Methods for simulating starquakes in neutron stars

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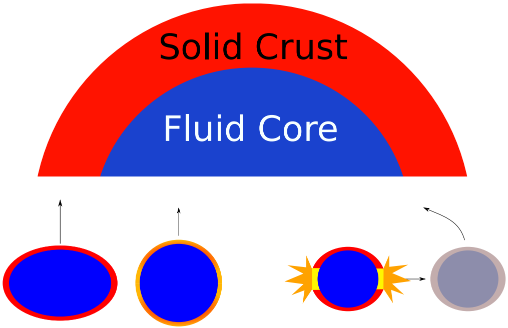
see C. Gundlach, I. Hawke, and SJE, CQG **29** 015055 (2012)

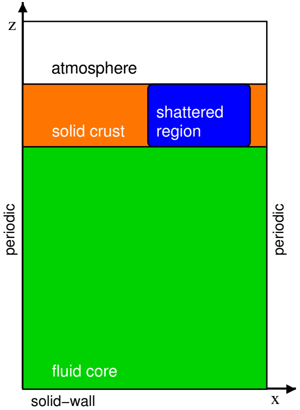
Astrophysical motivation



- NS has crust
- Crust breaks
  - spindown
  - magnetic fields
  - tidal forces
- Starquakes observable
  - precursors to sGRB's
  - pulsar glitches

Goal: Investigate dynamics of neutron star quakes

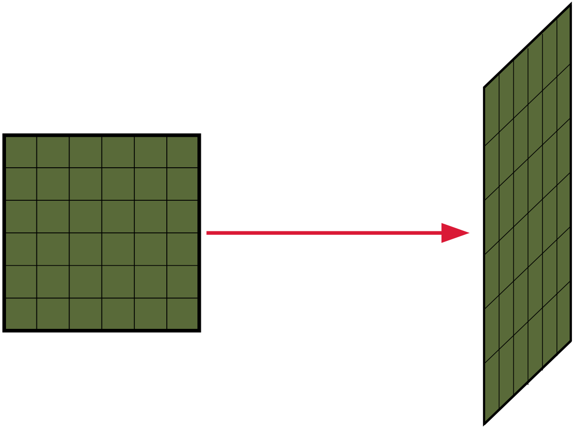
Outline



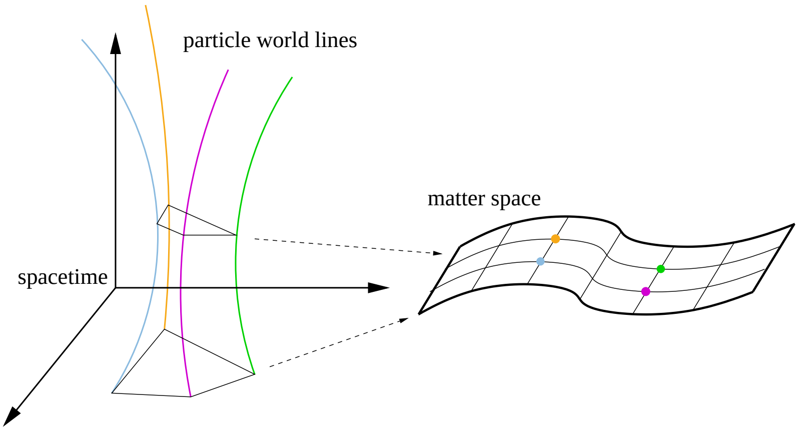
Technical aspects to address:

- Elasticity
- Interfaces
- Stellar surface
- Starquakes

Elasticity formulation



Elasticity formulation



Elasticity formulation

- Map χ : spacetime $\rightarrow$ matter-space
- *Configuration gradient*:

$$\psi^A{}_a := \frac{\partial \chi^A}{\partial x^a} \quad \text{with} \quad \psi^A{}_{[a,b]} = 0$$

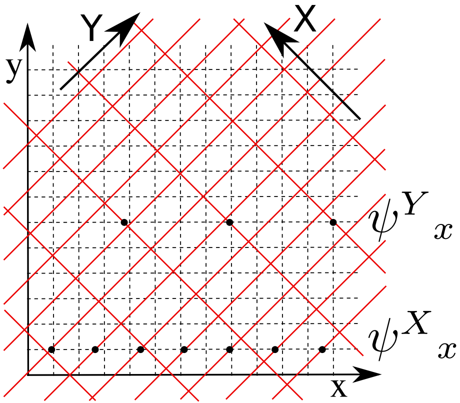
- Particle labels dragged with particles $(u^a \psi^A{}_a = 0)$ so

$$\psi^A{}_t = -\hat{v}^i \psi^A{}_i$$

- So we get an *evolution equation* and a *constraint*:

$$\psi^A{}_{i,t} + \left(\hat{v}^j \psi^A{}_j\right)_{,i} = 0 \quad \text{and} \quad \psi^A{}_{[i,j]} = 0.$$

Physical meaning of $\psi^A{}_i$



- Integrate to find conserved quantity
- Count up matter-space lines crossed in a particular direction
- Represent *crystal axes*

Jump Conditions

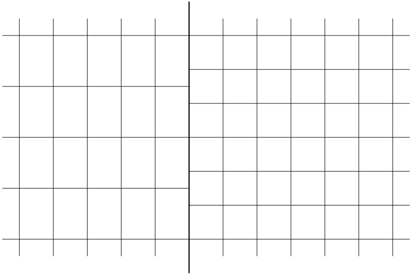
$$\psi^{A}{}_{[i,j]} = 0 \quad \text{and} \quad \psi^{A}{}_{i,t} + \left(\hat{v}^{j}\psi^{A}{}_{j}\right)_{,i} = 0$$

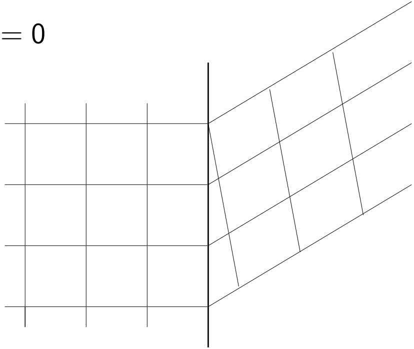
- Covector normal to the shock n_i
- Shock velocity s^i and normal shock speed $s = s^i n_i$
- Projector into surface tangent: $||^{i}{}_{j} := \delta^{i}{}_{j} - n^{i} n_{j}$
- Jump conditions become

$$[\psi^{A}{}_{||k}] = 0 \quad \text{and} \quad [\psi^{A}{}_{n}(\hat{v}^{n} - s)] + \psi^{A}{}_{||i}[\hat{v}^{||i}] = 0$$

Jump Conditions: Constraint Examples

$$[\psi^{A}{}_{||k}] = 0$$

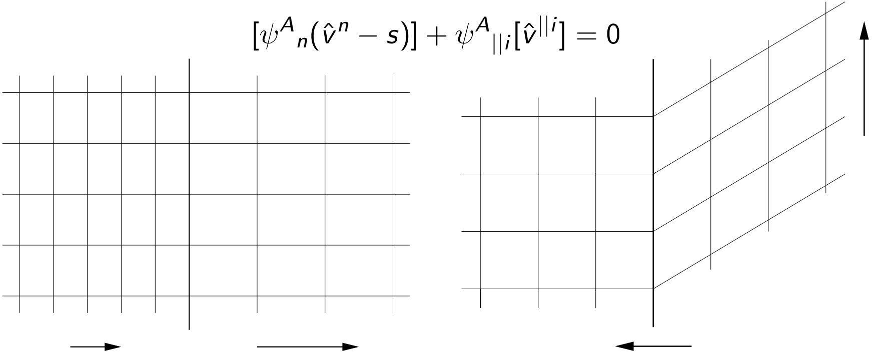




$[\psi^{Y}{}_{y}] \neq 0$ $[\psi^{X}{}_{y}] \neq 0$

Jump Conditions: Evolution Equation Examples

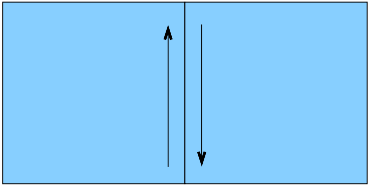
$$[\psi^A{}_n(\hat{v}^n - s)] + \psi^A{}_{||i}[\hat{v}^{||i}] = 0$$



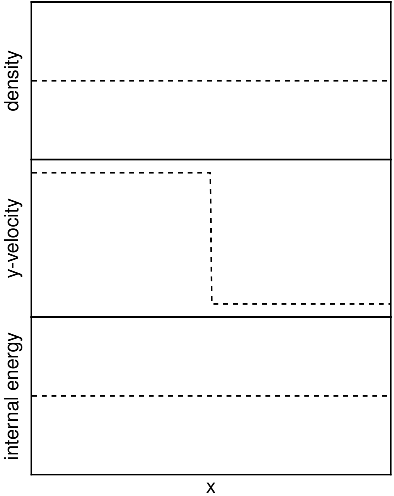
$$\hat{v}^{||i} = 0, \quad s = 0, \quad [\psi^A{}_n \hat{v}^n] = 0 \qquad \hat{v}^n = 0, \quad \psi^A{}_{||i}[\hat{v}^{||i}] - s[\psi^A{}_n] = 0$$

Elasticity: Shear Stresses

Perfect Fluid

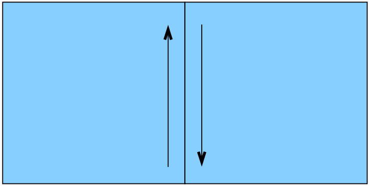


Initial contact stays stationary with evolution in time.

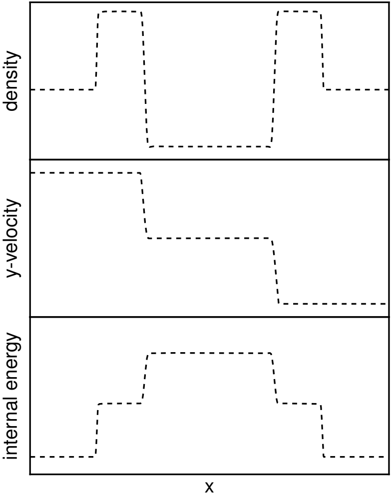


Elasticity: Shear Stresses

Elastic Solid



Initial discontinuity in velocity produces shear waves as time evolves.



How is the stress-energy tensor changed?

- More general stress-energy tensor

$$T^{ab} = eu^a u^b + p^{ab} = eu^a u^b + ph^{ab} + \pi^{ab}$$

- Anisotropic stress π^{ab} comes from

$$\pi_{ab} := \psi^A{}_a \psi^B{}_b \pi_{AB}$$

- On matter space, π_{AB} relates k_{AB} to g^{AB}
- Relaxed state at $k_{AB} = n^{2/3} g_{AB}$
- Have $u^a h^{bc} T_{ab} = 0 \rightarrow$ heat-flow terms are zero

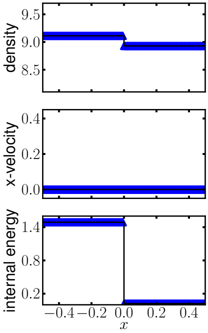
Matter evolution equations

	Conserved Quantity	Flux
	$S_i = \text{fluid} + \pi_{ij} v^j$	$\mathcal{F}(S_j)^i = \text{fluid} + \pi^i{}_j$
	$\tau = \text{fluid} + \pi$	$\mathcal{F}(\tau)^i = \text{fluid} - \pi\alpha^{-1}\beta^i + \gamma^{ij}\pi_{jk}v^k$
Add	$\psi^A{}_i$	$\mathcal{F}(\psi^A{}_j)^i = \hat{v}^k \psi^A{}_k \delta^i{}_j$
Remove	$\cancel{D}$	$\cancel{\mathcal{F}(D)}$

Advection equation: $k_{AB,t} + \hat{v}^i k_{AB,i} = 0$

Elasticity results

- Newtonian limit of our relativistic code matches published Newtonian Riemann tests
- 2D tests match 1D tests and exact solutions where available
- 2D cylindrical coordinates demonstrates that we can use a general metric



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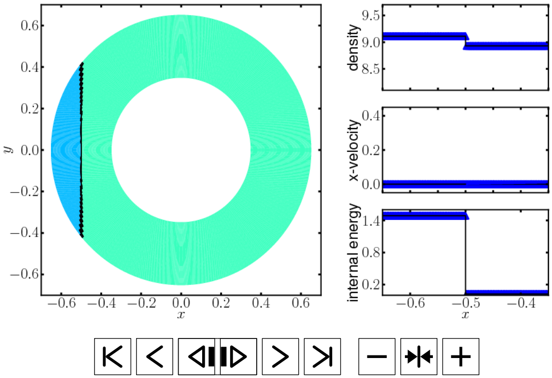

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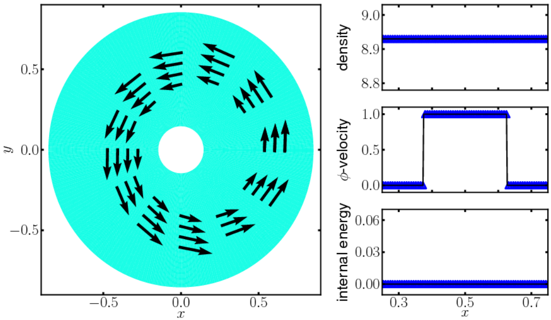
Elasticity results



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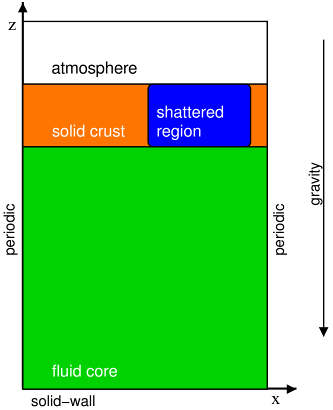
Outline



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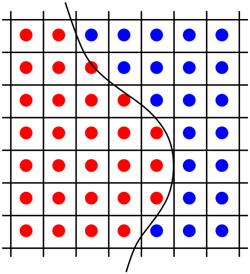
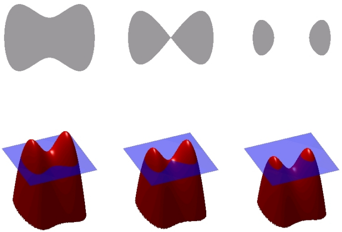
Interfaces



- Separate grid into regions governed by different physical models
- Track the moving boundary using a *level-set function*, ϕ
- Level set is associated with particles, so $u^a \nabla_a \phi = u^a \phi_{,a} = 0$
- In $3 + 1$ split: advection

Interfaces



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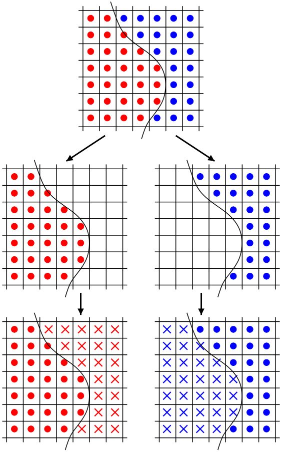
Interfaces: What happens at the interface?

- Apply appropriate boundary conditions
- Use approximate solution of multimaterial Riemann problem to determine behavior at the boundary
- Extension of ghost fluid method (GFM)

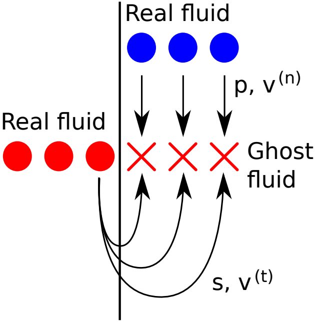
Example: Original GFM



Ghost fluid method:

- Continuous across contact: p, $v^{(n)}$
- Discontinuous across contact: s, $v^{(t)}$
- Calculate $n = n(s, p)$

Example: Original GFM



Ghost fluid method:

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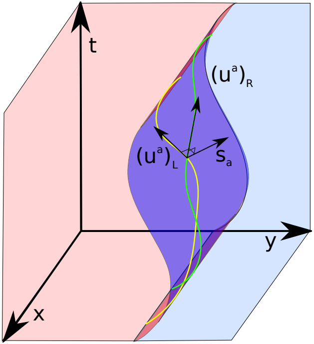
In general relativity



- Say interface has normal covector s_a and $s_a u^a = 0$
- In $3+1$ split $s_a = v_\perp n_a + k_a$ so

$$[v^a k_a] = [v_\perp] = 0$$

- We have either of the following boundary conditions

$$\begin{aligned} [T^{ab} s_a s_b] &\leftrightarrow [p^{ab} s_a s_b] = 0 \quad \text{Slip} \\ [T^{ab} s_a] &\leftrightarrow [p^{ab} s_a] \quad\; = 0 \quad \text{Stick} \end{aligned}$$

In general relativity

- We split p_{ab} into isotropic and anisotropic stress

$$p_{ab} = ph_{ab} + \pi_{ab}$$

- Interface conditions become

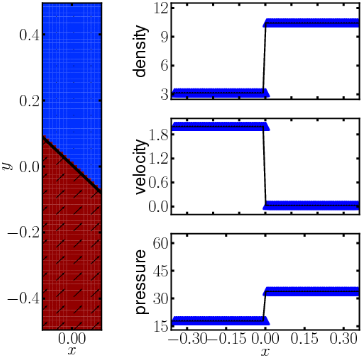
$$\begin{aligned} [p(1 - v_\perp^2) + \pi^{ab}(k_a - v_\perp v_a)(k_b - v_\perp v_b)] &= 0 \quad \text{Slip} \\ [ps^b + \pi^{ab}(k_a - v_\perp v_a)] &= 0 \quad \text{Stick} \end{aligned}$$

- In the Newtonian Limit

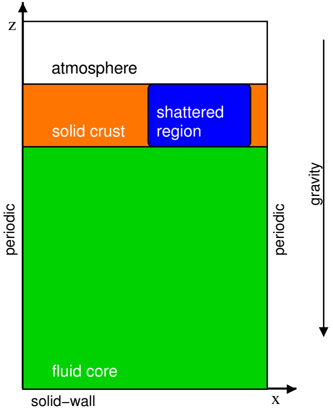
$$[p + \hat{\pi}^{ij} k_i k_j] = 0 \quad \text{or} \quad [pk^i + \hat{\pi}^{ij} k_j] = 0$$

Interface results

- Newtonian and relativistic interfaces in 1D
- Moved to *multimodel* code for 2D infrastructure
- 2D Newtonian interfaces



Outline



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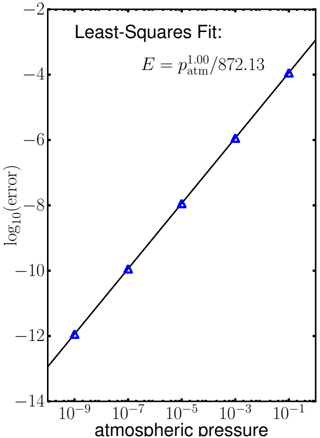
Why do we need an atmosphere?

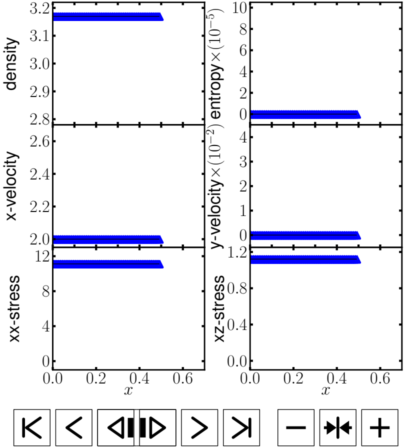
- In terrestrial solids, when $p \to 0$, n and ϵ do not
- In NS, shear and thermal terms *both* scale with n:
  - $n \to 0$ and $\epsilon \to 0$ as $p \to 0$
  - All zero at the surface
- Small fluctuations at the surface can cause these to go negative, which causes numerical problems
- Positivity preserving schemes have worked for fluids (Radice et al. 2014), but unclear how to extend to elasticity

What is the atmosphere?

- Treat the atmosphere as another *model* in our code
- Model consists of only a pressure, p_{atm}
- Don't evolve
- Just use to apply boundary conditions at the surface

Atmosphere Results





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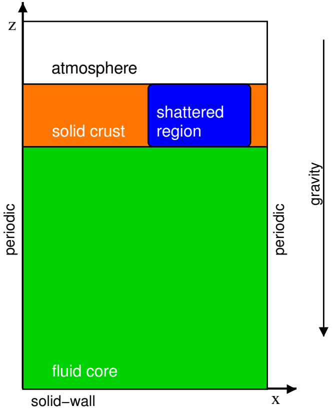

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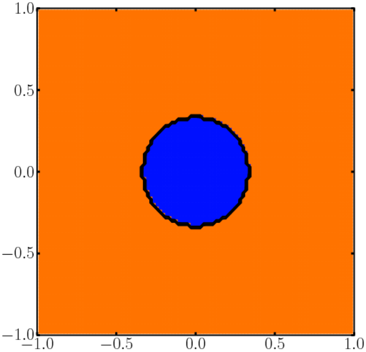
Starquake mechanisms

Cracking

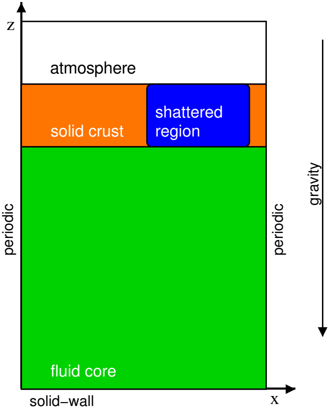
- Material breaks and slips along a surface, handled using interfaces (previous slide)
- Suppressed by pressure in NS?

Shattering

- Instantaneous relaxation, matter-space metric proportional to spacetime metric
- Suggested by molecular dynamics simulations



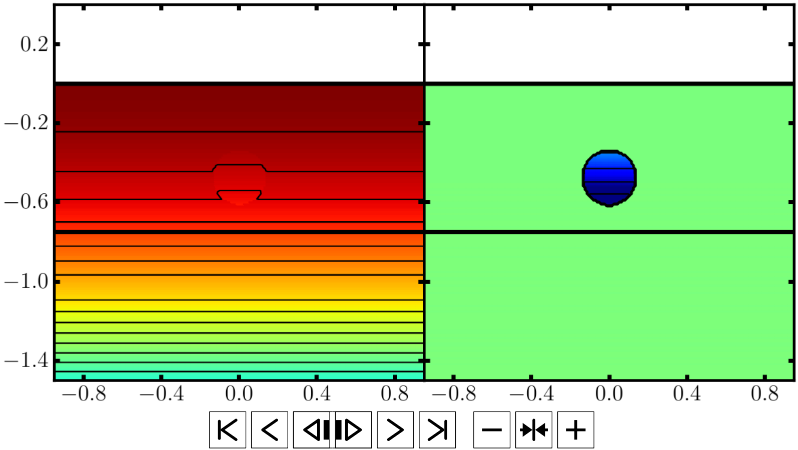
2D toy model



Combine technical aspects:

- Elasticity
- Interfaces
- Stellar surface
- Starquakes

2D toy model



Future Work

- Assess effect of atmosphere
- Toy model in GR
- Elasticity and interfaces in a 3D fully relativistic code
- Magnetic fields