Region with Spherical Light Rays
  - Calculating and Viewing the Shadow
- Moving Observers: Aberration
- Observation
- Summary & Outlook

Kerr–Newman–NUT Metric

$$
\begin{aligned}
g = &\Sigma\left(\tfrac{1}{\Delta}\mathrm{d}r^2 + \mathrm{d}\vartheta^2\right) + \tfrac{1}{\Sigma}\left((\Sigma + a\chi)^2 \sin^2\vartheta - \chi^2\Delta\right)\mathrm{d}\varphi^2 \\
&+ \tfrac{2}{\Sigma}\left(\Delta\chi - a(\Sigma + a\chi)\sin^2\vartheta\right)\mathrm{d}t\,\mathrm{d}\varphi - \tfrac{1}{\Sigma}\left(\Delta - a^2\sin^2\vartheta\right)\mathrm{d}t^2
\end{aligned}
$$

$$
\begin{aligned}
\Sigma &= r^2 + (\ell + a\cos\vartheta)^2 \\
\chi &= a\sin^2\vartheta - 2\ell\cos\vartheta \\
\Delta &= r^2 - 2mr + a^2 - \ell^2 + q_e^2
\end{aligned}
$$

- a spin
- q_e electric charge
- ℓ NUT parameter

$\ell = 0$	$a = 0$	$a \neq 0$
$q_e = 0$	Schwarzschild	Kerr
$q_e \neq 0$	Reissner-Nordström	Kerr-Newman

Taub-NUT	
$\ell \neq 0$	$a = 0$
	$q_e = 0$

Kerr–Newman–NUT with Λ

$$g = \Sigma\left(\frac{1}{\Delta_r}\mathrm{d}r^2 + \frac{1}{\Delta_\vartheta}\mathrm{d}\vartheta^2\right) + \frac{1}{\Sigma}\left((\Sigma + a\chi)^2\Delta_\vartheta \sin^2\vartheta - \Delta_r\chi^2\right)\mathrm{d}\varphi^2$$
$$+\frac{2}{\Sigma}\left(\Delta_r\chi - a(\Sigma + a\chi)\Delta_\vartheta \sin^2\vartheta\right)\mathrm{d}t\,\mathrm{d}\varphi - \frac{1}{\Sigma}\left(\Delta_r - a^2\Delta_\vartheta \sin^2\vartheta\right)\mathrm{d}t^2$$

$$\Sigma = r^2 + (\ell + a\cos\vartheta)^2$$
$$\chi = a\sin^2\vartheta - 2\ell\cos\vartheta$$
$$\Delta = r^2 - 2mr + a^2 - \ell^2 + q_e^2$$
$$\Delta_r = \Delta - \Lambda\left((a^2 - \ell^2)\ell^2 + \left(\tfrac{1}{3}a^2 + 2\ell^2\right)r^2 + \tfrac{1}{3}r^4\right)$$
$$\Delta_\vartheta = 1 + \Lambda\left(+\tfrac{4}{3}a\ell\cos\vartheta + \tfrac{1}{3}a^2\cos^2\vartheta\right)$$

a spin

q_e electric charge

ℓ NUT parameter

Λ cosmological constant

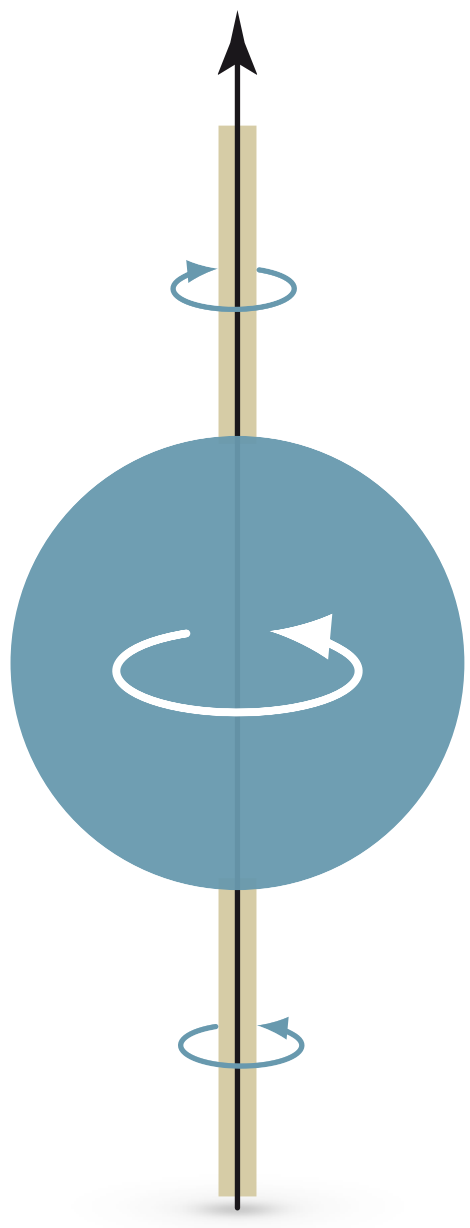
Kerr–Newman–NUT with Λ
Properties

- Horizons: $\Delta_r = 0$
- Ring singularity: $\Sigma = 0$ $\quad \exists$ if $|a| \geq |\ell|$
- Axial singularity: $g^*(\mathrm{d}t, \mathrm{d}t) \longrightarrow \infty$

  2 semi-infinite singularities on half-axis

  → massless rotating rods

- Causality violation: $g_{\varphi\varphi} = g(\partial_\varphi, \partial_\varphi) < 0$
- Ergosphere: $g_{tt} = g(\partial_t, \partial_t) > 0$



Kerr–Newman–NUT with Λ

Equations of Motion for Light

$$\Sigma \dot{t} = \frac{\chi(L_z - E\chi)}{\Delta_\vartheta \sin^2\vartheta} + \frac{(\Sigma + a\chi)\big((\Sigma + a\chi)E - aL_z\big)}{\Delta_r}$$

$$\Sigma \dot{\varphi} = \frac{L_z - E\chi}{\Delta_\vartheta \sin^2\vartheta} + \frac{a\big((\Sigma + a\chi)E - aL_z\big)}{\Delta_r}$$

$$\Sigma^2 \dot{\vartheta}^2 = \Delta_\vartheta K - \frac{(\chi E - L_z)^2}{\sin^2\vartheta}$$

$$\Sigma^2 \dot{r}^2 = \big((\Sigma + a\chi)E - aL_z\big)^2 - \Delta_r K$$

sphere condition

$$\dot{r} = 0, \; \ddot{r} = 0$$

Constants of Motion for Light

- Energy $E := -\frac{\partial \mathcal{L}}{\partial \dot{t}}$
- Angular Momentum $L_z := \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$
- Lagrangian $\mathcal{L} = 0$
- Carter Constant K

Kerr–Newman–NUT with Λ

Region with Spherical Light Rays

For $\Sigma^2 \dot{r}^2$ & $\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\Sigma^2 \dot{r}^2\right)$ provide $\dot{r} = 0,\ \ddot{r} = 0$:

$$a\frac{L_z}{E} = (\Sigma + a\chi) - \frac{2r\Delta_r}{r-m}$$

$$\frac{K}{E^2} = \frac{4r^2\Delta_r}{(r-m)^2}$$

$\rightarrow \frac{L_z}{E}$ & $\frac{K}{E^2}$

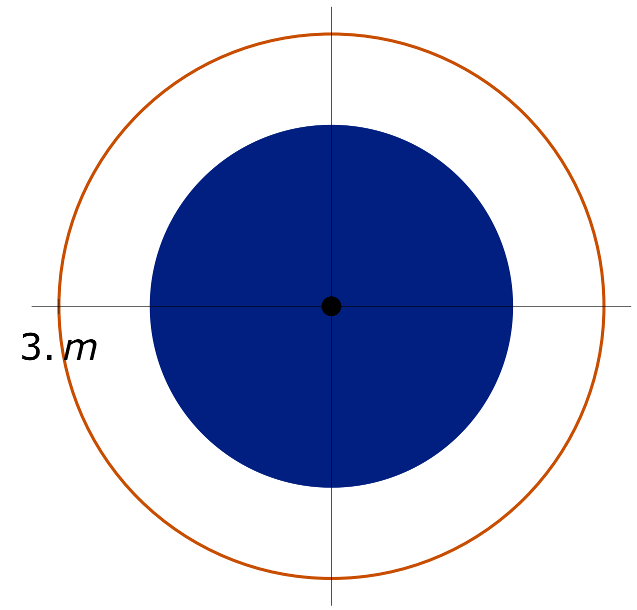
Inserting in $0 \le \Sigma^2 \dot{\vartheta}^2 = K - \frac{(\chi E - L_z)^2}{\sin^2\vartheta}$

reveals region with spherical light rays:

$$\mathcal{K}\colon\ \left(4r\Delta_r - \Sigma\Delta_r'\right)^2 \le 16a^2r^2\Delta_r\Delta_\vartheta \sin^2\vartheta$$

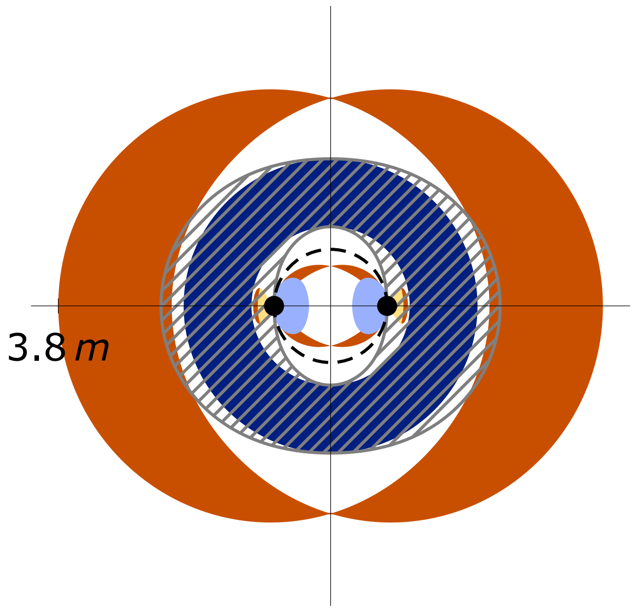
Region with Spherical Light Rays

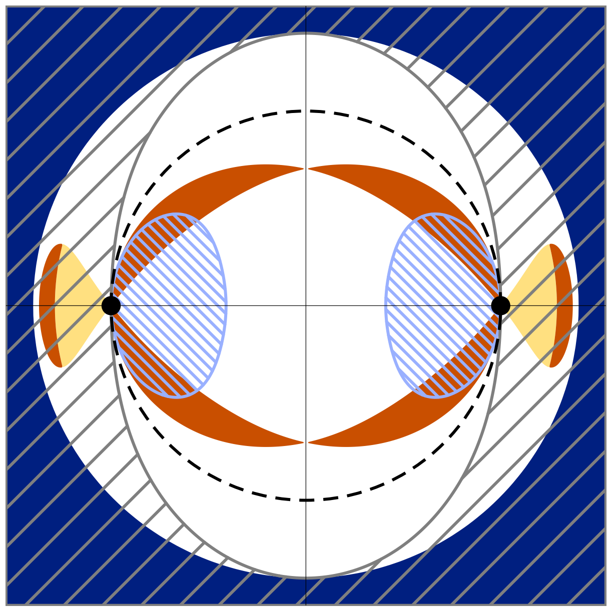
Schwarzschild (ϑ, r)



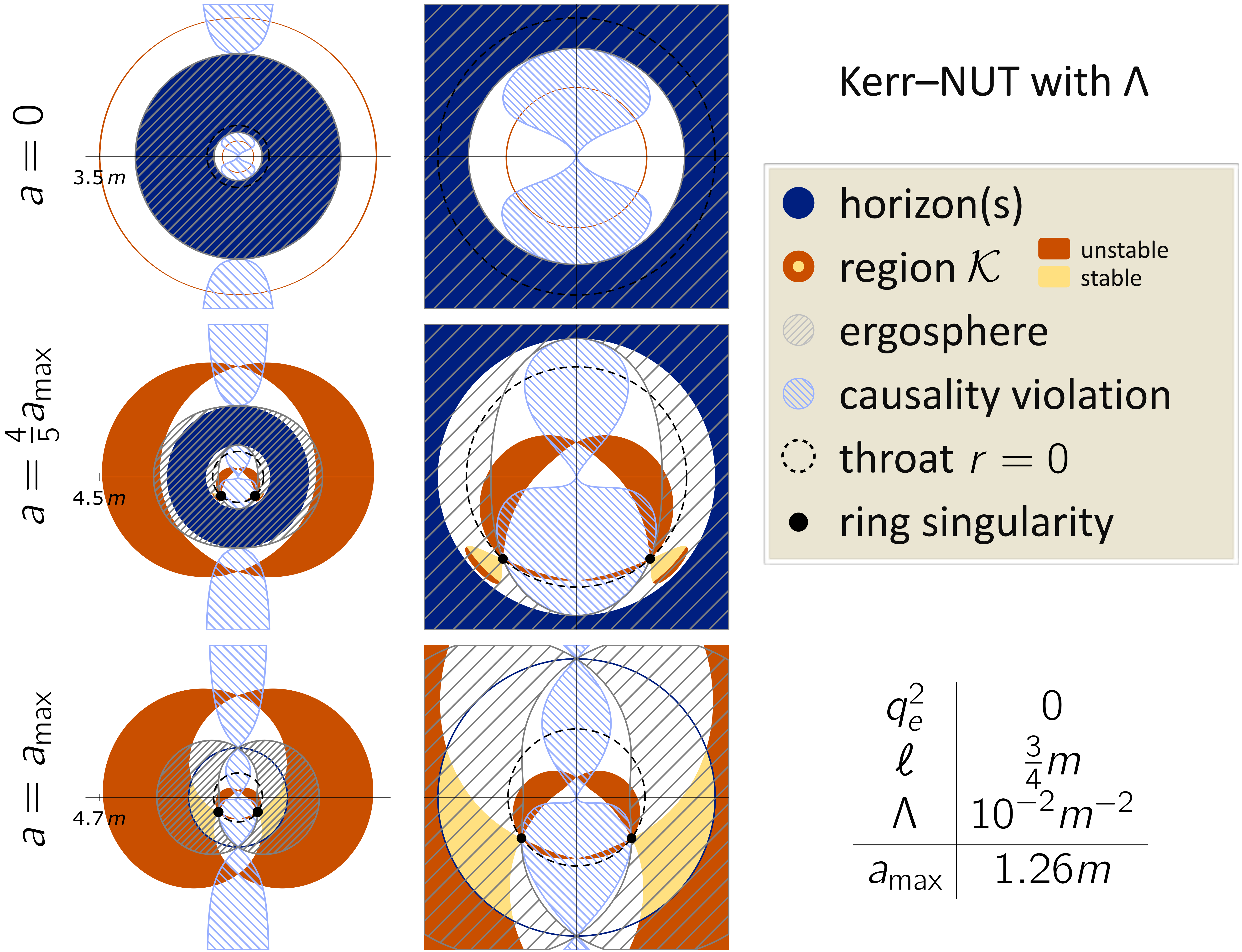
- horizon(s)
- region $\mathcal{K}$ — unstable / stable
- ergosphere
- causality violation
- throat $r = 0$
- ring singularity

Kerr

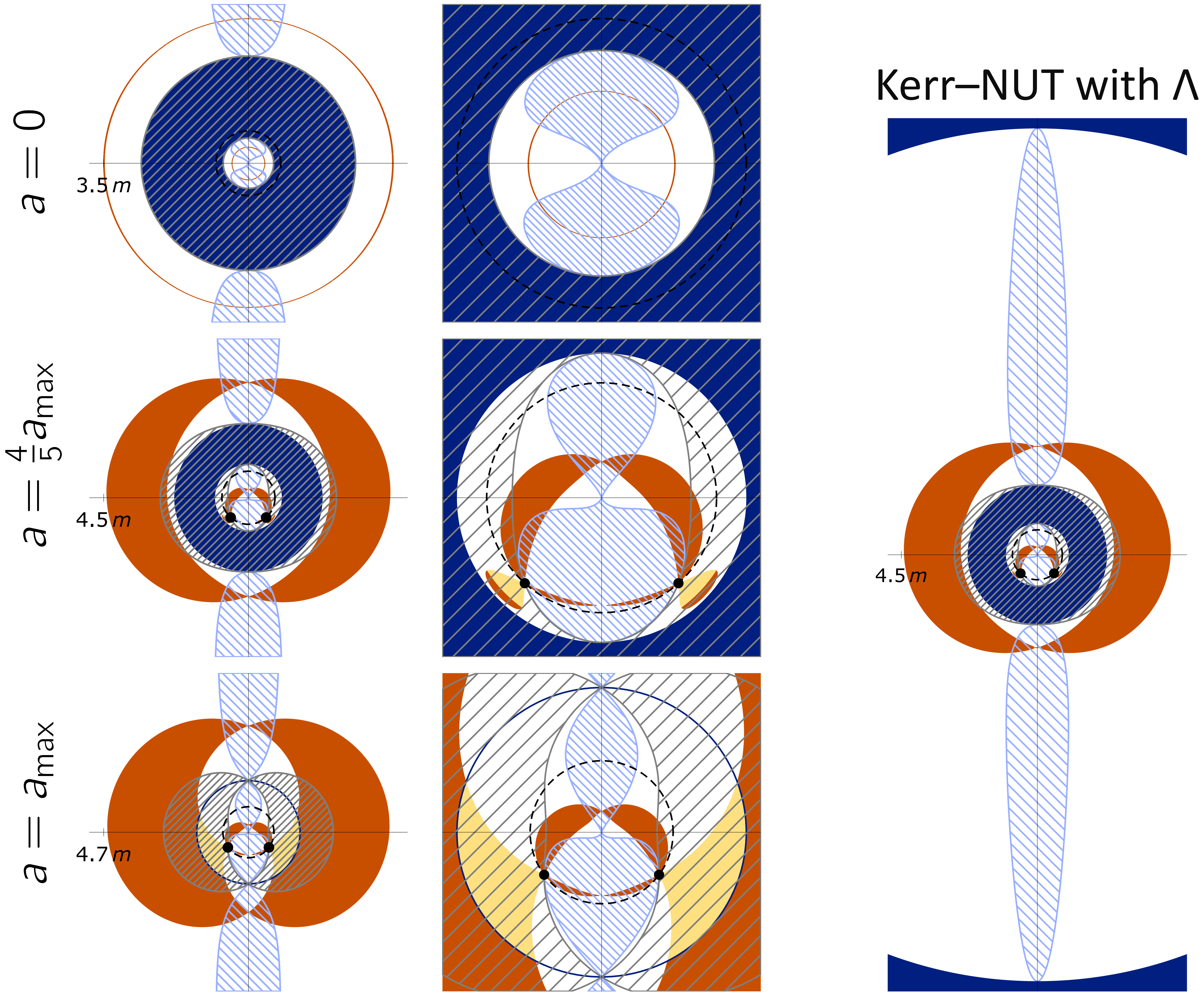




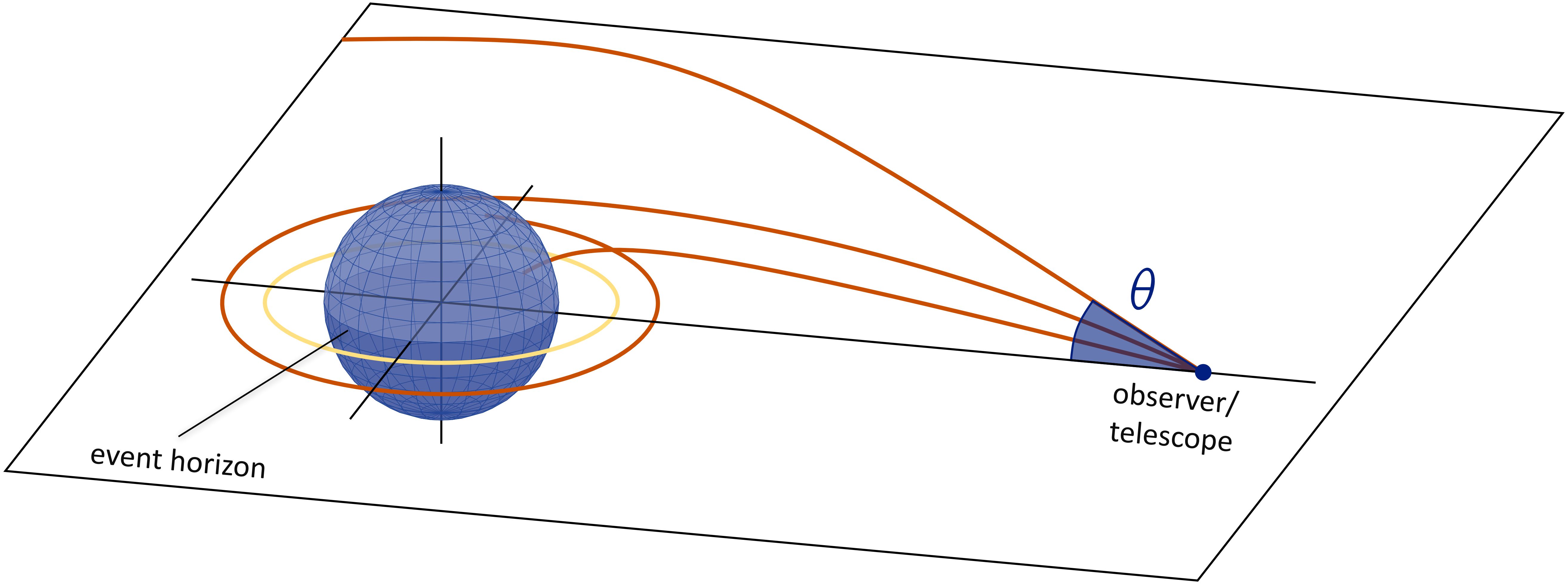
$$\left(\vartheta,\ \begin{matrix} e^r, & \text{for } r \leq 0 \\ r+1, & \text{for } r > 0 \end{matrix}\right)$$



$a = 0$
3.5 m
$a = \frac{4}{5} a_{\max}$
4.5 m
$a = a_{\max}$
4.7 m
Kerr–NUT with Λ
horizon(s)
region $\mathcal{K}$
unstable
stable
ergosphere
causality violation
throat $r = 0$
ring singularity
q_e^2 | 0
ℓ | $\frac{3}{4} m$
Λ | $10^{-2} m^{-2}$
$a_{\max}$ | $1.26 m$



Kerr–NUT with Λ
$a = 0$
$3.5\,m$
$a = \frac{4}{5} a_{\max}$
$4.5\,m$
$a = a_{\max}$
$4.7\,m$
$4.5\,m$



Shadow of a Black Hole

The region on the celestial sphere around an observer which is left dark if all light sources are at infinity

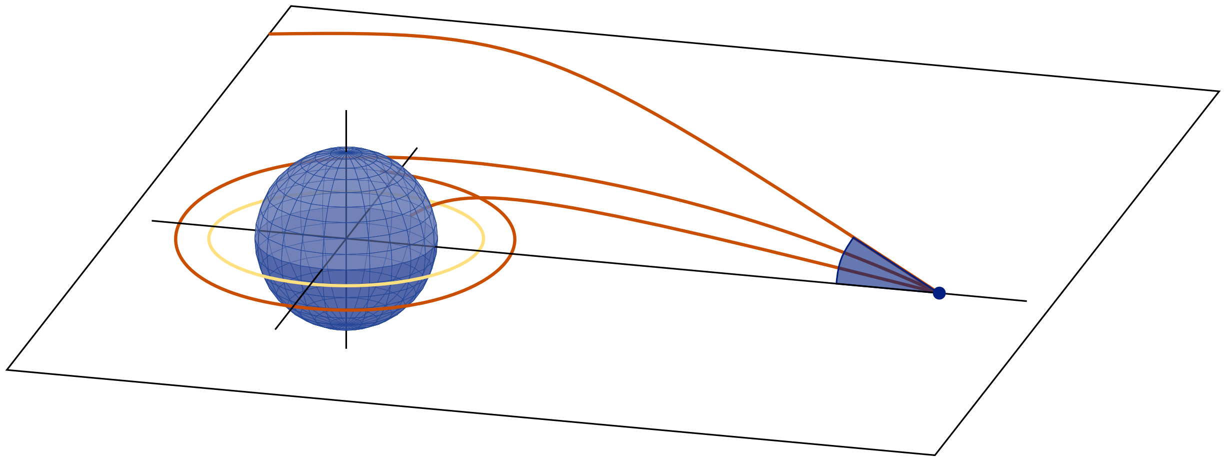
limiting case

$\dot{r} = 0,\ \ddot{r} = 0$

→ (θ, ψ)

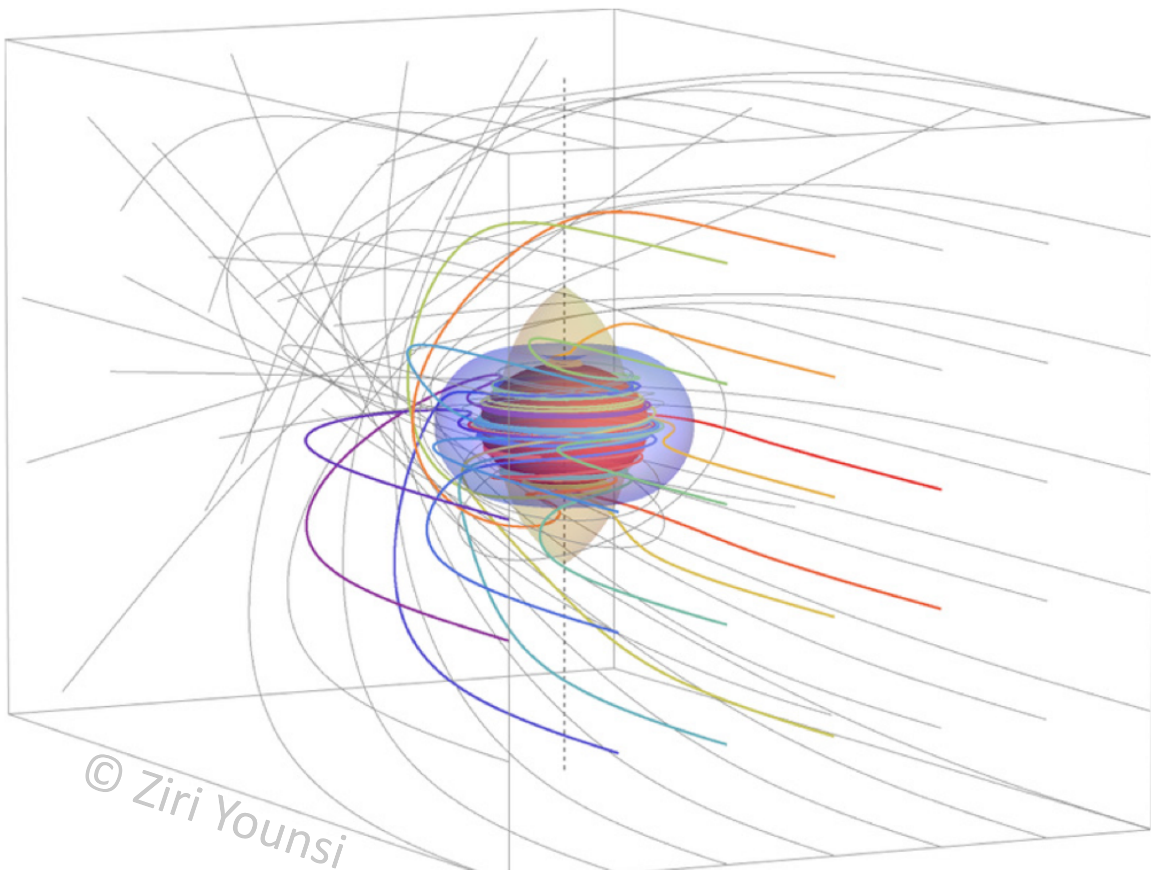
Geometric construction

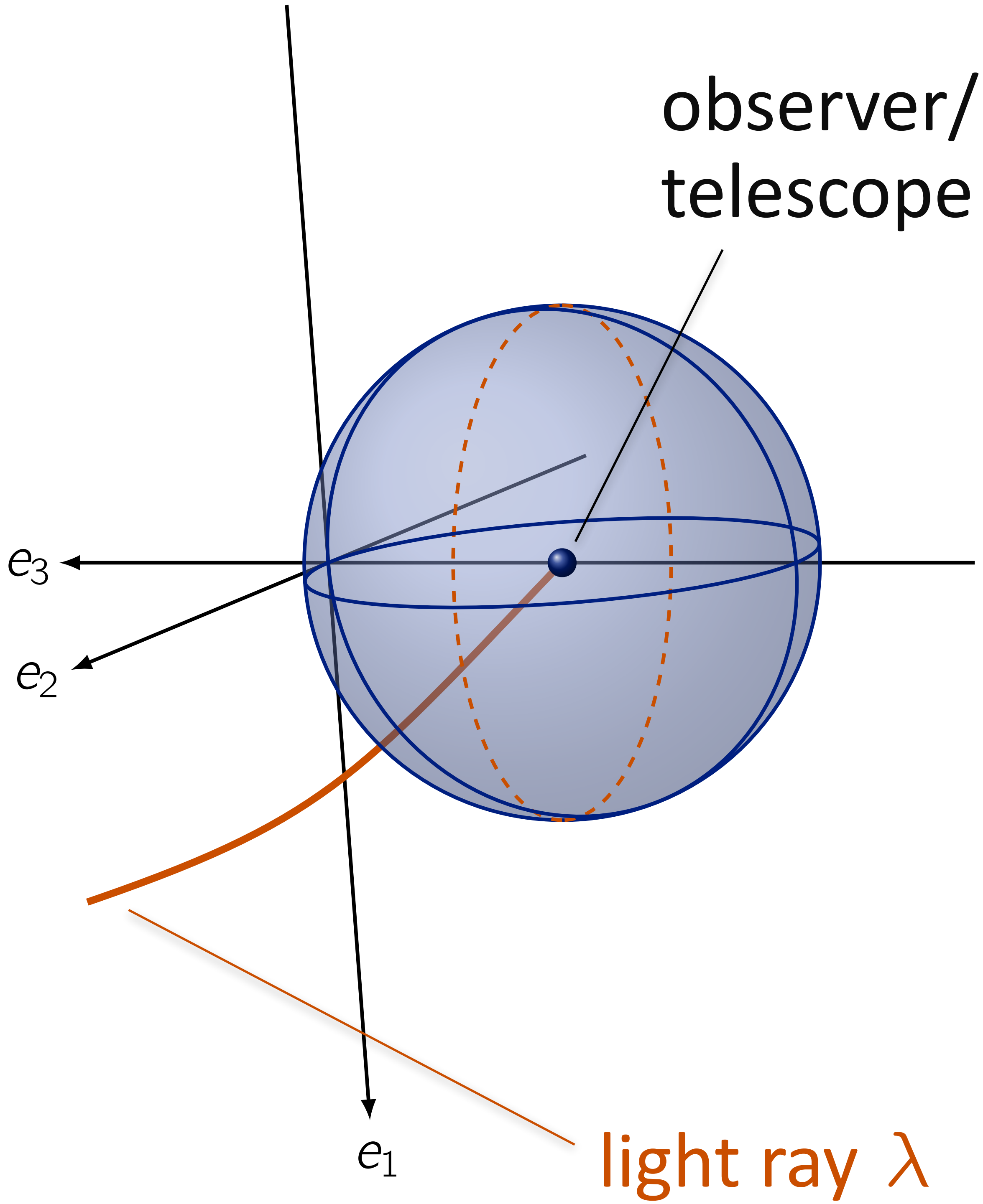
- observer: point
- only consider lightlike geodesics
- do not consider variability of light source(s)
- do not consider matter



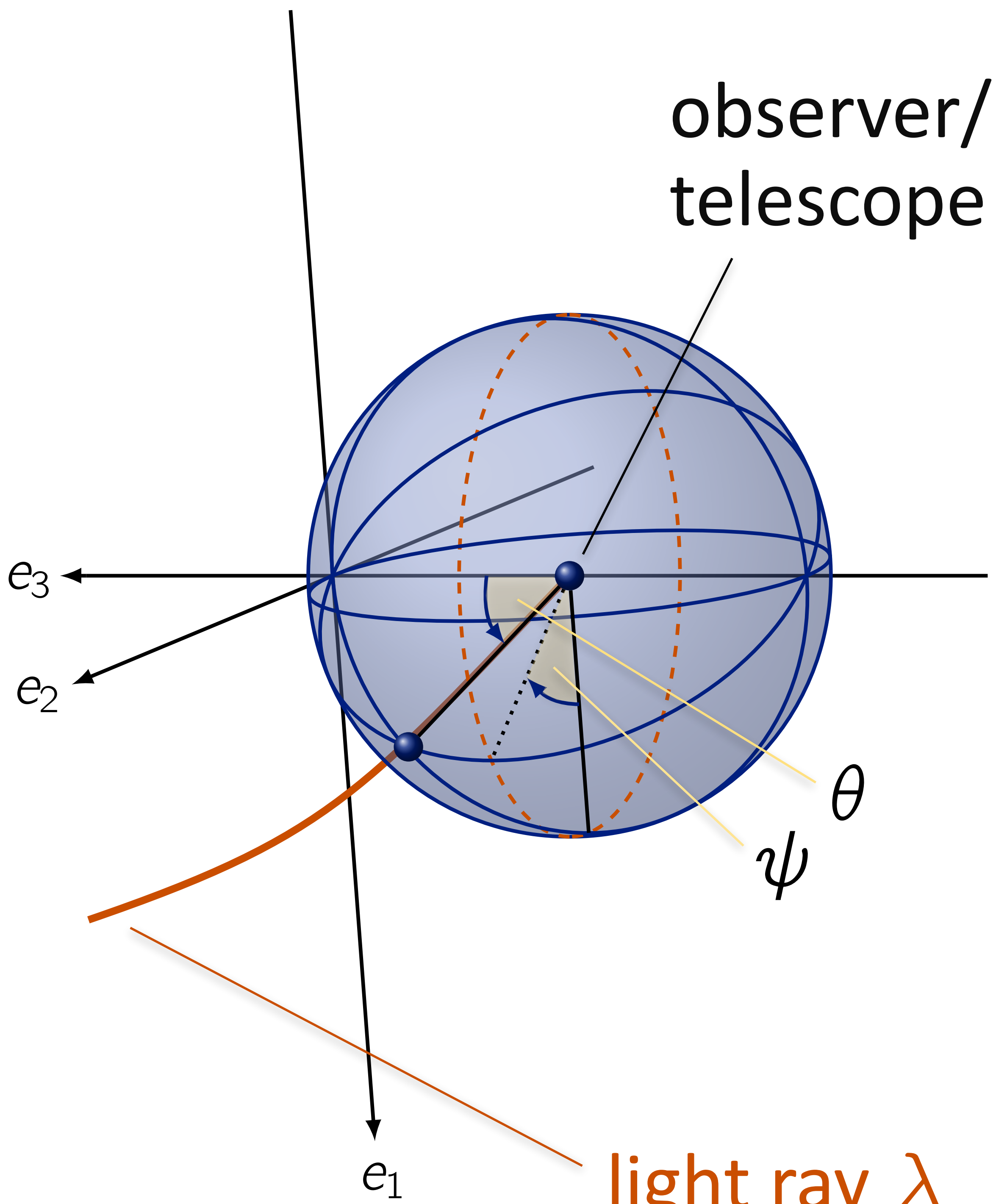
Ray-Tracing

- observer: grid
- matter described by absorption and emission
- can take into account variability of light source(s)





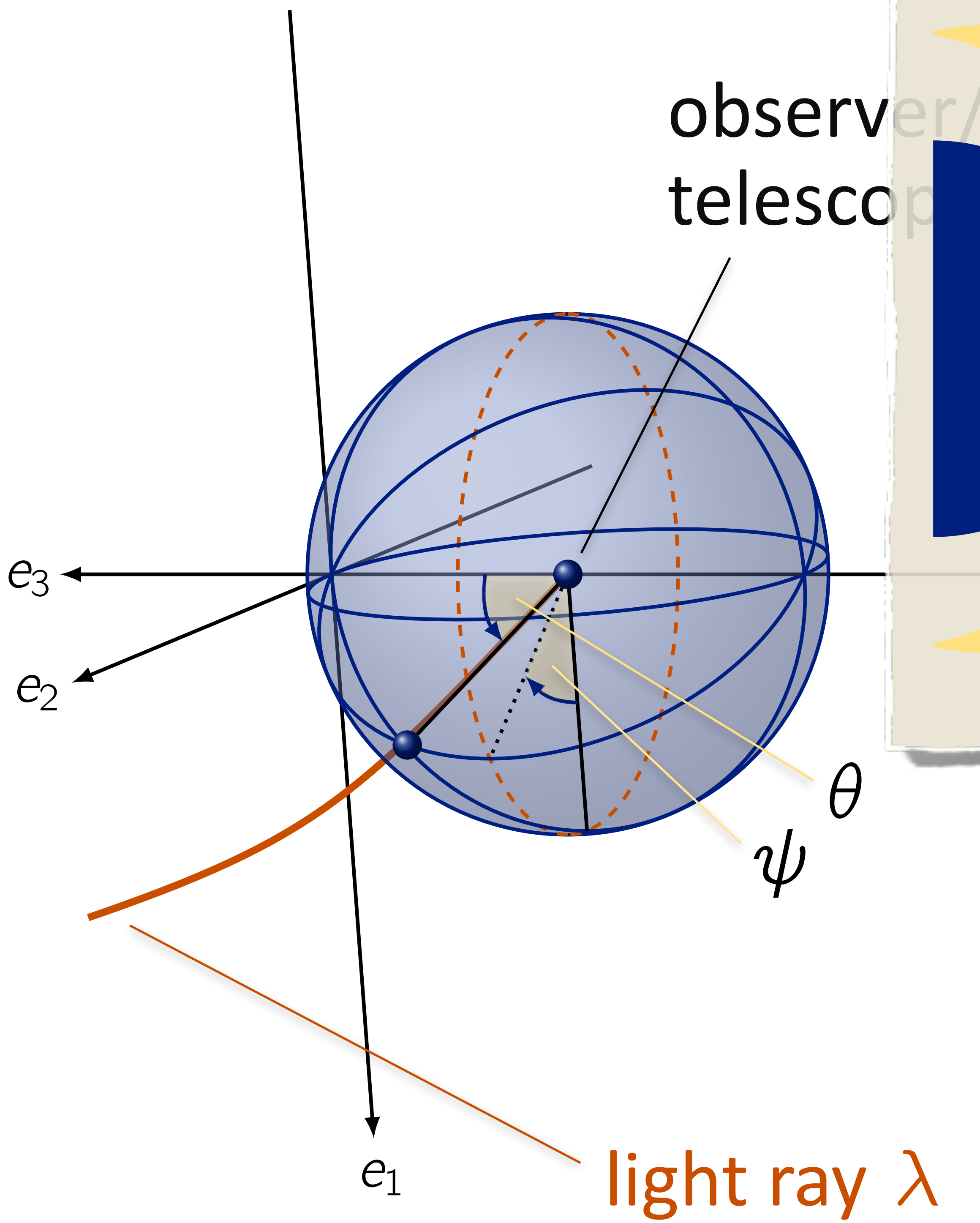
- Consider light which is sent into the past from the observer's position into the direction (θ, ψ)

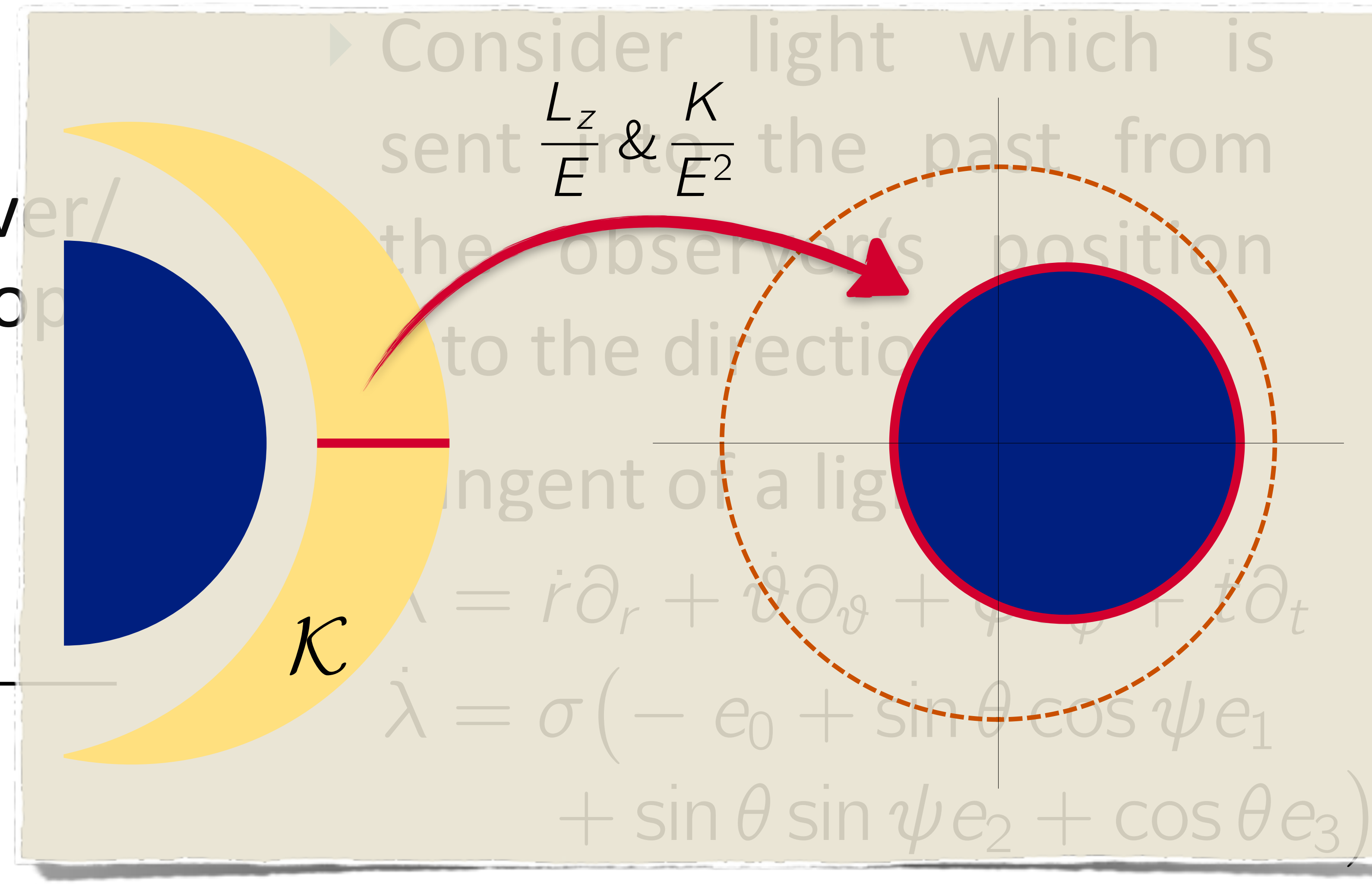


- Consider light which is sent into the past from the observer‘s position into the direction (θ, ψ)
- Tangent of a light ray

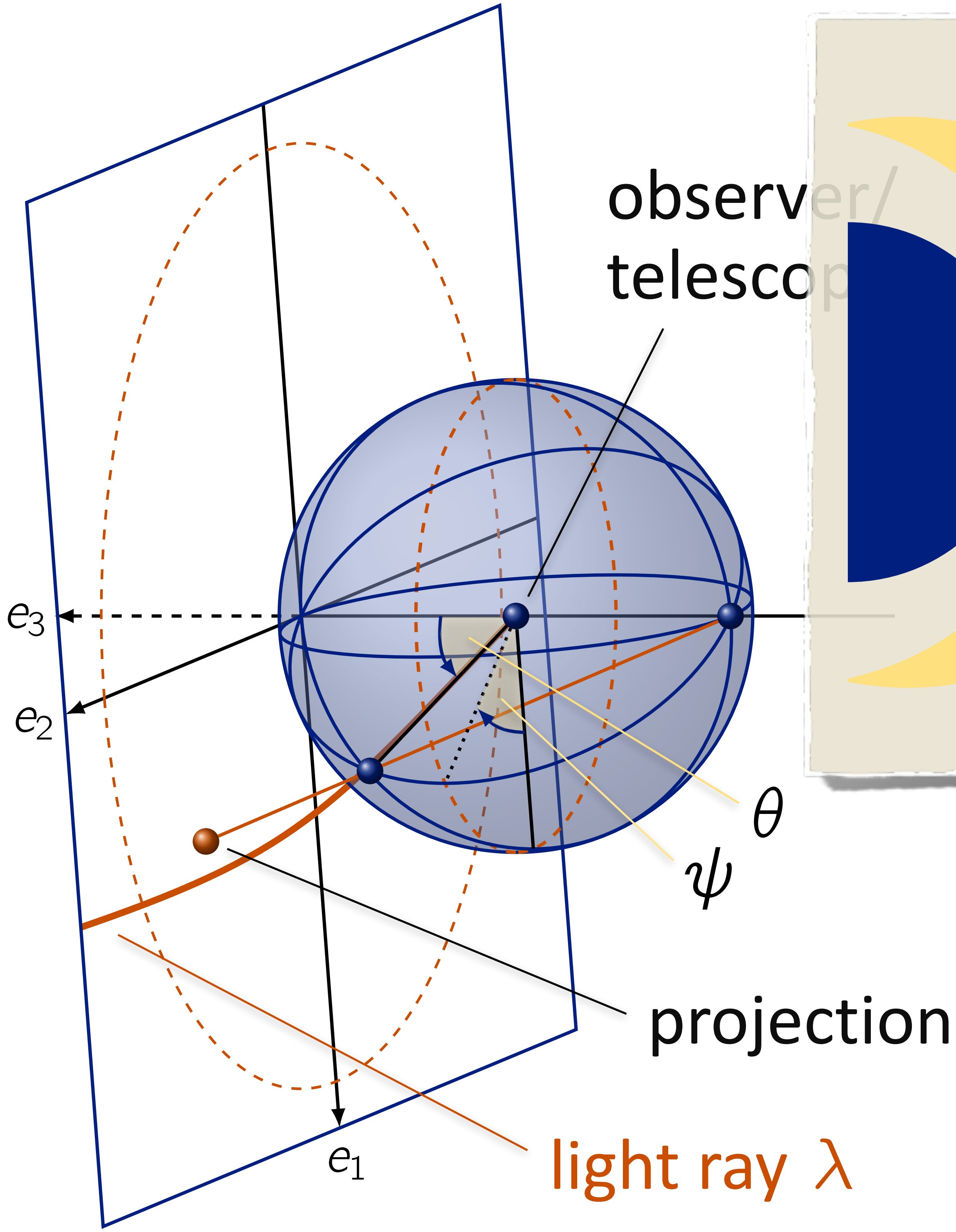
$$\dot{\lambda} = \dot{r}\partial_r + \dot{\vartheta}\partial_\vartheta + \dot{\varphi}\partial_\varphi + \dot{t}\partial_t$$

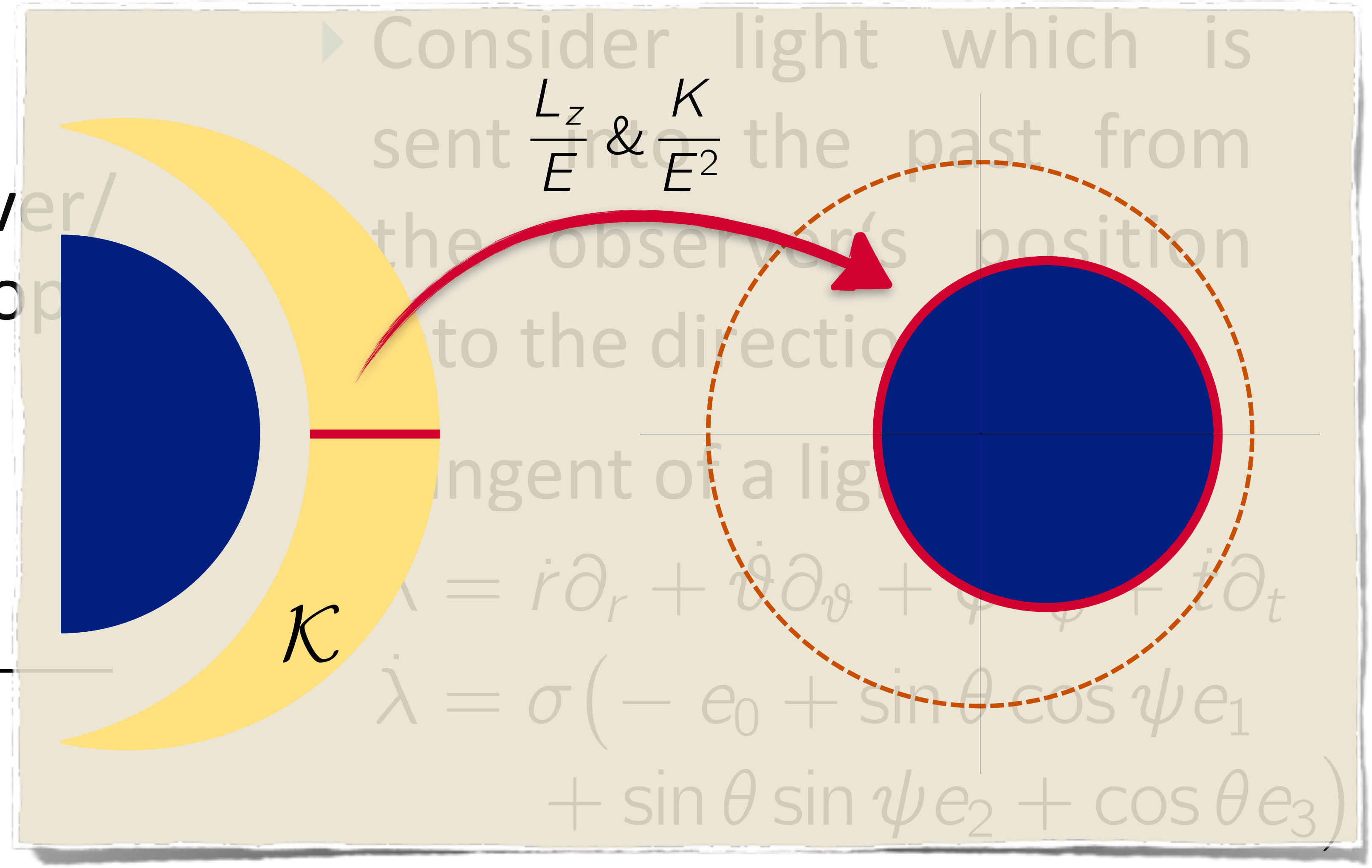
$$\dot{\lambda} = \sigma\big(-e_0 + \sin\theta\cos\psi\, e_1 + \sin\theta\sin\psi\, e_2 + \cos\theta\, e_3\big)$$





- With L_z/E, K/E^2 one gets
  $\sin\theta$ & $\sin\psi$



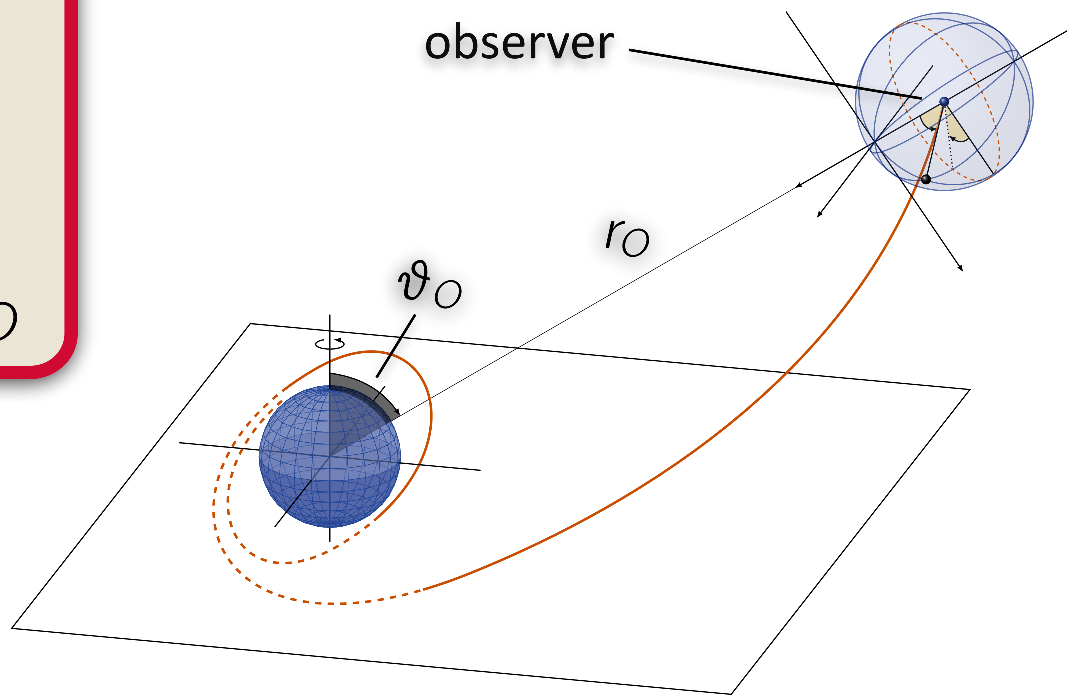


- With L_z/E, K/E^2 one gets
  $\sin\theta$ & $\sin\psi$
- The Shadow is viewed via stereographic projection

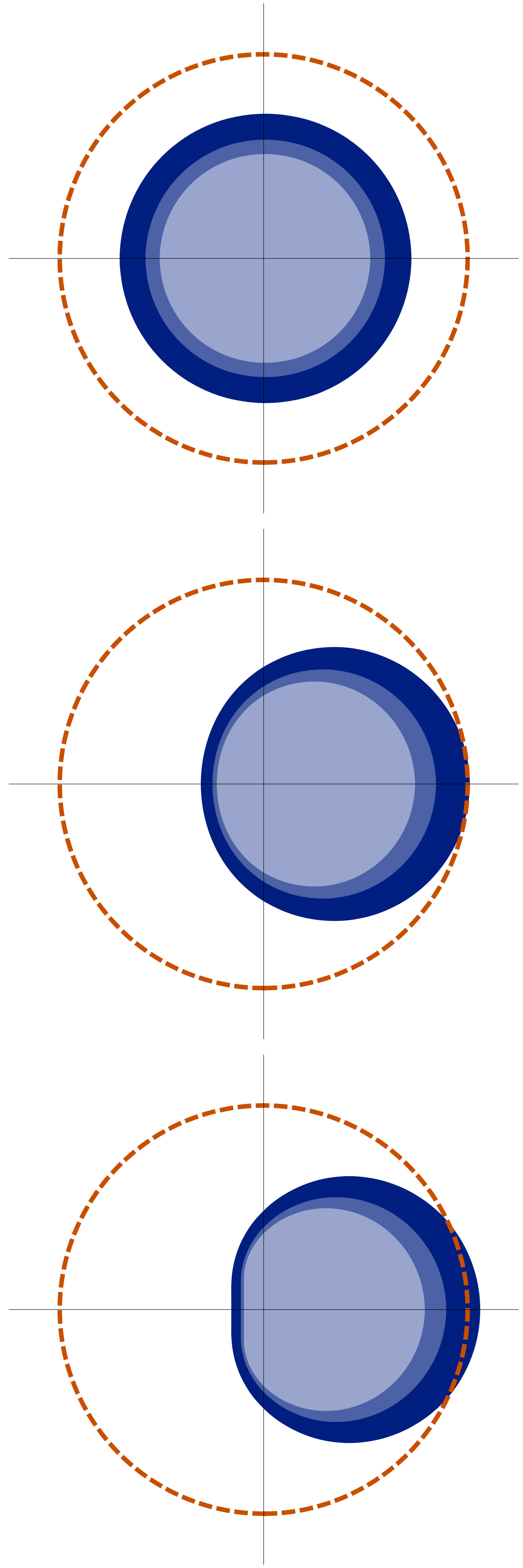
Celestial Coordinates of the Boundary of the Shadow

$$\sin\theta = \left.\frac{\sqrt{\Delta_r K_E}}{r^2 + \ell^2 - aL_E}\right|_{r_O}$$

$$\sin\psi = \left.\frac{L_E + a\cos^2\vartheta + 2\ell\cos\vartheta}{\sqrt{\Delta_\vartheta K_E}\sin\vartheta}\right|_{\vartheta_O}$$



with $K_E = \dfrac{K}{E^2}, \quad L_E = \dfrac{L_z}{E} - a$

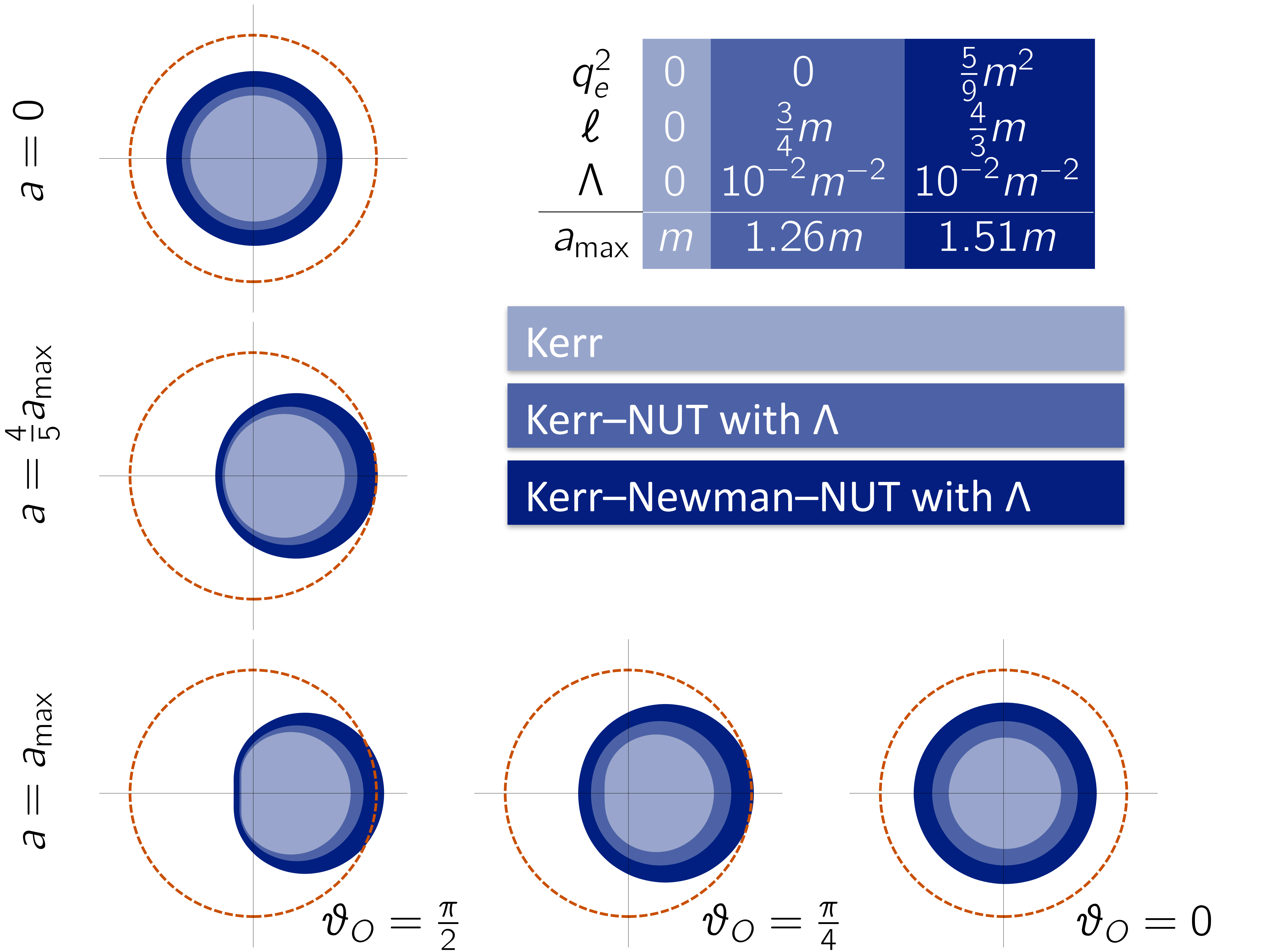


q_e^2	0	0	$\frac{5}{9} m^2$
ℓ	0	$\frac{3}{4} m$	$\frac{4}{3} m$
Λ	0	$10^{-2} m^{-2}$	$10^{-2} m^{-2}$
$a_{\max}$	m	$1.26 m$	$1.51 m$

Kerr

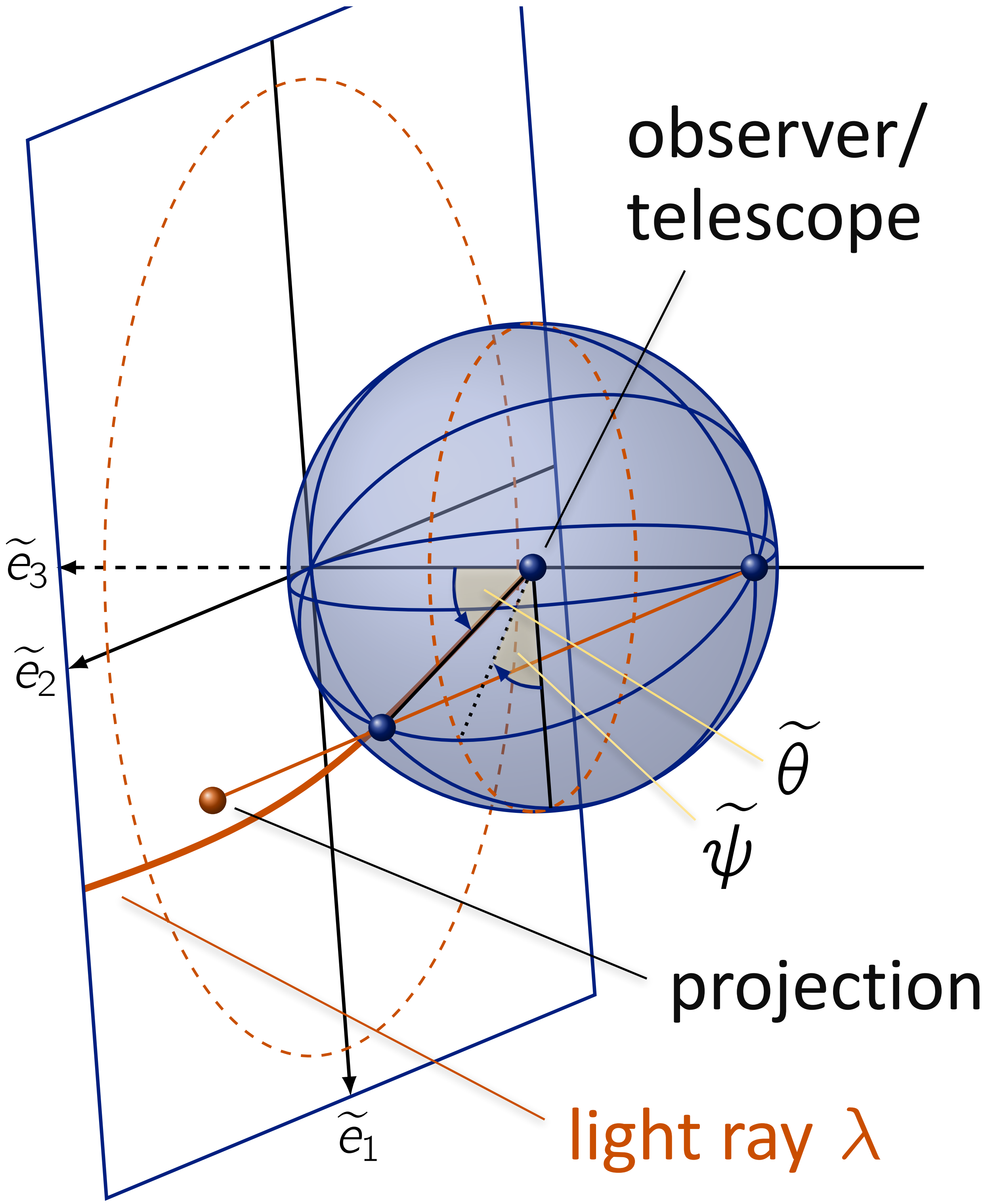
Kerr–NUT with Λ

Kerr–Newman–NUT with Λ



q_e^2 0 0 $\frac{5}{9}m^2$
ℓ 0 $\frac{3}{4}m$ $\frac{4}{3}m$
Λ 0 $10^{-2}m^{-2}$ $10^{-2}m^{-2}$
$a_{\max}$ m $1.26m$ $1.51m$
Kerr
Kerr–NUT with Λ
Kerr–Newman–NUT with Λ
$a = 0$
$a = \frac{4}{5}a_{\max}$
$a = a_{\max}$
$\vartheta_O = \frac{\pi}{2}$
$\vartheta_O = \frac{\pi}{4}$
$\vartheta_O = 0$

Moving Observer



- Velocity: $\vec{v} = (v_1, v_2, v_3),\ v = |\vec{v}|$
- Tetrad for moving observer

$$\tilde{e}_0 = \frac{v_1 e_1 + v_2 e_2 + v_3 e_3 + e_0}{\sqrt{1-v^2}}$$

$$\tilde{e}_1 = \frac{(1-v_2^2)e_1 + v_1(v_2 e_2 + e_0)}{\sqrt{1-v_2^2}\,\sqrt{1-v_1^2-v_2^2}}$$

$$\tilde{e}_2 = \frac{e_2 + v_2 e_0}{\sqrt{1-v_2^2}}$$

$$\tilde{e}_3 = \frac{(1-v_1^2-v_2^2)e_3 + v_3(v_1 e_1 + v_2 e_2 + e_0)}{\sqrt{1-v_1^2-v_2^2}\,\sqrt{1-v^2}}$$

- Tangent of a light ray

$$\dot{\lambda} = \dot{r}\partial_r + \dot{\vartheta}\partial_\vartheta + \dot{\varphi}\partial_\varphi + \dot{t}\partial_t$$

$$\dot{\lambda} = \sigma\big(-\tilde{e}_0 + \sin\tilde{\theta}\cos\tilde{\psi}\,\tilde{e}_1 + \sin\tilde{\theta}\sin\tilde{\psi}\,\tilde{e}_2 + \cos\tilde{\theta}\,\tilde{e}_3\big)$$

Celestial Coordinates of the Boundary of the Shadow

Moving Observer

$$\cos\widetilde{\theta} = \frac{\frac{1}{\sigma}\dot{r} + k_{0r}}{k_{3r}}$$

$$\sin\widetilde{\psi} = \frac{k_{3r}\left(\frac{1}{\sigma}\dot{\varphi} + k_{0\varphi} - \frac{k_{1\varphi}}{k_{1\vartheta}}\left(\frac{1}{\sigma}\dot{\vartheta} + k_{0\vartheta}\right)\right) - \left(k_{3\varphi} - \frac{k_{3\vartheta}}{k_{1\vartheta}}k_{1\varphi}\right)\left(\frac{1}{\sigma}\dot{r} + k_{0r}\right)}{k_{2\varphi}\sqrt{k_{3r}^2 - \left(\frac{1}{\sigma}\dot{r} + k_{0r}\right)^2}}$$

where $\widetilde{e}_\mu = k_{\mu\nu}\partial_\nu$

Motion in Radial Direction: Aberration Formula

$$\cos\widetilde{\theta} = \frac{v + \cos\theta}{1 + v\cos\theta}$$

Celestial Coordinates of the Boundary of the Shadow

Moving Observer

$$\cos\tilde{\theta} = \frac{\frac{1}{\sigma}\dot{r} + k_{0r}}{k_{3r}}$$

$$\sin\tilde{\psi} = \frac{k_{3r}\left(\frac{1}{\sigma}\dot{\varphi} + k_{0\varphi} - \frac{k_{1\varphi}}{k_{1\vartheta}}\left(\frac{1}{\sigma}\dot{\vartheta} + k_{0\vartheta}\right)\right) - \left(k_{3\varphi} - \frac{k_{3\vartheta}}{k_{1\vartheta}}k_{1\varphi}\right)\left(\frac{1}{\sigma}\dot{r} + k_{0r}\right)}{k_{2\varphi}\sqrt{k_{3r}^2 - \left(\frac{1}{\sigma}\dot{r} + k_{0r}\right)^2}}$$

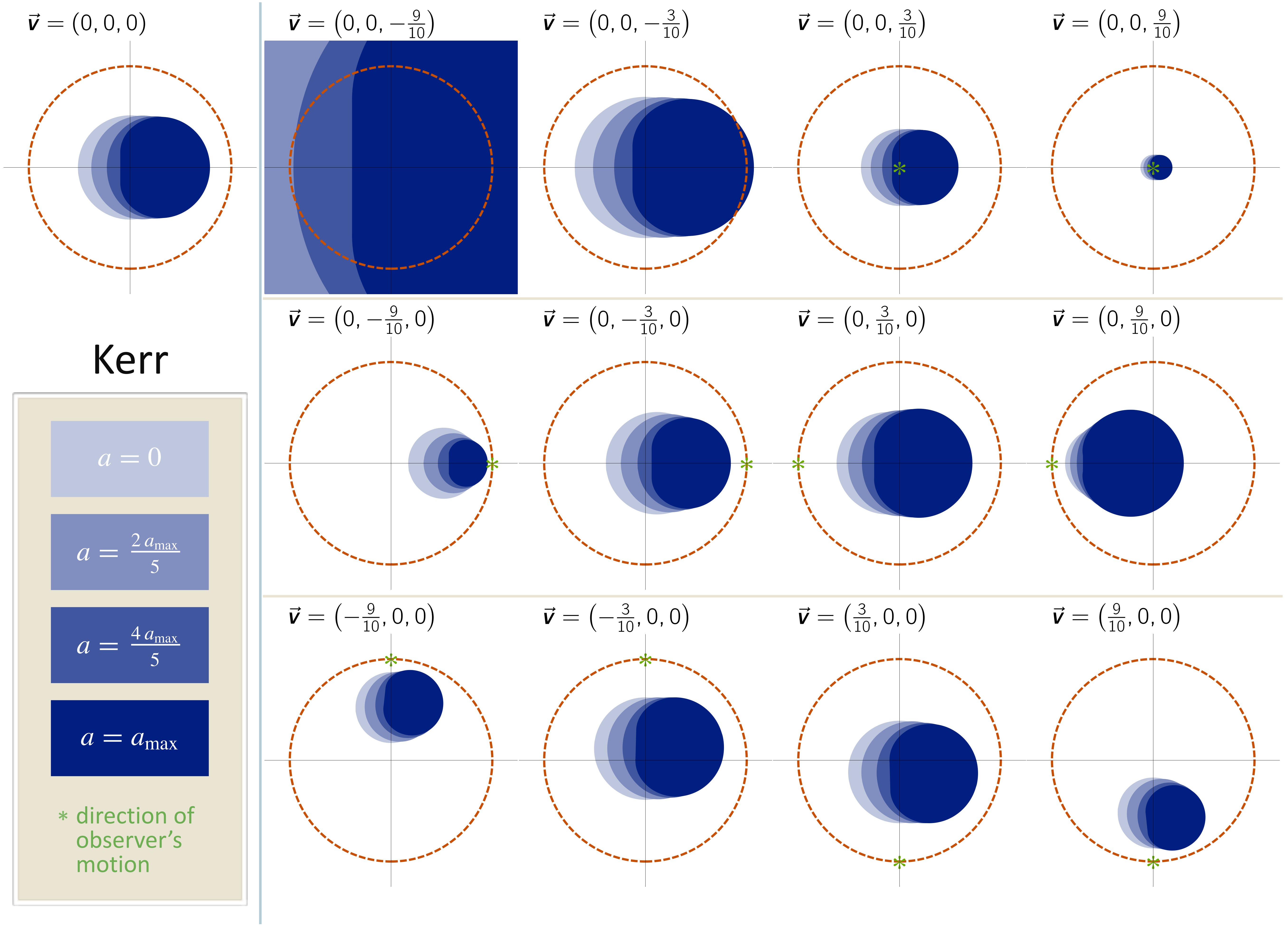
where $\tilde{e}_\mu = k_{\mu\nu}\partial_\nu$

Motion in Radial Direction: Aberration Formula

$$\tan\frac{\tilde{\theta}}{2} = \sqrt{\frac{1-v}{1+v}}\tan\frac{\theta}{2}$$

towards BH ($v > 0$): contraction

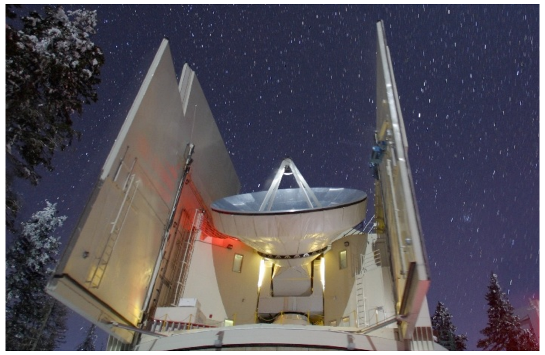
away from BH ($v < 0$): expansion



$\vec{v} = (0, 0, 0)$
$\vec{v} = (0, 0, -\frac{9}{10})$
$\vec{v} = (0, 0, -\frac{3}{10})$
$\vec{v} = (0, 0, \frac{3}{10})$
$\vec{v} = (0, 0, \frac{9}{10})$
Kerr
$\vec{v} = (0, -\frac{9}{10}, 0)$
$\vec{v} = (0, -\frac{3}{10}, 0)$
$\vec{v} = (0, \frac{3}{10}, 0)$
$\vec{v} = (0, \frac{9}{10}, 0)$
$a = 0$
$a = \frac{2\,a_{\max}}{5}$
$a = \frac{4\,a_{\max}}{5}$
$a = a_{\max}$
$\vec{v} = (-\frac{9}{10}, 0, 0)$
$\vec{v} = (-\frac{3}{10}, 0, 0)$
$\vec{v} = (\frac{3}{10}, 0, 0)$
$\vec{v} = (\frac{9}{10}, 0, 0)$
* direction of observer's motion

Observation of the Shadow of Sgr A*

Black Hole Cam | Event Horizon Telescope



ARO/SMT (AZ/USA)

APEX (Chile)

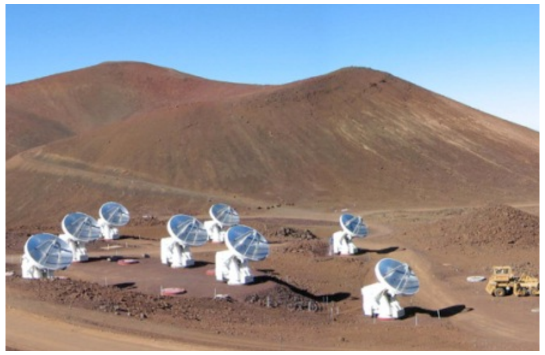
ASTE (Chile)

CARMA (CA/USA)

CSO (HI/USA)

IRAM 30m (Spanien)

JCMT (HI/USA)

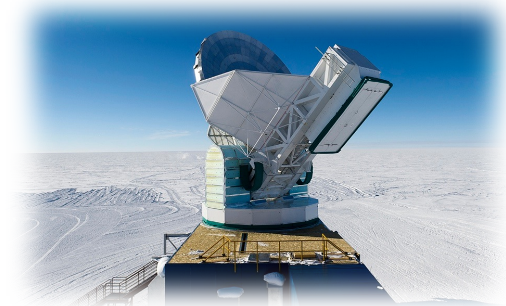


SMA (HI/USA)

ALMA (Chile)

LMT (Mexiko)

PdBI (Frankreich)

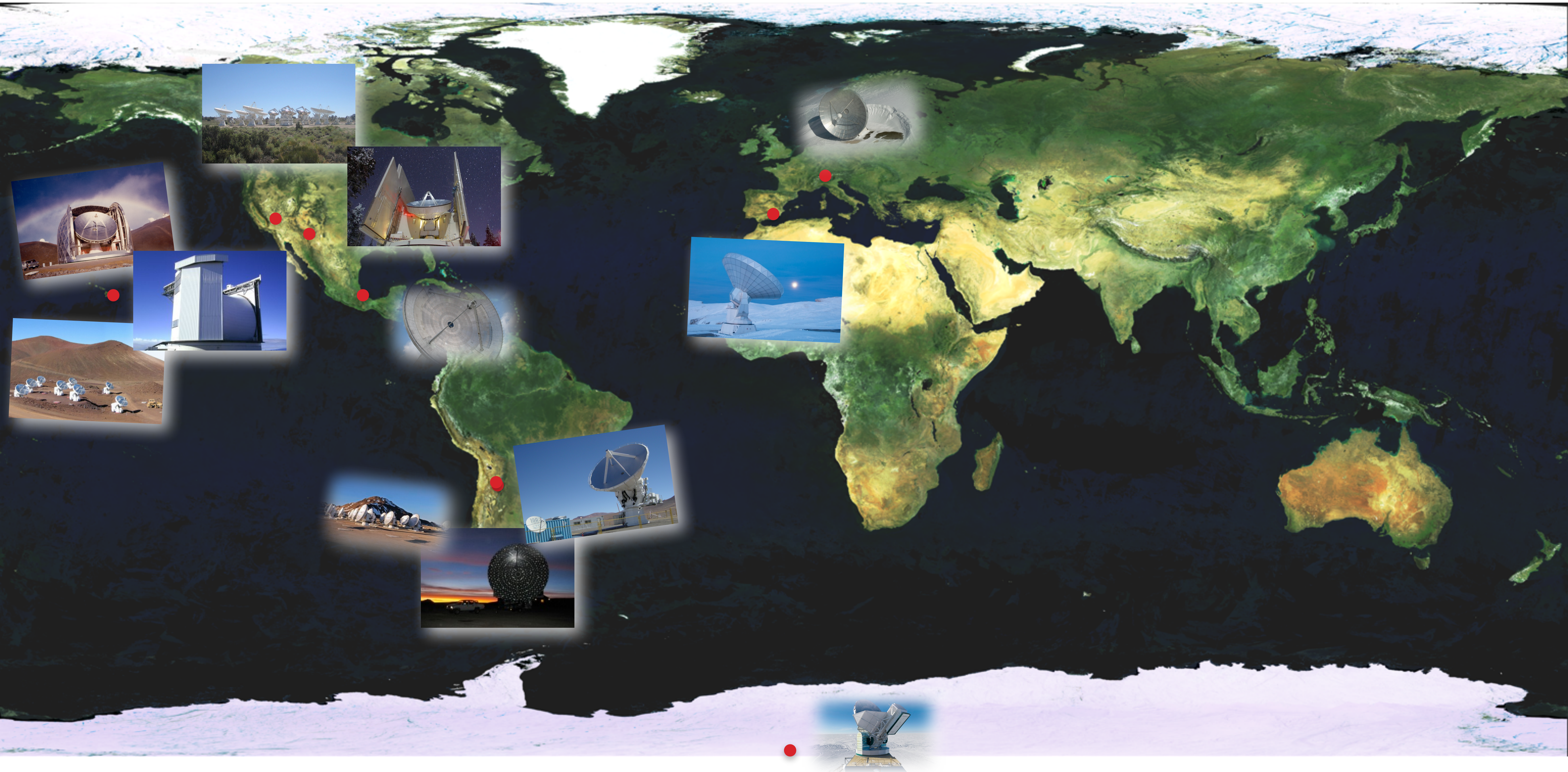


SPT (Antarktis)

www.eventhorizontelescope.org/array/

Observation of the Shadow of Sgr A*

Black Hole Cam | Event Horizon Telescope

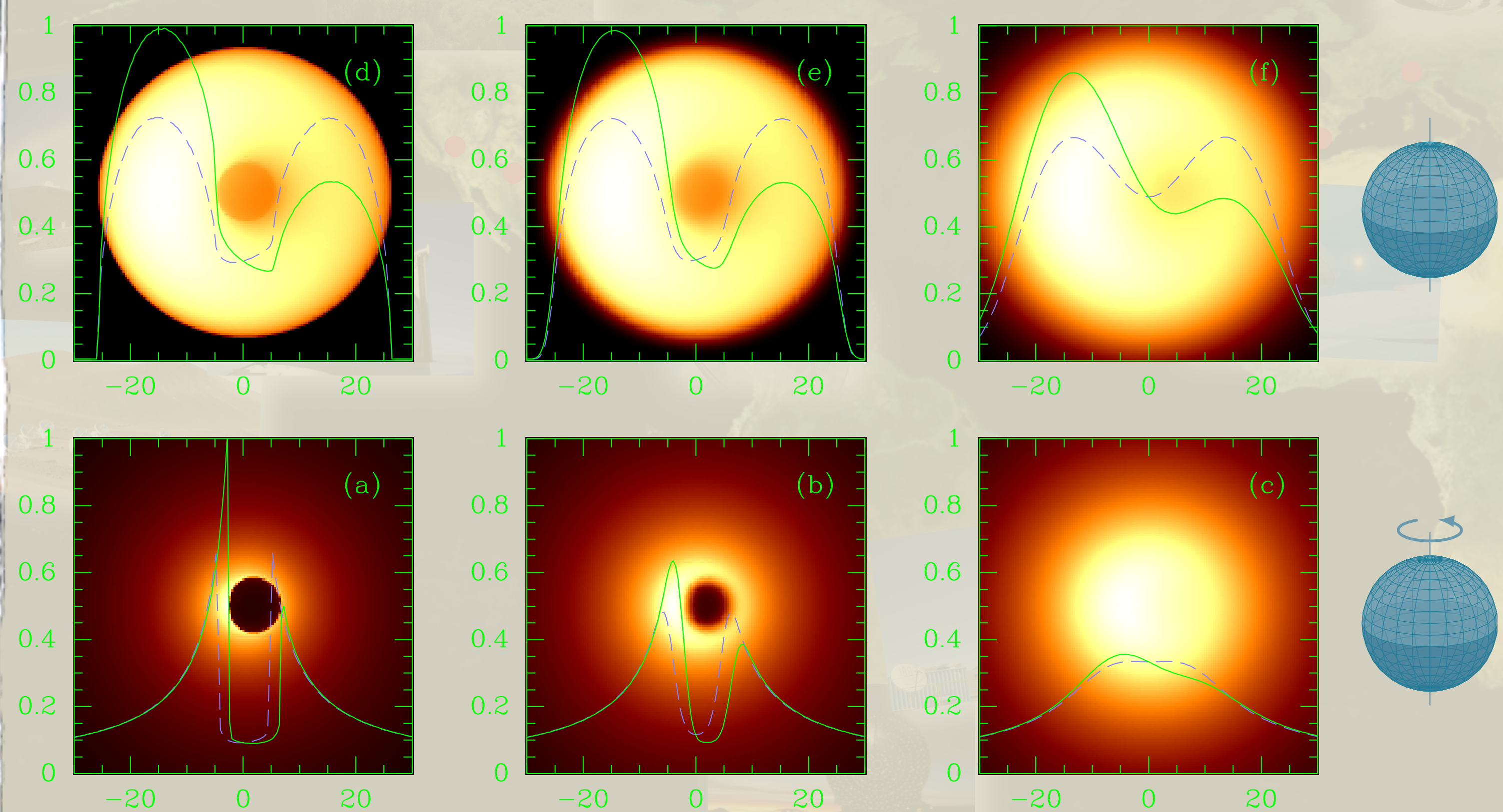


Observation of the Shadow of Sgr A*

Black Hole Cam | Event Horizon Telescope

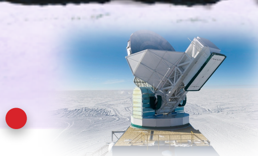
Simulations: Ray-Tracing

[Falcke, Melia, Agol (2000)]

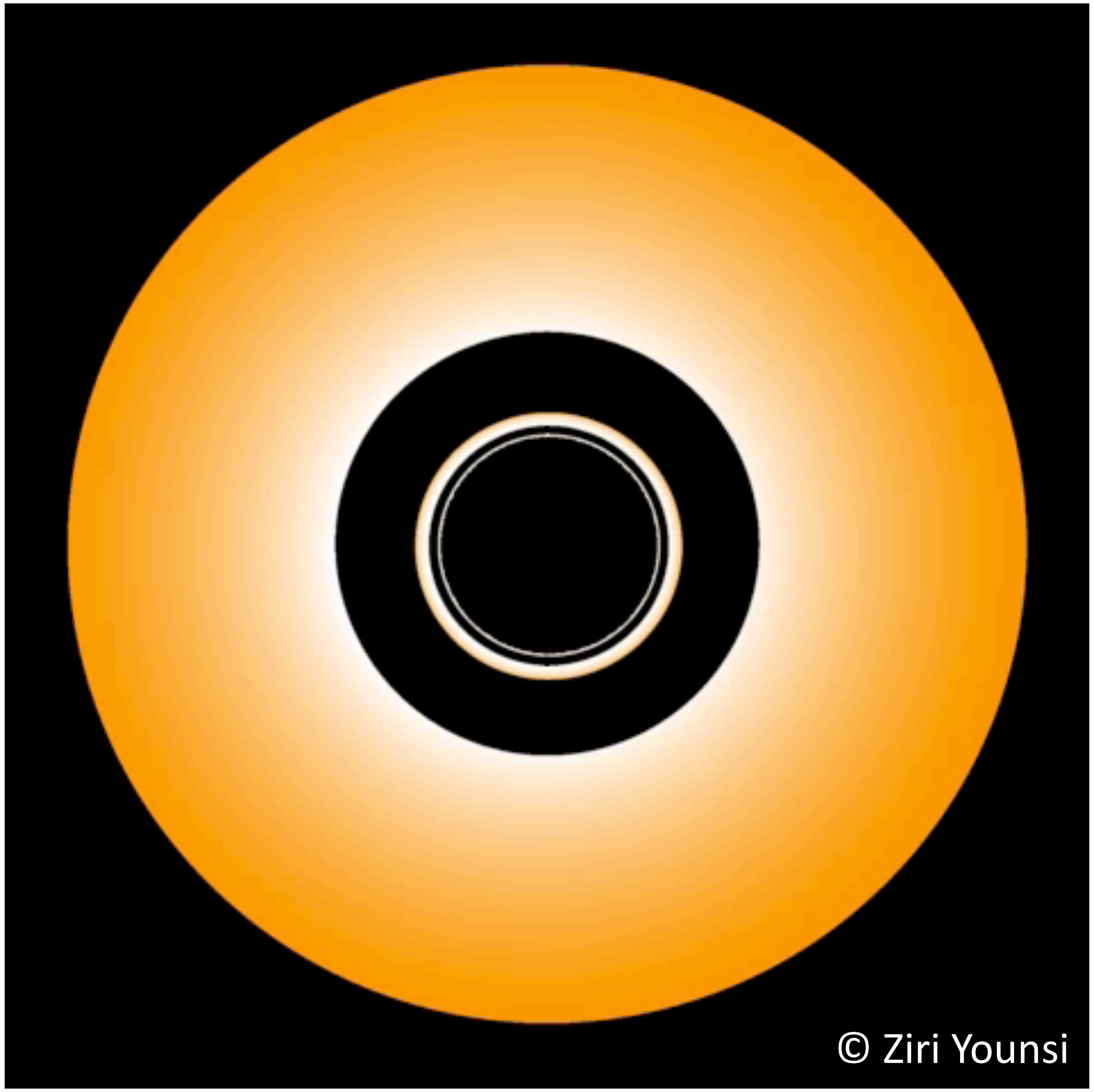


wavelength: 0.6 mm 1.3 mm

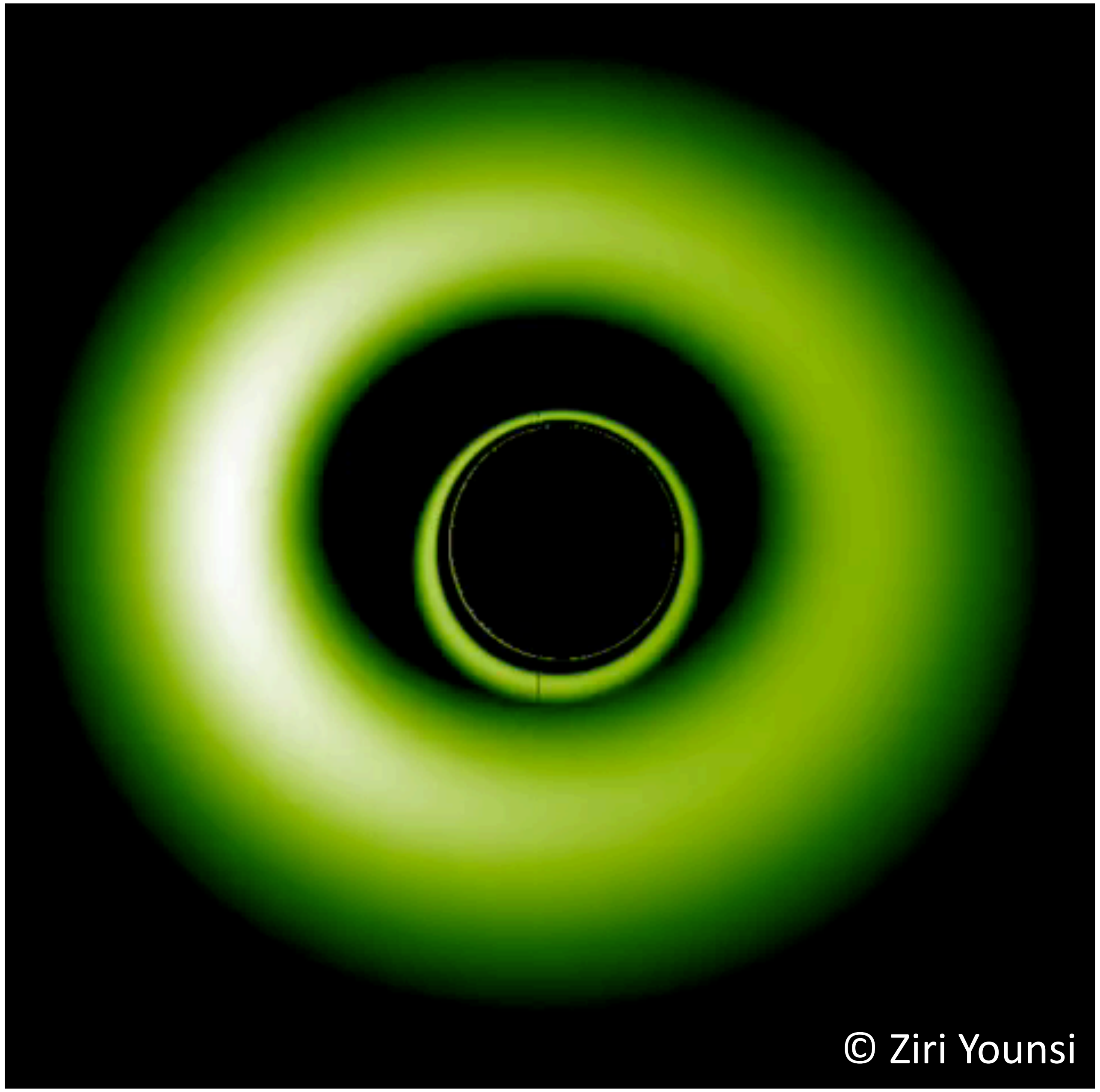
- combine radio telescopes via computers (VLBI)
- 2000 times more detailed than Hubble
- Simulations with
  - interstellar scattering
  - accretion disc/torus
  - jets
  - finite telesc. resolution



Ray-Tracing: Accretion Torus



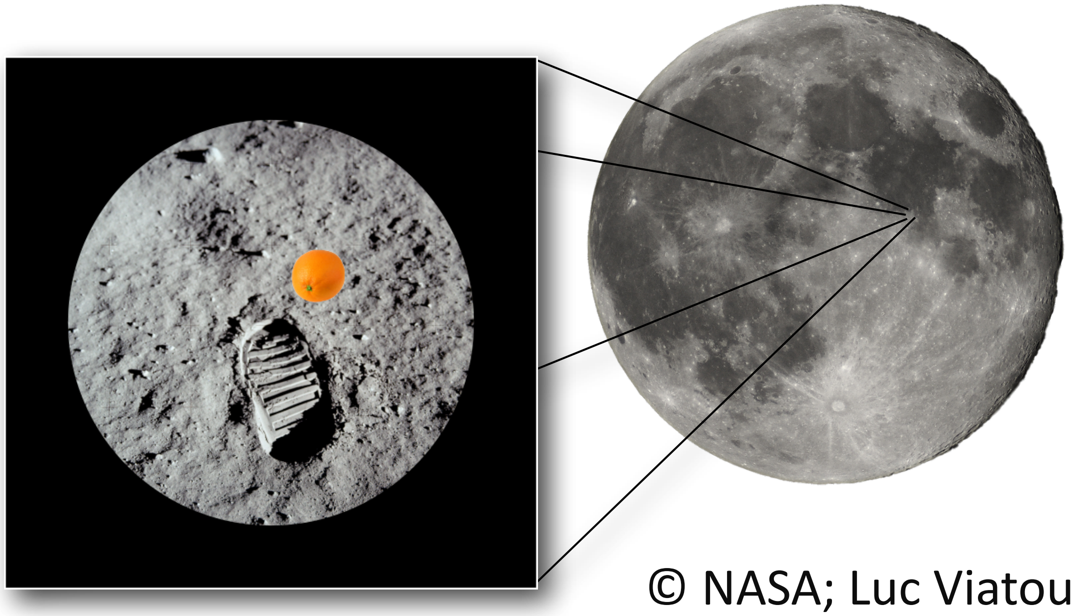
Optically Thick



Optically Thin

Observation of the Shadow of Sgr A*

Expected radius:
≈ 25 μas



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	1.3 mm	0.87 mm
CARMA – SMT	300 μas	200 μas
Hawaii – SMT	58 μas	39 μas
Hawaii – ALMA	28 μas	19 μas
PdBI – SPT	23 μas	15 μas

www.eventhorizontelescope.org/technology/building_a_larger_array.html

Summary

- Analytic formula for the boundary curve of the shadow
- Extr. rot. Black Hole: distorted shadow
- Observable: only shape of shadow, no axes

A. Grenzebach, V. Perlick, C. Lämmerzahl

Photon Regions and Shadows of Kerr–Newman–NUT Black Holes with Cosmological Constant

arXiv:1403.5234 | PRD 89 124004

Outlook

- Plebański–Demiański spacetime, C-metric
- Recover parameters of black hole from the shape of the shadow?
- Influences of variability of light on the shadow

A. Grenzebach (to appear)

Aberrational Effects for Shadows of Black Holes

Summary

- Analytic formula for the boundary curve of the shadow
- Extr. rot. Black Hole: distorted shadow
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Photon Regions and Shadows of Kerr–Newman–NUT Black Holes with Cosmological Constant

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Aberrational Effects for Shadows of Black Holes

Thank You!