Radial oscillations of compact stars and collapse to black holes

FIAS Frankfurt Institute for Advanced Studies

Alessandro Brillante, Prof. Igor N. Mishustin, FIAS

Outline

1 Motivation

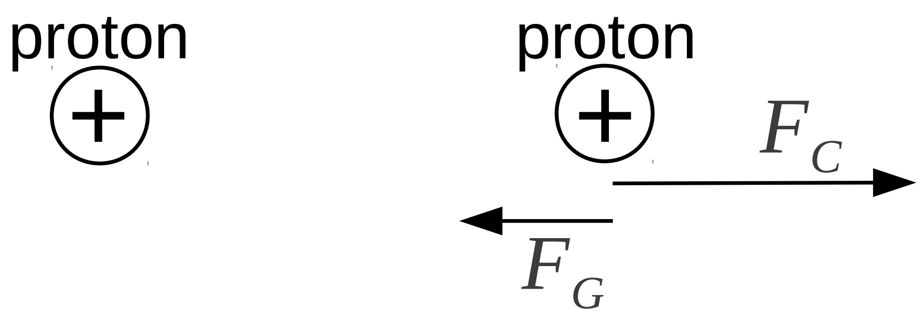
2 Derivation of oscillation equation

3 Results on neutral hybrid stars

4 Results on charged strange and hybrid stars

5 Non-linear effects, critical phenomena

How much charge allowed?



$$\frac{F_C}{F_G} = \frac{e^2}{G m_p^2} \approx 10^{36}$$

Number of baryons in one neutron star: $N_B \approx 3 \cdot 10^{57}$

Number of net unit charges allowed to build
“reasonable” charged compact stars: $N_c < 10^{-18} N_B$

assumption: EOS for charged compact stars
calculated at charge neutrality (only 1 independent
chemical potential)

prescription to find oscillation equation

- time-dependent spherically symmetric metric

- equations: $G_{\mu\nu} = 8\pi T_{\mu\nu}$ $T^{\mu\nu}{}_{;\nu} = 0$ $(nu^{\mu})_{;\mu} = 0$

$$\partial_\mu \left[\sqrt{-g} F^{\nu\mu} \right] = 4\pi \sqrt{-g}\, j^\nu$$

- decompose variables: $A\,(r,t) = A_0\,(r) + \delta A\,(r,t)$

- linearize nonlinear equations

- subtract equilibrium equations from time-dependent equations and get perturbations: $\delta A\,(r,t)$

- substitute perturbations in $T^{\mu\, r}{}_{;\,\mu} = 0$ and get pulsation equation

Derivation of oscillation equation 1

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

$$G_0{}^0 = -e^{-2\Lambda}\left[2r^{-1}\Lambda' - \left(1 - e^{2\Lambda}\right)r^{-2}\right]$$

$$G_1{}^1 = e^{-2\Lambda}\left[2r^{-1}\Phi' + r^{-2}\right] - r^{-2}$$

$$G_2{}^2 = e^{-2\Lambda}\left[\Phi'' - \Phi'\Lambda' + \Phi'^2 + r^{-1}\left(\Phi' - \Lambda'\right)\right]$$
$$+ e^{-2\Phi}\left[\dot{\Phi}\dot{\Lambda} - \ddot{\Lambda} - \dot{\Lambda}^2\right]$$

$$G_0{}^1 = 2r^{-1}e^{-2\Lambda}\dot{\Lambda}$$

$$T_\mu{}^\nu = (\rho + P)\,u_\mu u^\nu + Pg_\mu{}^\nu + \frac{1}{4\pi}\left[F_{\mu\alpha}F^{\alpha\nu} - \frac{1}{4}g_\mu{}^\nu F^{\beta\gamma}F_{\beta\gamma}\right]$$

Derivation of oscillation equation 2

$$\delta\Lambda = -(\Phi_0{}' + \Lambda_0{}')\,\xi$$

$$\delta\rho = -\xi\rho_0{}' - (\rho_0 + P_0)\,\frac{e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

$$\delta\Phi' = 4\pi r e^{2\Lambda_0}\delta P + 2\Phi_0{}'\delta\Lambda + r^{-1}\delta\Lambda - \frac{Q_0\delta Q e^{2\Lambda_0}}{r^3}$$

$$\delta P = \frac{dP_0}{d\rho_0}\delta\rho = -\xi P_0{}' - \frac{\gamma P_0 e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

Energy-momentum conservation:

$$e^{2\Lambda_0 - 2\Phi_0}(\rho_0 + P_0)\,\dot{v} + \delta P' + \frac{Q_0 Q_0{}'\xi'}{4\pi r^4} + \frac{Q_0 Q_0{}''\xi}{4\pi r^4}$$
$$+ \frac{Q_0{}'^2\xi}{4\pi r^4} + \Phi_0{}'(\delta\rho + \delta P) + (\rho_0 + P_0)\,\delta\Phi' = 0$$

The oscillation equation

Chandrasekhar, APJ, 140 (1964) 417

$$\sigma^2 e^{\lambda_0-\nu_0}(\overset{1}{p_0}+\epsilon_0)\,\xi = \frac{4}{r}\frac{dp_0}{dr}\overset{2}{\xi} - e^{-(\lambda_0+2\nu_0)/2}\overset{3}{\frac{d}{dr}}\left[e^{(\lambda_0+3\nu_0)/2}\frac{\gamma p_0}{r^2}\frac{d}{dr}\left(r^2 e^{-\nu_0/2}\xi\right)\right] + \frac{8\pi G}{c^4}e^{\lambda_0}\overset{4}{p_0}(p_0+\epsilon_0)\,\xi - \frac{1}{p_0+\epsilon_0}\overset{5}{\left(\frac{dp_0}{dr}\right)^2}\xi\,.$$

AB & Igor N. Mishustin, EPL, 105 (2014) 39001

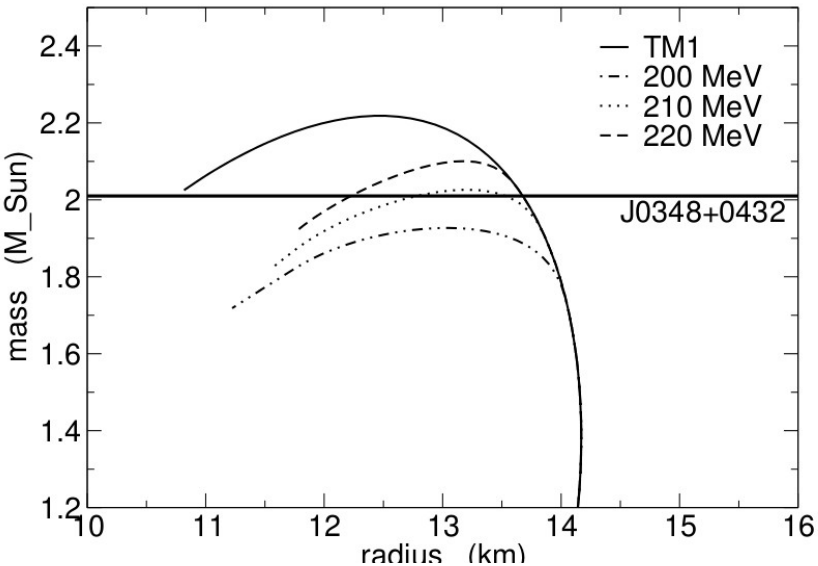
$$\omega^2 e^{2\Lambda_0-2\Phi_0}(\overset{1}{\rho_0}+P_0)\,\xi = -e^{-\overset{3}{\Lambda_0}-2\Phi_0}\left[e^{\Lambda_0+3\Phi_0}\frac{\gamma P_0}{r^2}\left(r^2e^{-\Phi_0}\xi\right)'\right]' - (\overset{5}{\rho_0}+P_0)\,\Phi_0'^2\xi + 4r^{-1}\overset{2}{\xi}P_0' + 8\pi(\overset{4}{\rho_0}+P_0)\,\xi e^{2\Lambda_0}P_0 + (\rho_0+P_0)\,r^{-4}\xi e^{2\Lambda_0}Q_0^2 \longleftarrow \text{CHARGE TERM}$$

Neutral hybrid stars – Gibbs constr.

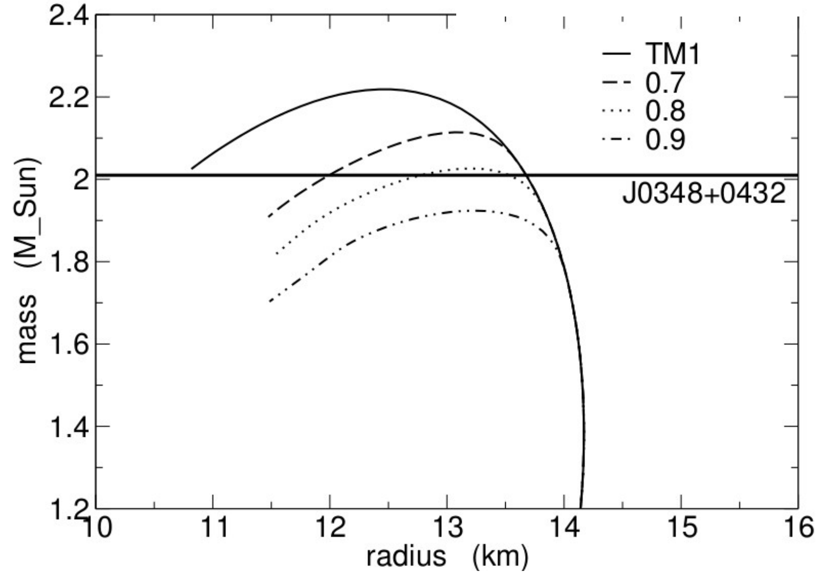
$$\Omega_{\mathrm{QM}} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\mathrm{eff}}$$

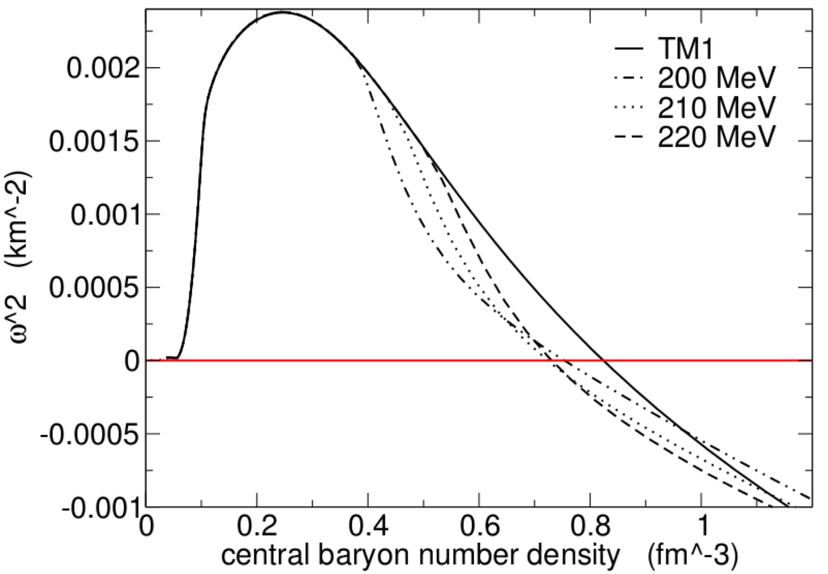
Weissenborn et al. Astrophys. J. 740, L14 (2011)

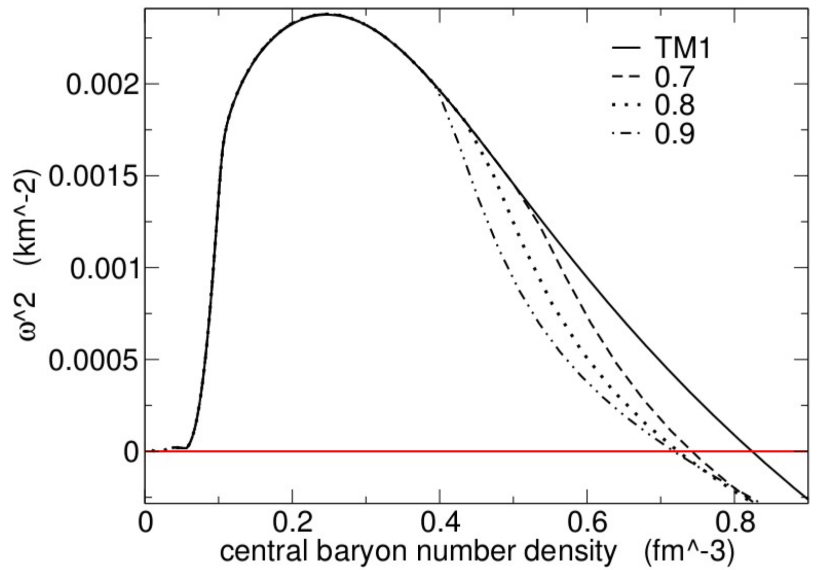
$m_s = 100\ MeV, a_4 = 0.8$



$m_s = 100\ MeV\ \ B^{\frac{1}{4}} = 210\ MeV$







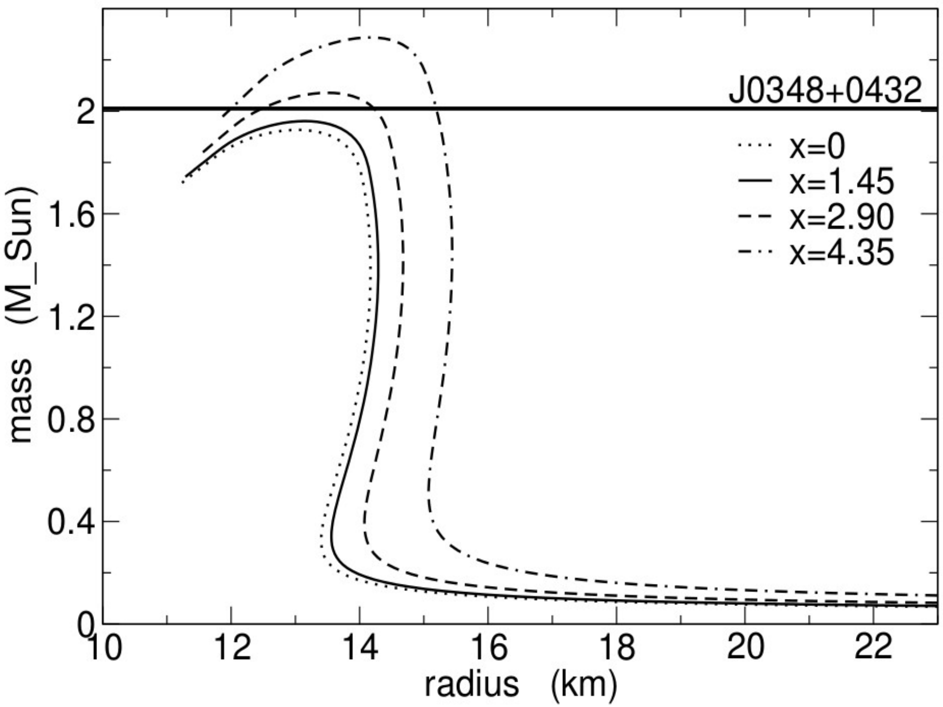
Charged hybrid stars – Gibbs constr.

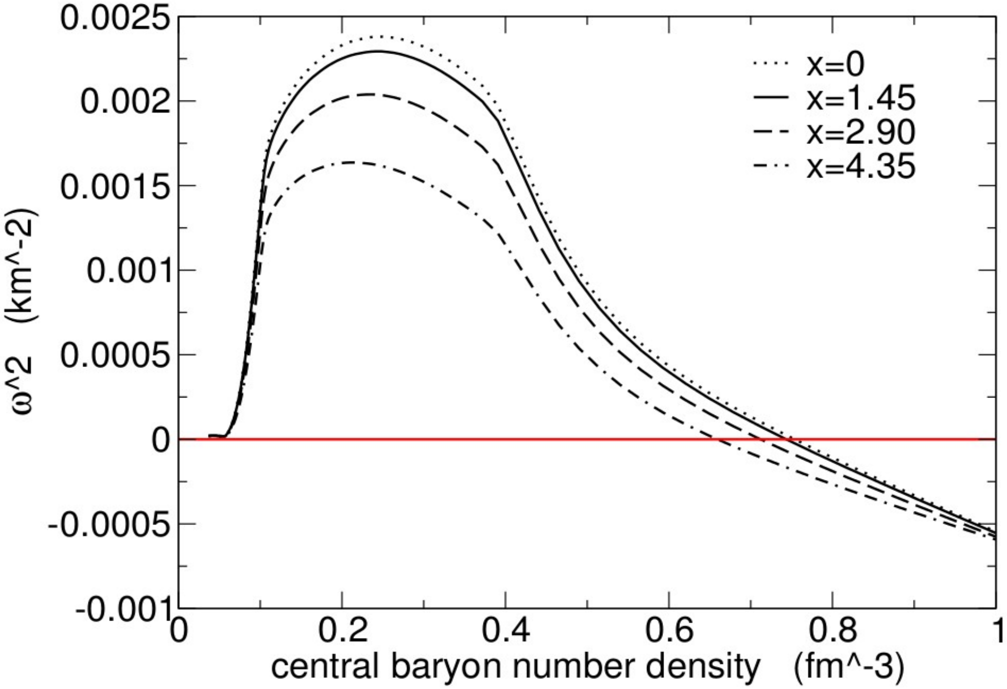
Hadronic phase: relativistic mean-field model, TM1 parameter set

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 0.8$, $B^{1/4} = 200\,MeV$

$$x = 10^{19}\frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)



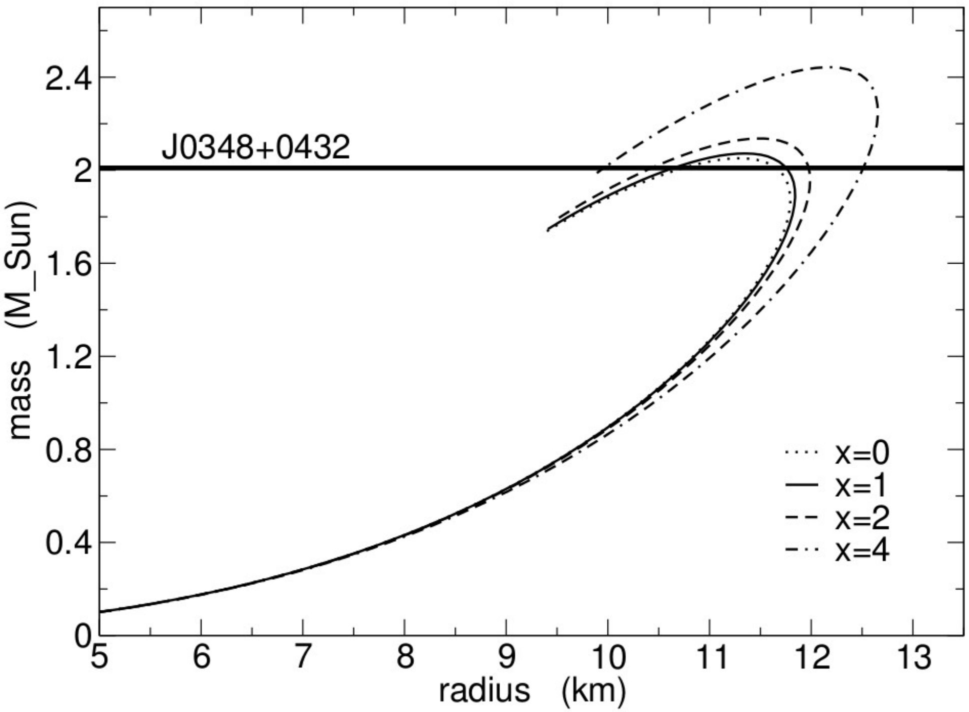


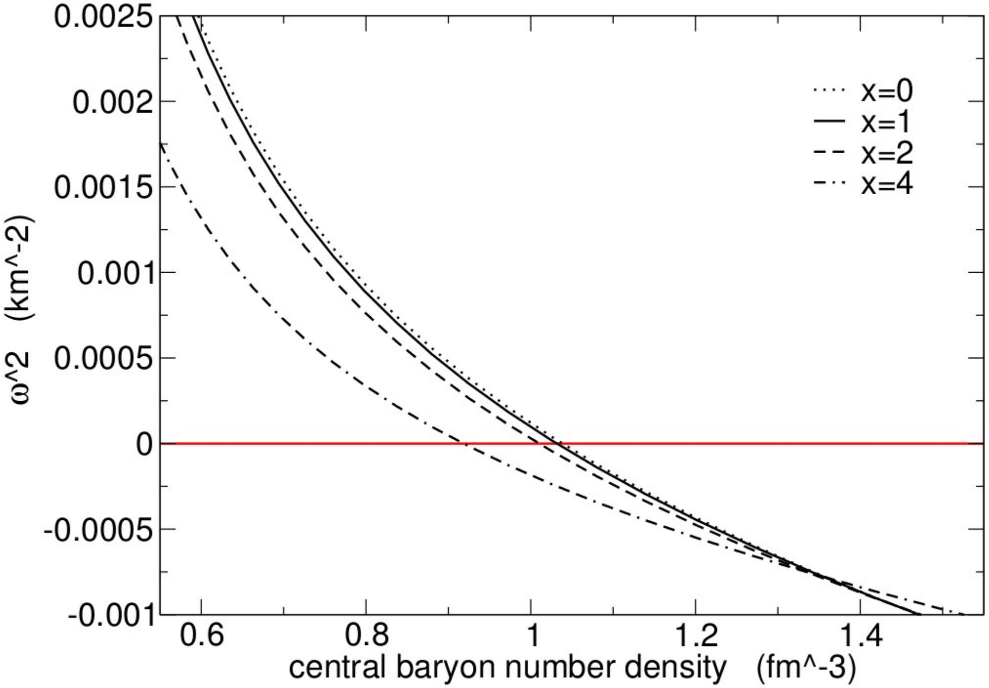
Charged strange stars

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 1.0$, $B^{1/4} = 140\,MeV$

$$x = 10^{19} \frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)





Schwarzschild-like coordinates

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad T^{\mu\nu}{}_{;\nu} = 0$$

constrain equations:

$$\Lambda' = 4\pi r\left(\rho + P\right)e^{4\Lambda}{u^1}^2 + 4\pi r\rho e^{2\Lambda} + \frac{1-e^{2\Lambda}}{2r} + \frac{Q^2 e^{2\Lambda}}{2r^3}$$

$$\Phi' = 4\pi r\left(\rho + P\right)e^{4\Lambda}{u^1}^2 + 4\pi r P e^{2\Lambda} + \frac{e^{2\Lambda}-1}{2r} - \frac{Q^2 e^{2\Lambda}}{2r^3}$$

evolution equations:

$$\frac{\partial P}{\partial t} + c_{11}\frac{\partial P}{\partial r} + c_{12}\frac{\partial u^1}{\partial r} + c_{13} = 0$$

$$\frac{\partial u^1}{\partial t} + c_{21}\frac{\partial P}{\partial r} + c_{22}\frac{\partial u^1}{\partial r} + c_{23} = 0$$

$$\xi = \frac{\partial\rho}{\partial P} - e^{2\Lambda}{u^1}^2 + \frac{\partial\rho}{\partial P}e^{2\Lambda}{u^1}^2$$

$$c_{11} = \frac{1}{\xi}\left(\frac{\partial\rho}{\partial P} - 1\right)e^{2\Phi}u^0u^1$$

$$c_{12} = \frac{\rho+P}{\xi u^0}$$

$$c_{13} = \frac{e^{\Phi}u^1}{2r^3\left(1+e^{2\Lambda}{u^1}^2\right)\xi}\Big[2e^{\Lambda}Qr\rho_{ch} + 2e^{3\Lambda}Qr\rho_{ch}{u^1}^2 + 5e^{\Phi}r^2\left(\rho+P\right)u^0$$
$$+ e^{2\Lambda+\Phi}\left(\rho+P\right)\left(Q^2 + r^2\left(-1 - 8\pi P r^2 + 4{u^1}^2\right)\right)u^0\Big]$$

$$c_{21} = \frac{1}{\left(\rho+P\right)\xi}\frac{\partial\rho}{\partial P}e^{2\Phi-2\Lambda}u^0$$

$$c_{22} = c_{11}$$

$$c_{23} = -\frac{e^{\Phi-2\Lambda}}{2r^3\left(\rho+P\right)\xi}\Big[\frac{\partial\rho}{\partial P}\Big(2e^{\Lambda}Qr\rho_{ch} + 2e^{3\Lambda}Qr\rho_{ch}{u^1}^2 + e^{\Phi}r^2\left(\rho+P\right)u^0$$
$$+ e^{2\Lambda+\Phi}\left(Q^2 - r^2 - 8\pi P r^4\right)\left(\rho+P\right)u^0\Big) + 4e^{2\Lambda+\Phi}r^2\left(\rho+P\right){u^1}^2u^0\Big]$$

1D code

- 2nd order time, 4th order space accuracy

- constrained evolution

- implicit discretization for evolution equations (iterated Crank-Nicolson)

- bicgstab algorithm for sparse linear system

- strengths: good tracking of stellar surface (Lagrangian coordinates), CFL condition always fullfilled, for spheres faster than multidimensional codes, better resolution possible

- weaknesses: unable to penetrate Schwarzschild radius (coordinate singularity), limited to barotropic fluids, limited to spheres

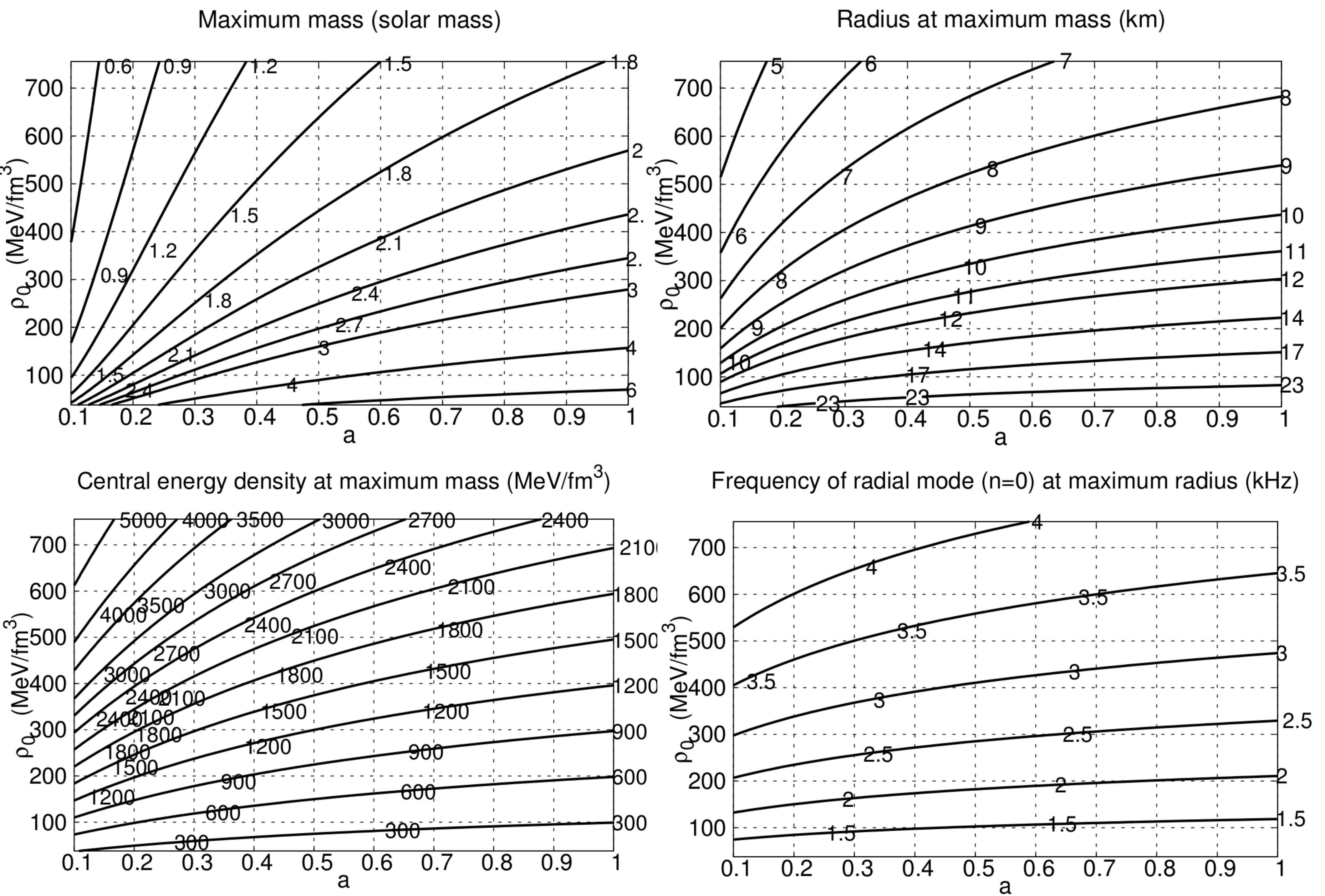
Generalized Zeldovich EoS

(1914-1987)

$P = a (\rho - \rho_0)$

Compactness x=2M/R of maximum mass and maximum radius configuration depends only on a.

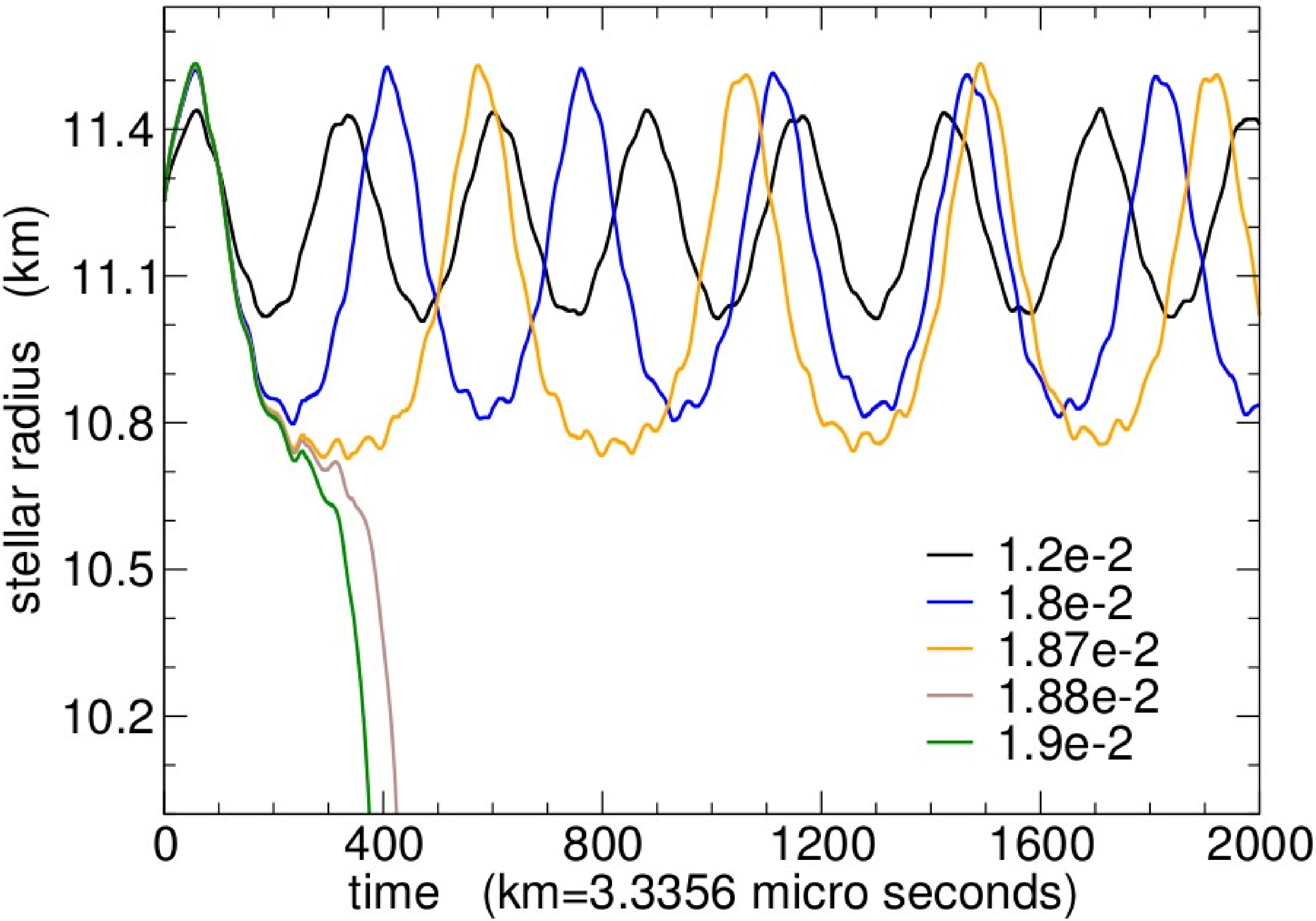
Different choices for rho_0 preserve compactness (for max m. and max r. configurations).



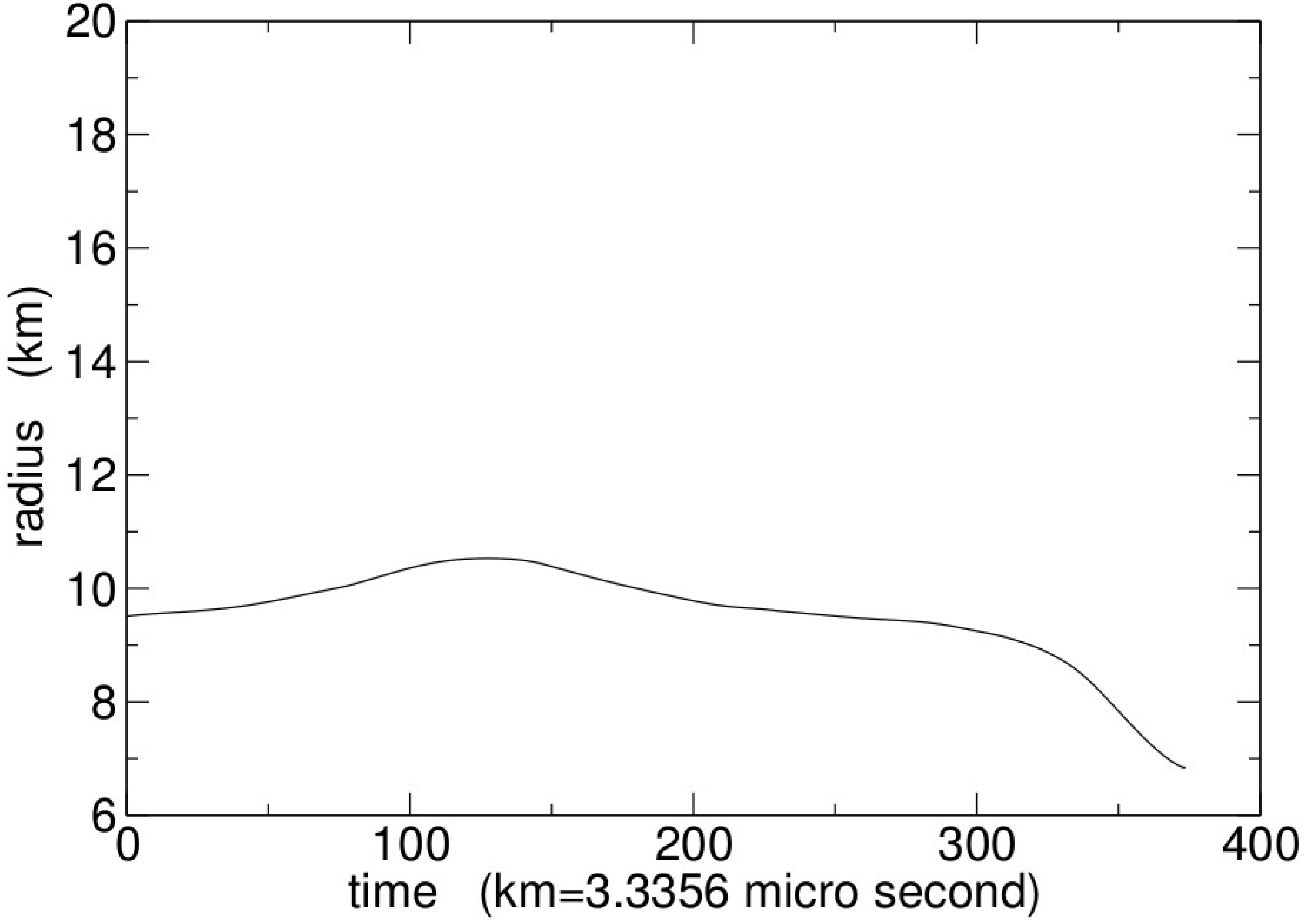
Maximum mass (solar mass)
Radius at maximum mass (km)
Central energy density at maximum mass (MeV/fm3)
Frequency of radial mode (n=0) at maximum radius (kHz)
ρ0 (MeV/fm3)
a

Critical phenomena

Maximum mass configuration (km): 2.984016494
Selected masses (km): 2.962963, 2.963477514, 2.9635508309, 2.96356153, 2.96358311021



Gravitational collapse to black hole, initially outgoing velocity



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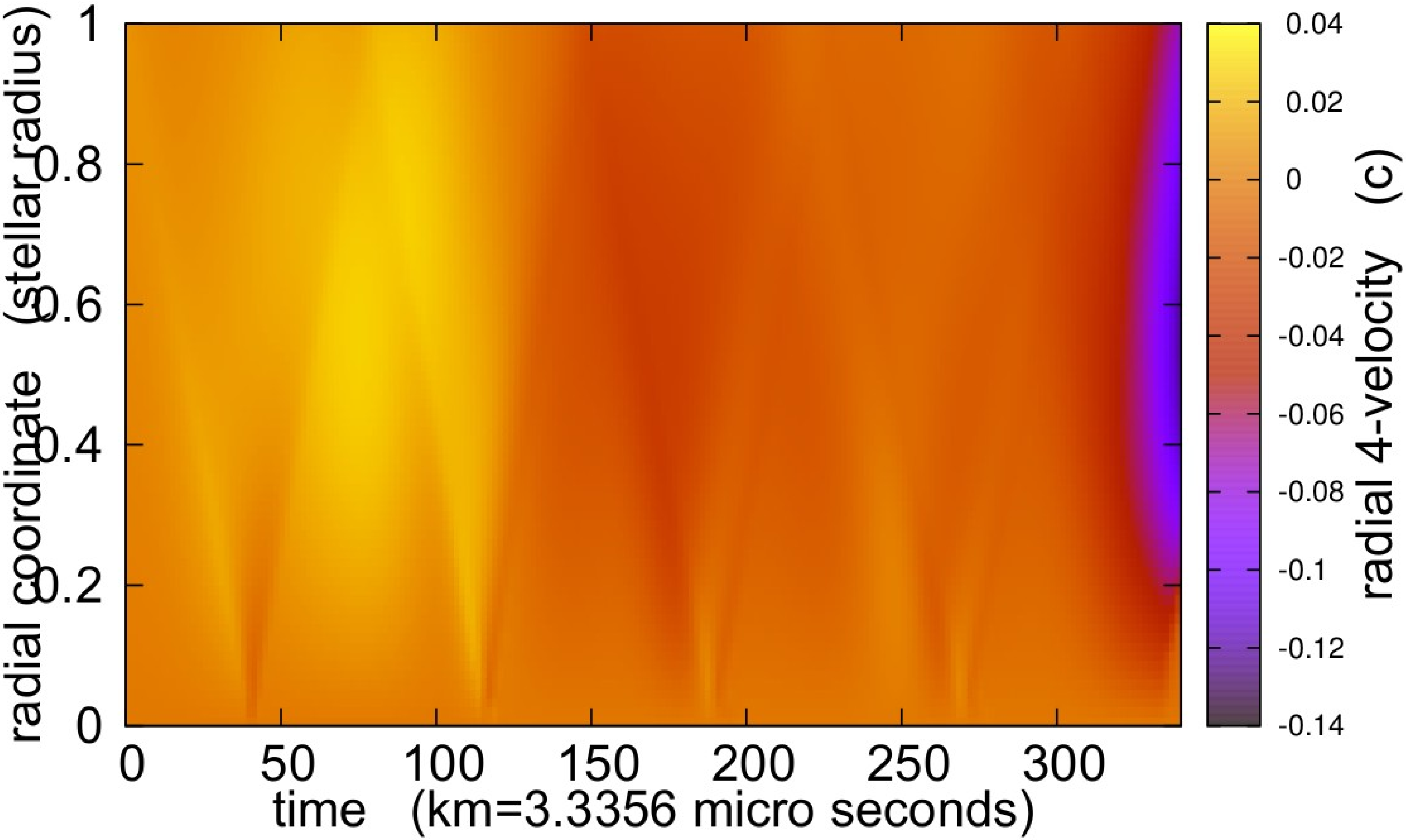

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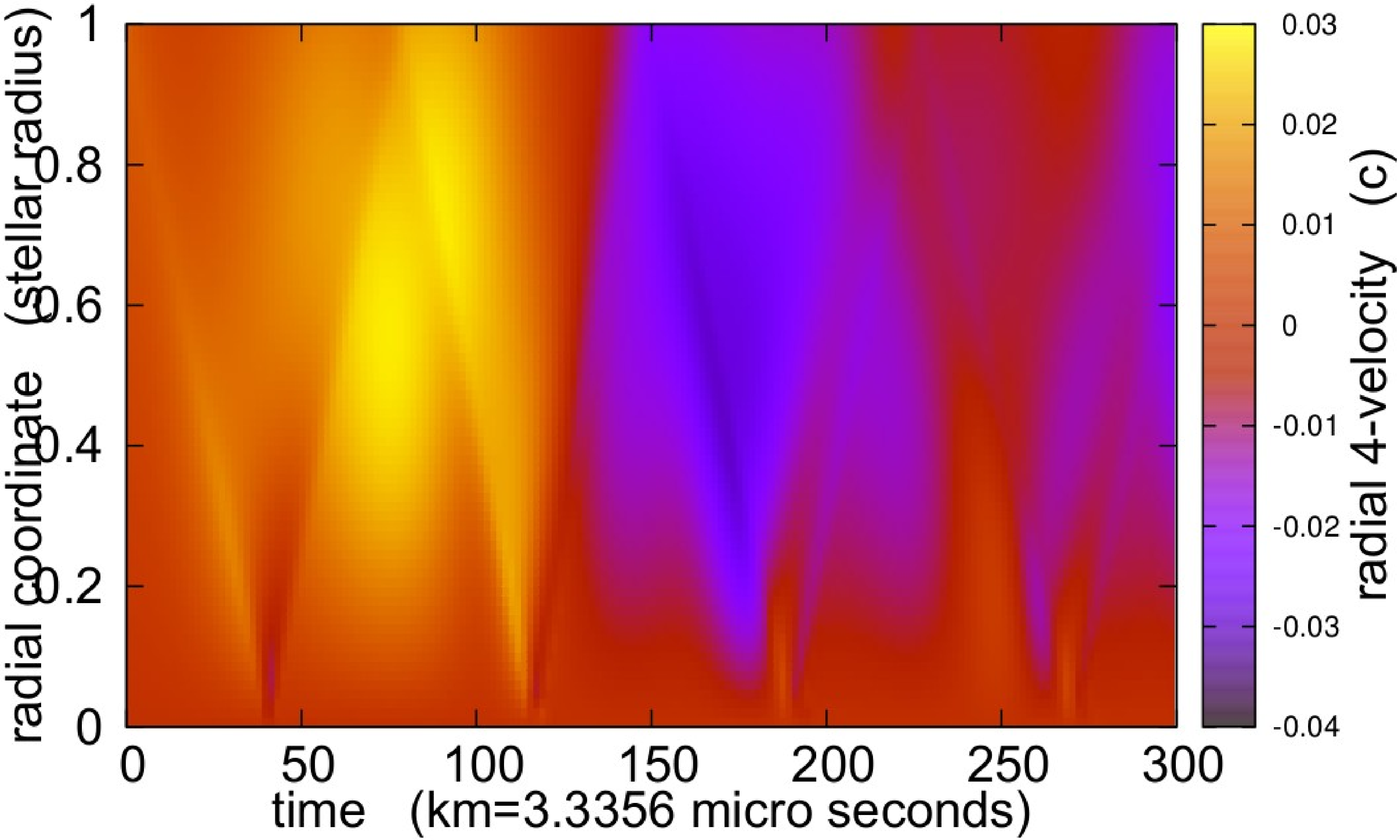
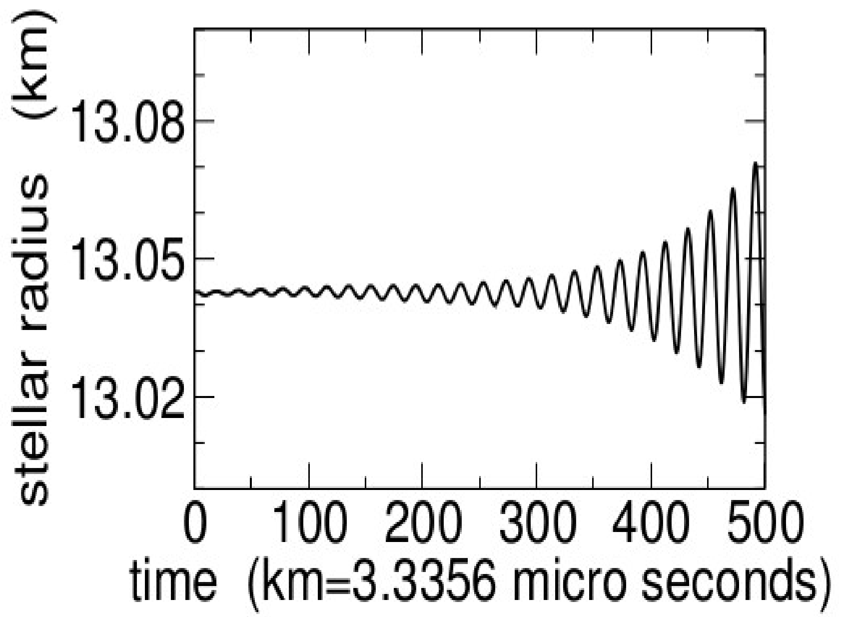


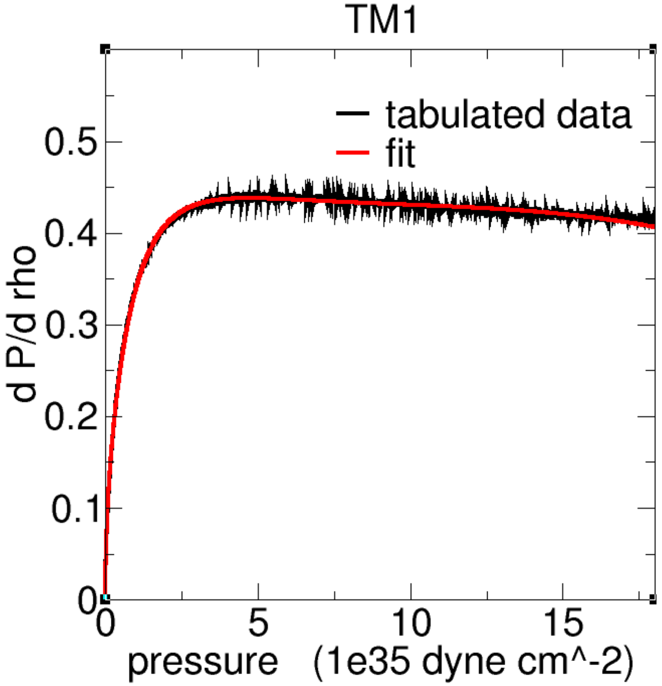
Parametrization for barotropic EoS

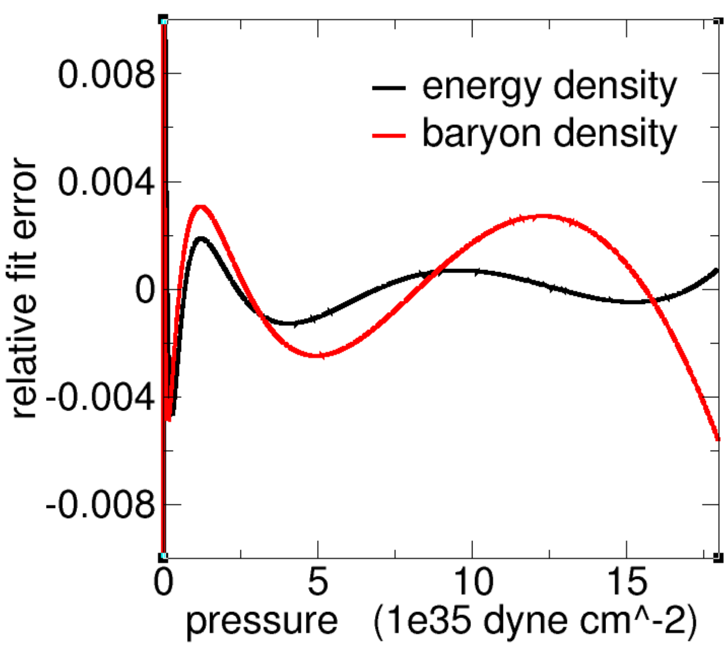
$$f = \sum_{k=1}^{3} \frac{a_{k1} + a_{k2}x + a_{k3}x^2}{\left(1 + e^{a_{k5}(a_{k6}+x)}\right)(1 + a_{k4}x)}$$

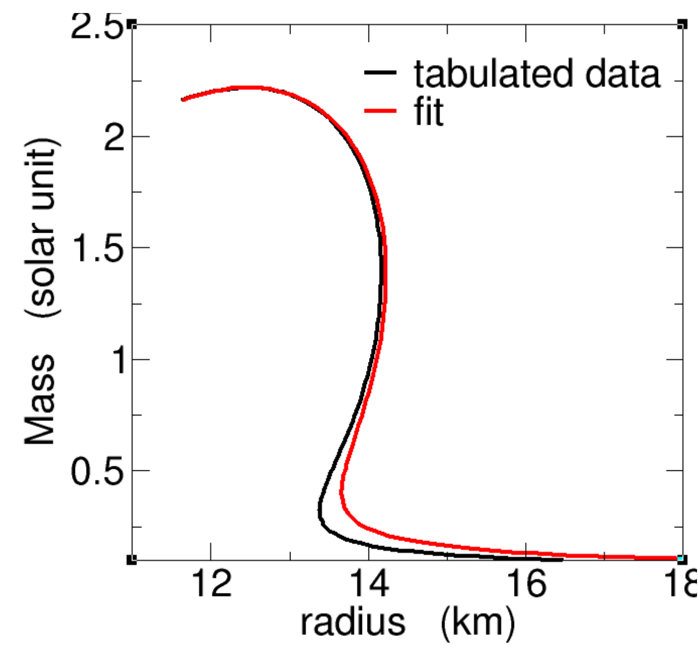
$$f = log(density)$$

$$x = log(pressure)$$









Results

- generalization of Chandrasekhar's equation to charge

- enhancement of masses of hybrid and strange stars due to Coulomb repulsion

- both Coulomb interaction and deconfinement lead to lower frequencies of radial eigenmodes at given central density

- A naïve application to stars with sharp density discontinuity contradicts with the *static stability criterion*.

Outlook

- Are there hollow charged spheres?

- introduce viscosity

- Critical phenomena with realistic microphysics

Radial oscillations in neutral and charged compact stars

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EMMI Workshop "Quark Matter in Compact Stars", FIAS, Frankfurt am Main, October 7-10, 2013

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Why care about charge in compact stars?

- even if the global charge is zero, there can be separation of charges inside / freedom to arrange charge within

- charge might prevent gravitational collapse and support supermassive stars

- charged balls might be natural candidates to form extremal black holes

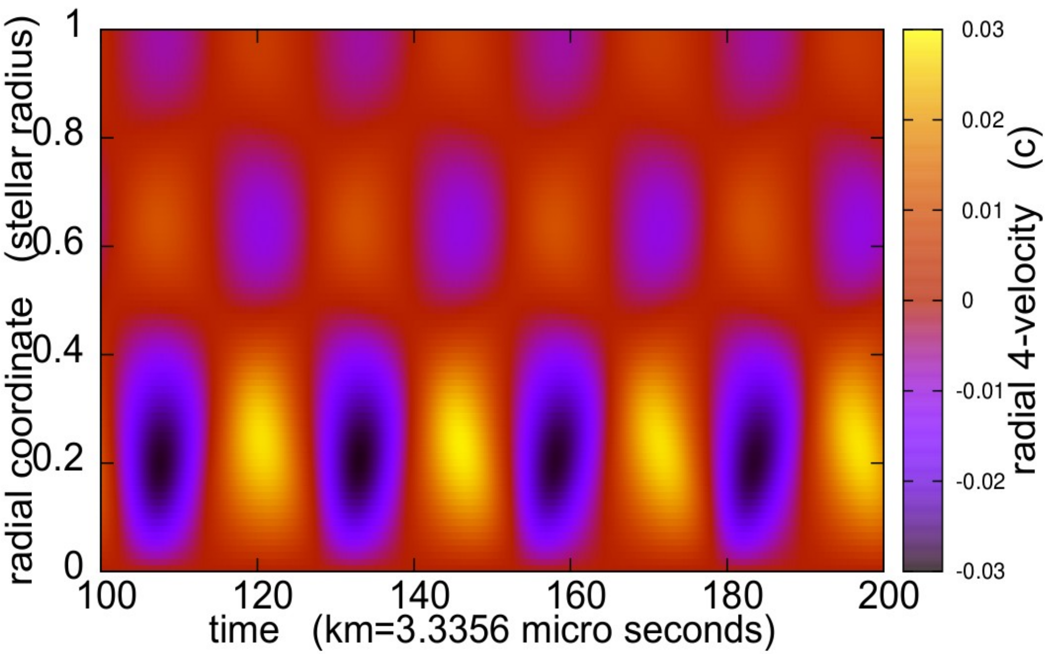
Radial oscillations

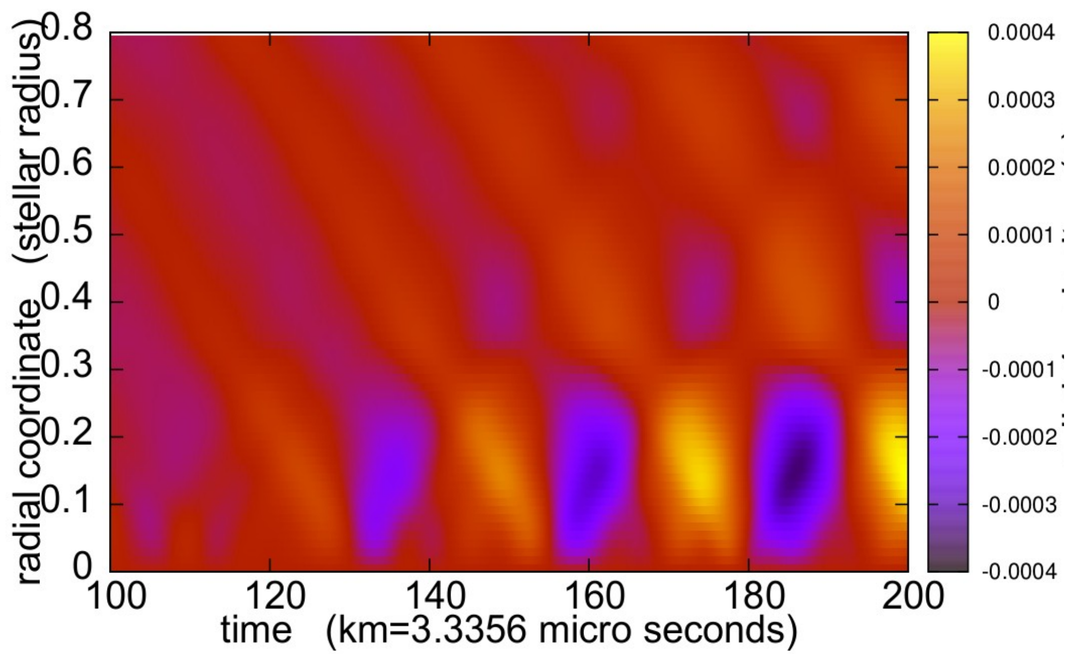
- spherical symmetry preserved

- type of oscillation described by number of nodes

- Sturm-Liouville equation as in Newtonian gravity

- discrete set of frequencies given by boundary conditions: $\xi(r=0)=0,\ \Delta P(r=R)=0$





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