Hybrid stars within a SU(3) chiral Quark Meson Model

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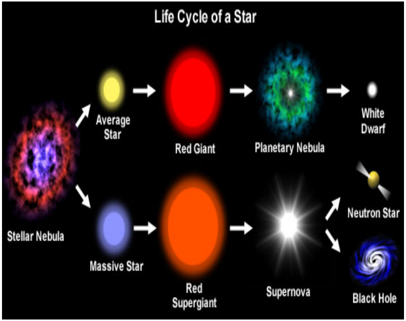
Astro coffee, May 24 2016

Abstract

1. Ultradense Matter in Compact Objects
2. Combining Micro and Macro
3. The Quark Meson model
4. Compact Stars
5. Summary and Outlook

Death of a star

- If the nuclear fuel is exhausted any stars life will end differently
- Stars with $M \leq 8M_\odot$ reject outer layers in a planetary nebula $\rightarrow$ White dwarf
- Stars with $M \geq 8M_\odot$ end in a Supernova explosion $\rightarrow$ Compact star
- Stars with $M \geq 20M_\odot$ end in a $\rightarrow$ Black hole



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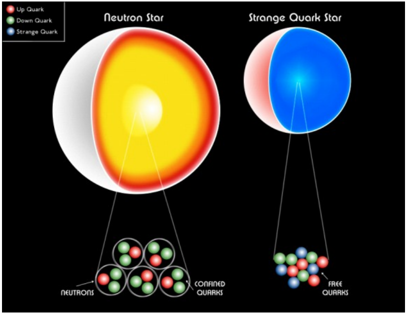
Supernova remnants: Compact stars

Compact stars are either:

- neutron stars (consisting of mainly neutrons)
- hybrid stars (consisting of a hadronic shell and a quark-matter core)
- quark stars (consisting of quark matter)
- Size: $R \approx 10 - 15\text{km}$
  Mass: $M \approx 1.5 M_\odot$
  Density:
  $\rho_0 \approx 2.5 \cdot 10^{14} \frac{g}{cm^3} \approx 145 \frac{MeV}{fm^3}$



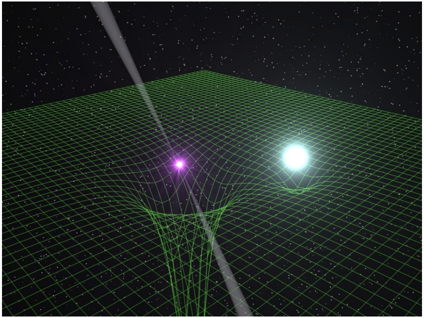
Credit: Chandra X-ray observatory (NASA)

New pulsar mass measurements

Recent measurements

1. Demorest et. al; 2010
   PSR J1614-2230 with
   $M = 1.97 \pm 0.04 M_{\odot}$
2. Antoniadis et. al; 2013
   PSR J0348+0432 with
   $M = 2.01 \pm 0.04 M_{\odot}$

set new constraints on thermodynamic quantities. Until 2010 the Hulse Taylor Pulsar with $M = 1.4411 \pm 0.00035 M_{\odot}$ was the heaviest.

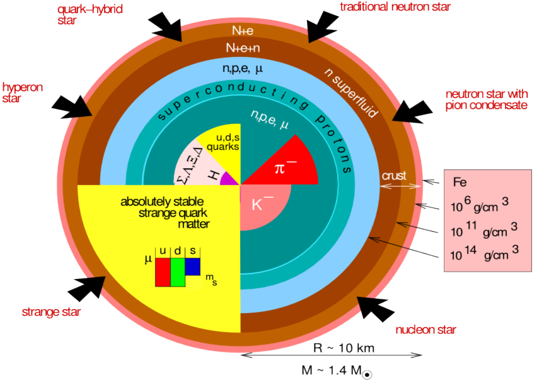


Credit: Science Magazine

The stars interior

What do certain models predict for the star to consist of?

1. ...just neutrons?
2. ...hyperons?
3. ...kaon condensates?
4. ...quark matter (QM)?
5. ...color superconducting QM?



Credit: Fridolin Weber

Micro: Calculation of an equation of state (EoS)

1. Polytropic EoS:
   - Fermi Gas - no interactions considered
   - $p(\epsilon) = K\epsilon^{\gamma}$ where $K = const.$ and e.g. $\gamma = \frac{4}{3}$;

   see: *Compact stars for undergraduates*, Sagert et. al 2005
2. Nuclear matter EoS:
   - respects n-n interactions
   - formation of clusters
   - density dependent

   see: *Composition and thermodynamics of nuclear matter with light clusters*, Typel et. al 2010
3. Quark matter EoS
   - Incorporation of QCD properties
   - Exchange of scalar- and vector mesons as mediators of strong interaction
   - Modelling confinement via vacuum energy density term B

   see: *Compact stars in a SU(3) Quark Meson Model*, Zacchi et. al 2015

Macro: How to compute compact stars

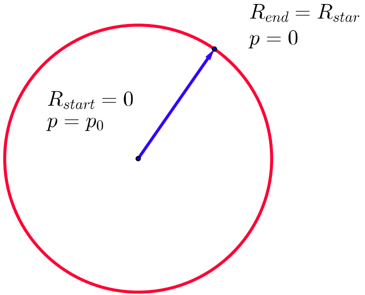
The Tolman-Oppenheimer-Volkoff equations (TOV) are:

- general relativistic equations to determine the mass-radius relations of compact stars
- Input is an Equation of State (EoS): $p(\epsilon)$ where $\epsilon(r)$

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \epsilon(r) \\ \frac{dp}{dr} &= -\frac{G\epsilon(r)m(r)}{r^2}\left(1+\frac{p(r)}{\epsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1} \end{aligned}$$

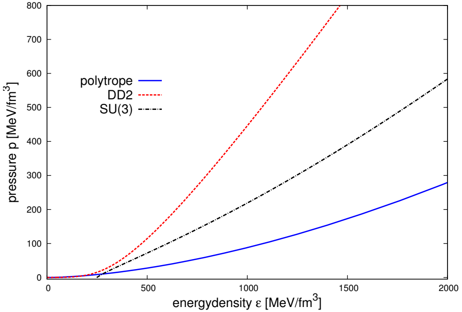
TOV equations: Boundary conditions

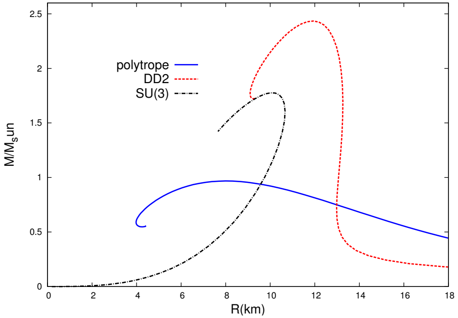
1. Start integration of the TOV equations at the center with boundary conditions $m(R_{start} = 0) = 0$
2. End integration of the TOV equations at the stars surface where the pressure vanishes $m(R_{end}) = M_{star}$



Combining Micro and Macro

From an EoS → TOV - equations → Mass-Radius Relations





Each EoS predicts a specific mass vs. radius line

- Quark stars: Selfbounded objects
- Neutron stars: Bounded by gravity

SU(3) Quark Meson Model

Setting up a realistic Quark model

Ultradense matter might be a phase of deconfined quarks

Computation

Compute the EoS and solve the TOV equations

Combination

Combine the QM EoS with a hadronic EoS (DD2) $\rightarrow$ Hybrid- and maybe even Twin stars

The SU(3) Lagrangian

Properties of hybrid stars depend on Quark Matter EoS derived by a Lagrangian density $\mathcal{L} = \mathcal{L}_{F_{n,s}} + \mathcal{L}_{\phi} + \mathcal{L}_{V}$

$$
\begin{aligned}
\mathcal{L}_{F_{n,s}} &= \bar{\Psi}_n \left(i\not{\partial} - g_\omega \gamma^0 \omega - g_\rho \vec{\tau} \gamma^0 \rho - g_n \sigma_n \right) \Psi_n \\
&+ \bar{\Psi}_s \left(i\not{\partial} - g_s \sigma_s - g_\phi \gamma^0 \phi \right) \Psi_s \\
\mathcal{L}_\phi &= tr(\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \lambda_1 [tr(\phi^\dagger \phi)]^2 - \lambda_2 tr(\phi^\dagger \phi)^2 \\
&- m_0^2 tr(\phi^\dagger \phi) - tr[\hat{H}(\phi + \phi^\dagger)] + c \left(\det(\phi^\dagger) + \det(\phi) \right) \\
\mathcal{L}_V &= -tr(\partial_\mu V)^\dagger (\partial^\mu V) - m_v^2 tr(V^\dagger V)
\end{aligned}
$$

The Equation of State: Solve the Lagrangian

With

$$\mathcal{Z} = \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{\left(\int_0^\beta d\tau \int_V d^3\vec{r}(\mathcal{L}+\bar{\Psi}\gamma^0\mu\Psi)\right)}$$

and

$$\begin{aligned} p &= \frac{ln\mathcal{Z}}{\beta} = -\Omega \\ \epsilon &= -p + \sum_{f=u,d,s} \mu_f n_f + Ts \end{aligned}$$

we have a relation for the necessary values.

EoS - Grandcanonical Potential Ω

Having performed the $T \to 0$ approximation the resulting grandcanonical potential is

$$\Omega = \mathcal{V} + \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^{k_F^f} dk \cdot k^2 \left(\sqrt{k_{n,s}^2 + \tilde{m}^2} - \tilde{\mu}_f \right)$$

where

$$\begin{aligned} \mathcal{V} &= -\frac{1}{2}\left(m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2\right) + \frac{\lambda_1}{4}(\sigma_n^2 + \sigma_s^2)^2 + \frac{\lambda_2}{4}(\sigma_n^4 + \sigma_s^4) \\ &+ \frac{m_0^2}{2}(\sigma_n^2 + \sigma_s^2) - \frac{2\sigma_n^2 \sigma_s}{\sqrt{2}} \cdot c - h_n \sigma_n - h_s \sigma_s + B \end{aligned}$$

The energy density and the pressure are then determined to

$$
\begin{aligned}
\epsilon &= \epsilon_e + \frac{\lambda_1}{4}(\sigma_n^2+\sigma_s^2)^2 + \frac{\lambda_2}{4}(\sigma_n^4+\sigma_s^4) + \frac{m_0^2}{2}(\sigma_n^2+\sigma_s^2) \\
&- \frac{2\sigma_n^2\sigma_s}{\sqrt{2}}\cdot c - h_n\sigma_n - h_s\sigma_s + B + \frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right) \\
&+ \frac{3}{\pi^2}\sum_{f=u,d,s}\int_0^{k_F^f} dk\cdot k^2\left(\sqrt{k_{n,s}^2+\tilde{m}^2}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
p &= -\frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right) + \frac{\lambda_1}{4}(\sigma_n^2+\sigma_s^2)^2 + \frac{\lambda_2}{4}(\sigma_n^4+\sigma_s^4) \\
&+ \frac{m_0^2}{2}(\sigma_n^2+\sigma_s^2) - \frac{2\sigma_n^2\sigma_s}{\sqrt{2}}\cdot c - h_n\sigma_n - h_s\sigma_s + B \\
&+ \frac{3}{\pi^2}\sum_{f=u,d,s}\int_0^{k_F^f} dk\cdot k^2\left(\sqrt{k_{n,s}^2+\tilde{m}^2} - \tilde{\mu}_f\right)
\end{aligned}
$$

Lagrangian parameters

- The vacuum expectation values

$$\begin{aligned}\langle\sigma_n\rangle &= f_\pi = 92.4 MeV\\ \langle\sigma_s\rangle &= \frac{2f_K - f_\pi}{\sqrt{2}} = 94.47 MeV\end{aligned}$$

- Scalar and vector couplings

$$\begin{aligned}g_n &= \frac{m_q}{f_\pi} \quad with \quad m_q = 300\ MeV\\ g_s &= \sqrt{2} g_n\\ g_\omega &= g_\rho = \frac{g_\phi}{\sqrt{2}}\end{aligned}$$

Lagrangian parameters

- Spontaneous breaking controlled by λ_1 via m_0^2, m_V^2 and m_σ λ_2 from meson masses (m_π,m_K,...) and decay constants (PDG)
- Such as c, which describes axial anomaly ($m_{\eta'}$)
- Explicit symmetry breakers

$$h_n = f_\pi m_\pi^2$$
$$h_s = \sqrt{2} f_K m_K^2 - \frac{h_n}{\sqrt{2}}$$

Parameter space

Constituent quark mass m_q

$g_n = \frac{m_q}{f_\pi}$ and $g_s = g_n\sqrt{2}$, where g_s is adopted from SU(3) symmetry considerations.

Vector coupling $g_\omega \sim g_n$

The ϕ-meson coupling is also fixed by SU(3) symmetry

Mass of the σ-meson

m_σ covers a range from 400 MeV $\leq m_\sigma \leq$ 800 MeV

The Bag constant B

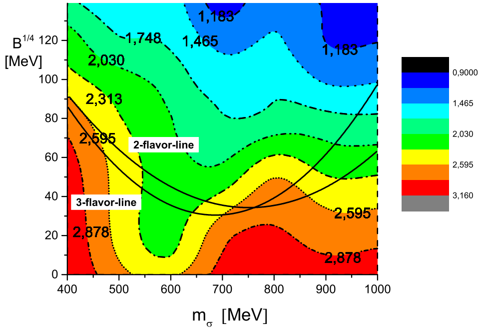
B models the confinement; 60 MeV $\leq B \leq$ 200 MeV.

Pure SU(3) quark stars within a tiny parameter space

Contour plot of vacuum pressure B vs. Sigma meson mass

1. Above the 2-flavour-line: Iron, i.e. hadronic matter more stable
2. Below the 3-flavour-line: Pure quark matter more stable

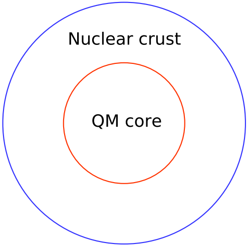
Pure quark-stars with $M_\odot \geq 2$ possible within a narrow area!

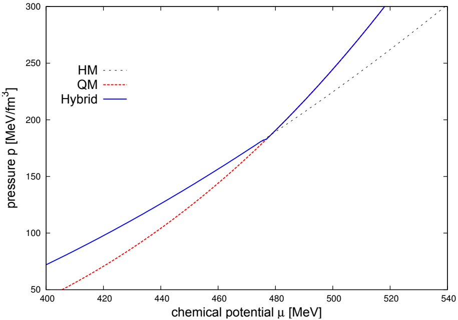


The other papameters:
$m_q = 300$ and $g_\omega = 2.0$

Hybrid stars: Maxwell construction

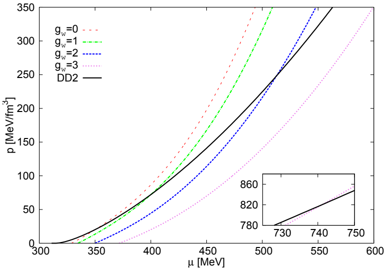
Combining a nuclear matter EoS and a Quark matter EoS:
Pressure p has to be dominant vs. chem. Potential μ

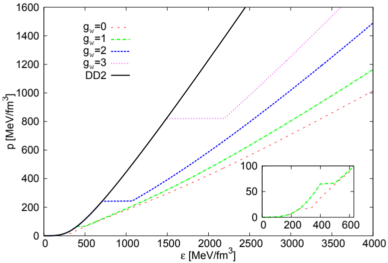




Results for $0 \leq g_\omega \leq 3$

From the intersecting point in the $p - \mu$ plane the EoS changes from the HM EoS to the QM EoS



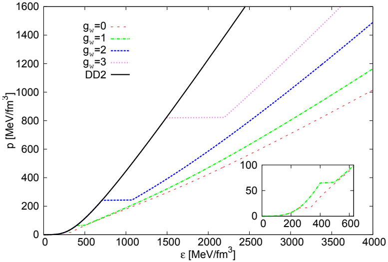


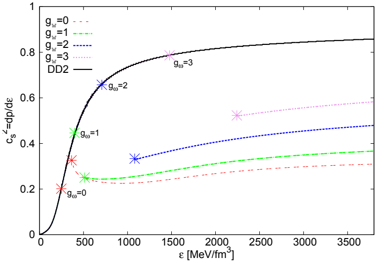
$$m_q = 300 \text{ MeV}, \ m_\sigma = 600 \text{ MeV}, \ B = 100 \text{ MeV}$$

The corresponding Speed of Sound (SoS)

The density within the star changes abruptly

$m_q = 300$ MeV, $m_\sigma = 600$ MeV, $B = 100$ MeV

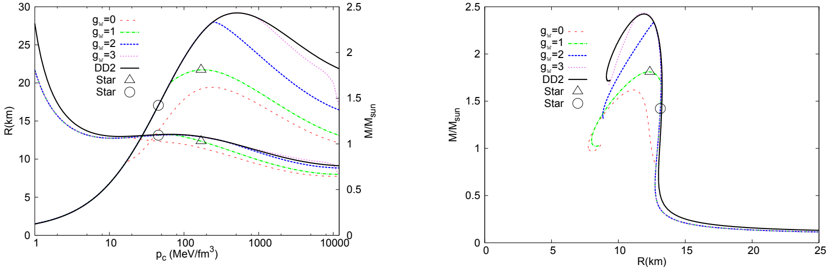




p_{trans} : transition pressure
ϵ_{trans} : transition energy density
$\Delta\epsilon$: Difference in energy density between the two EoS

Results for $0 \leq g_\omega \leq 3$

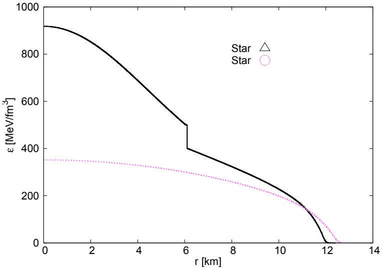
At a certain central pressure the star configurations get unstable

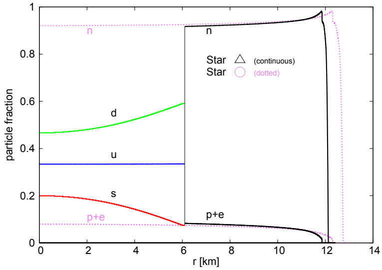


$$m_q = 300 \text{ MeV},\ m_\sigma = 600 \text{ MeV},\ B = 100 \text{ MeV}$$

Results for $0 \leq g_\omega \leq 3$

Particle composition of two individual stars:
$\triangle$(hybrid $1.8 M_\odot$) and $\bigcirc$(hadronic $1.4 M_\odot$)

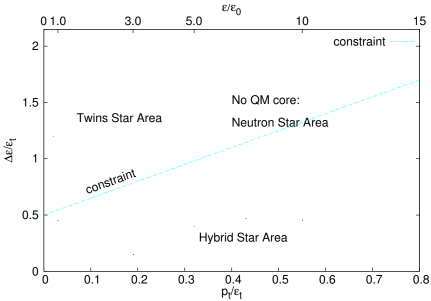




$m_q = 300$ MeV, $m_\sigma = 600$ MeV, $B = 100$ MeV

Criterion for stable hybrid stars

$$\frac{\Delta\epsilon_{crit}}{\Delta\epsilon} = \frac{1}{2} + \frac{3}{2}\frac{p_{trans}}{\epsilon_{trans}}$$



- Effect of QM core on the star is determined by $\Delta\epsilon$
- Small $\Delta\epsilon$: Quark matter has similar energydensity than nuclear matter
- Large $\Delta\epsilon$: Star becomes unstable, since QM-core is unable to counteract gravitational attraction

Twin stars: Same mass - different radii

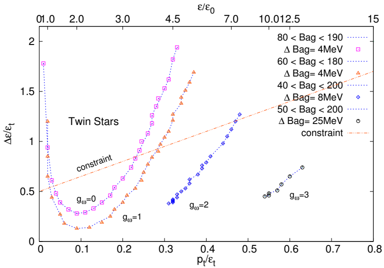
Considering a perturbation causing the star to collapse:

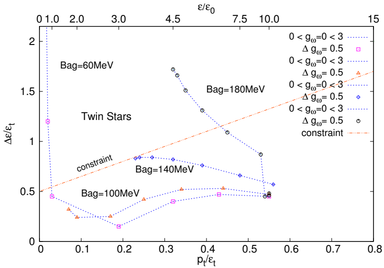
Possible scenarii:

1. Star collapses into a black hole
2. Star stabilizes again

One EoS may yield two stable branches.

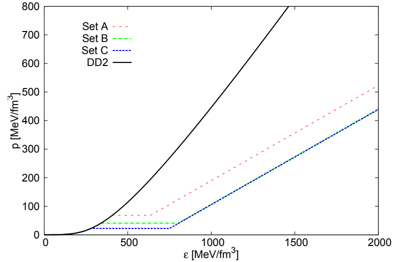
Twin stars: Hard to find

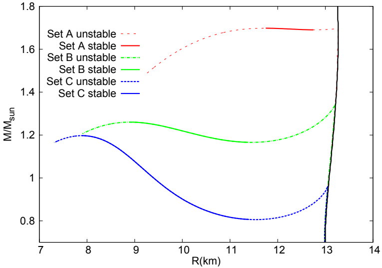




$$m_q = 300 \text{ MeV and } m_\sigma = 600 \text{ MeV}$$

The influence of the speed of sound



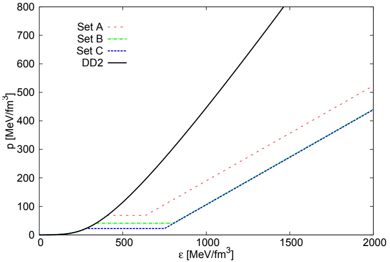


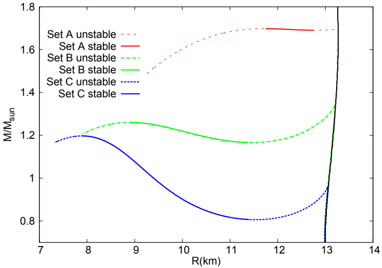
Twin Star Area scanned with SoS dependent EoS

$$p(\epsilon) = c_s^2 \left(\epsilon - \epsilon_*\right) \ , \quad \text{with: } \epsilon_* := \epsilon_t + \Delta\epsilon - \frac{1}{c_s^2} p_t \, ,$$

p_t/ϵ_t and $\Delta\epsilon/\epsilon_t$ under direct influence, $c_s^2 = \frac{1}{3}$

The influence of the speed of sound





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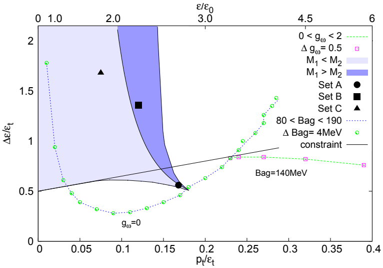
p_t/ϵ_t and $\Delta\epsilon/\epsilon_t$ under direct influence, $c_s^2 = \frac{1}{3}$

Twin stars with $2M_\odot$ hard to model



Star sequence	p_t/ϵ_t	$\Delta\epsilon/\epsilon_t$	M_1	R_1	M_2	R_2
● Set A	0.17	0.56	1.69	13.26	1.70	11.72
■ Set B	0.12	1.36	1.35	13.21	1.26	8.91
▲ Set C	0.08	1.68	0.96	13.05	1.20	7.90

Conclusions on Hybrids

Hybrid stars

1. m_q and m_σ show little effect.
2. Large g_ω and B:
   $\rightarrow 2M_\odot$ but tiny QM core, star configuration soon unstable
3. Small g_ω and B:
   $\rightarrow$ less then $2M_\odot$ with acceptable QM core, hybrid star configuration remarkably stable
4. No direct influence on p_t and ϵ_t

Conclusions on Twins

Twin stars

1. Twins within our model hard to achieve, swerve on speed of sound (SoS) independent EoS
2. Parametrization via SoS of the EoS leads to Twin stars: Large c_s^2 advantageous for Twins
3. Chances are best for low transition pressure p_t and large jump in energydensity $\Delta\epsilon$
   $\rightarrow$ The appearing QM core does not destabilize the star
4. Twin stars explain the two component structure of Short Radio Bursts

Neutron star merger, both stars with 1.4 $M_{\odot}$

Animation by Filippo Galeazzi, Goethe University Frankfurt

Summary and Outlook

Summary

1. Combination of Hadronic and Quark Matter EoS $\rightarrow$ solve the TOV equations
2. Maxwell construction to study hybrid stars
3. Do the Mass Radius relations exhibit $2M_{\odot}$ Twin Star solutions ?
4. Generally: Twin Star solutions $\sim 2M_{\odot}$ hard to find

Outlook

1. More properties - more sophisticated QM-EoS, e.g. inclusion of self enery loops?
2. Gibbs construction
3. Finite T calculation (Supernova EoS)
4. Cooling process via neutrino emission
5. Compact star merger
6. Gravitational wave emission