Compact Stars within a SU(3) chiral Quark Meson Model

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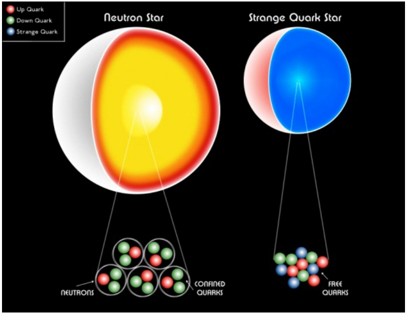
Astro coffee, January 17 2017

Abstract

Supernova remnants: Compact stars

Compact stars are either:

- Neutron stars (consisting mainly of neutrons)
- Hybrid stars (consisting of a hadronic shell and a quark-matter core)
- Quark stars (consisting of quark matter only)
- Size: $R \approx 10 - 15$km
  Mass: $M \approx 1.5 M_{\odot}$
  Density:
  $\rho_0 \approx 2.5 \cdot 10^{14} \frac{g}{cm^3} \approx 145 \frac{MeV}{fm^3}$

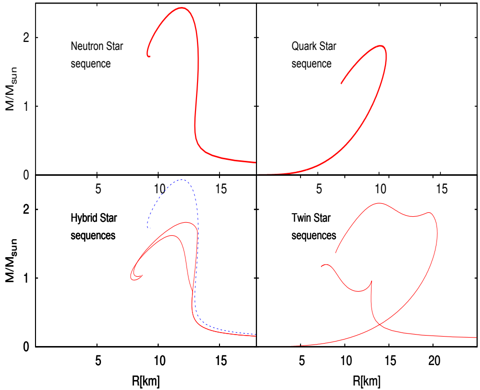


Credit: Chandra X-ray observatory (NASA)

Different types of compact stars

Characteristic mass radius relations of

1. Neutron Stars ($M \propto R^{-3}$)
2. Quark Stars ($M \propto R^{3}$)
3. Hybrid Stars (Can masquerade as Neutron star)
4. Twin Stars (Two stable branches)

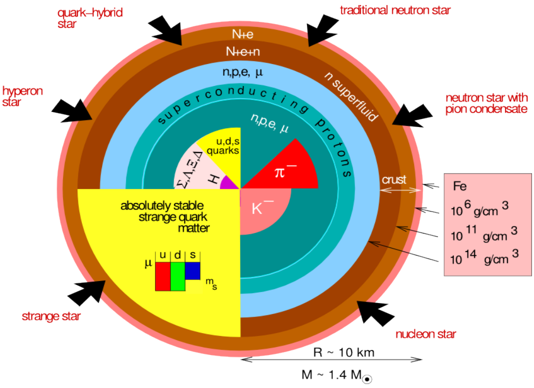


The stars interior

What do certain models predict for the star to consist of?

1. ...just neutrons?
2. ...hyperons?
3. ...kaon condensates?
4. ...quark matter (QM)?
5. ...color superconducting QM?

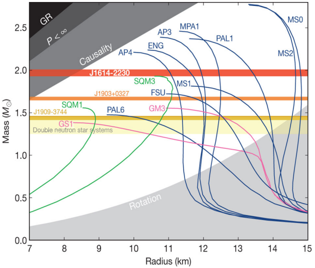
→ Equation of state (EoS)



Credit: Fridolin Weber

Constraints on different solutions

1. Schwarzschild Radius $\rightarrow R > 2GM/c^2$
2. $P < \infty \rightarrow R > 9GM/4c^2$
3. Causality $\rightarrow R > 2.9GM/c^2$
4. Star can not spin faster than 716Hz
   $\rightarrow \frac{1}{\nu} \simeq (0.96 \pm 0.03) \left(\frac{M_\odot}{M_{nr}}\right)^{1/2} \left(\frac{R_{nr}}{10\text{km}}\right)^{3/2}$
5. Solutions need to fulfill the $2M_\odot$ constraint
   $\rightarrow$ next slide



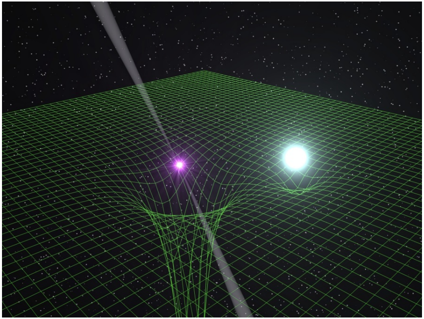
Credit: Lattimer and Prakash

New pulsar mass measurements

Recent measurements

1. Demorest et. al; 2010
   PSR J1614-2230 with
   $M = 1.97 \pm 0.04 M_{\odot}$
2. Fonseca et. al; 2016
   PSR J1614-2230 with
   $M = 1.928 \pm 0.017 M_{\odot}$
3. Antoniadis et. al; 2013
   PSR J0348+0432 with
   $M = 2.01 \pm 0.04 M_{\odot}$

...also a constraint for quark matter based stars?



Credit: Science Magazine

Skalar and vector nonet

Skalar and vector nonet as a base to constuct Lagrangian density

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_n + a_0^0}{\sqrt{2}} & a_0^+ & K_s^+ \\ a_0^- & \frac{\sigma_n - a_0^0}{\sqrt{2}} & K_s^0 \\ K_s^- & \bar{K}_s^0 & \sigma_s \end{pmatrix}$$

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_n^\mu + \rho^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} & K^{*\mu +} \\ \rho^{\mu -} & \frac{\omega_n^\mu - \rho^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} \\ K^{*\mu -} & \bar{K}^{*\mu 0} & \phi^\mu \end{pmatrix}$$

The SU(3) Quark Meson Model

SU(3) Lagrangian $\mathcal{L} = \mathcal{L}_{F_{n,s}} + \mathcal{L}_{\varphi} + \mathcal{L}_V$

$$
\begin{aligned}
\mathcal{L}_{F_{n,s}} &= \bar{\Psi}_{n,s}\left(i\partial\!\!\!/ - g_\omega\gamma^0\omega - g_\rho\vec{\tau}\gamma^0\rho - g_\phi\gamma^0\phi - g_{n,s}\sigma_{n,s}\right)\Psi_{n,s} \\
\mathcal{L}_{\varphi} &= tr(\partial_\mu\varphi)^\dagger(\partial^\mu\varphi) - \lambda_1[tr(\varphi^\dagger\varphi)]^2 - \lambda_2 tr(\varphi^\dagger\varphi)^2 \\
&- m_0^2 tr(\varphi^\dagger\varphi) - tr[\hat{H}(\varphi+\varphi^\dagger)] + c\left(\det(\varphi^\dagger)+\det(\varphi)\right) \\
\mathcal{L}_V &= tr(\partial_\mu V)^\dagger(\partial^\mu V) - m_v^2 tr(V^\dagger V)
\end{aligned}
$$

- couples quarks to mesons via Yukawa type coupling
- effective quark masses generated by the σ_n and σ_s-fields
- ω, ρ and ϕ: vector mesons as repulsive mediators

Symmetries of QCD realized in the SU(3) Quark Meson Model

$\mathcal{L} = \mathcal{L}_{F_{n,s}} + \mathcal{L}_{\varphi} + \mathcal{L}_{V}$ respects symmetries of QCD

Apart from color- and flavour symmetry, $\mathcal{L}$ exhibits chiral symmetry

- spontaneously broken due to chiral condensate $< \bar{\Psi}_{n,s}\Psi_{n,s} >$
- explicitly broken due to non vanishing quark masses $m_q \neq 0$

Restoration of chiral symmetry

- Chiral symmetry restored at high densities, i.e. chemical potential μ_q
- Quarks (nearly) massless
- Right handed and left handed quarks are (nearly) indistinguishable

The EoS from the SU(3) Quark Meson model at $T = 0$

Once the pressure is known...

$$\begin{aligned} p = \; - \; & \frac{\lambda_1}{4}(\sigma_n^2 + \sigma_s^2)^2 - \frac{\lambda_2}{4}(\sigma_n^4 + \sigma_s^4) - \frac{m_0^2}{2}(\sigma_n^2 + \sigma_s^2) + \frac{\sigma_n^2 \sigma_s}{2\sqrt{2}} c \\ & + h_n \sigma_n + h_s \sigma_s - B + \frac{1}{2}\left(m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2\right) \\ & - \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^{k_F^f} dk \cdot k^2 \left(\sqrt{k^2 + \tilde{m}_f^2} - \tilde{\mu}_f\right) \end{aligned}$$

... the energy density follows from the Gibbs-Duhem relation, $\Omega = \epsilon + \sum_f \mu_f n_f$, where $n_f = (k_F^f)^3/\pi^2$ is the density associated to each quark flavour.

Important Parameters of the model

Repulsive vector meson coupling $g_\omega = g_\rho = \sqrt{2} g_\phi$

Models the repulsive interaction ($0 \leq g_\omega \leq 10$)

The Bag constant $B^{1/4}$

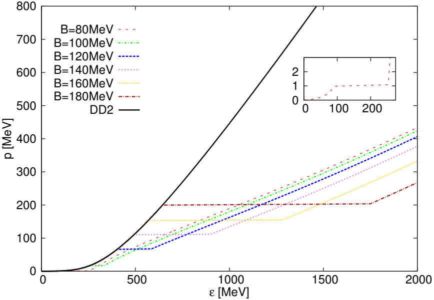
$B^{1/4}$ acts as an additive vacuum pressure ($0 \leq B^{1/4} \leq 150$ MeV)
(Depending on the used model)

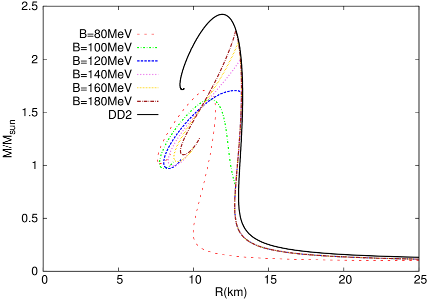
Sigma meson mass m_σ experimentally not well determined

Variation: $400 \leq m_\sigma \leq 1000$ MeV

Hybrid stars: Maxwell construction

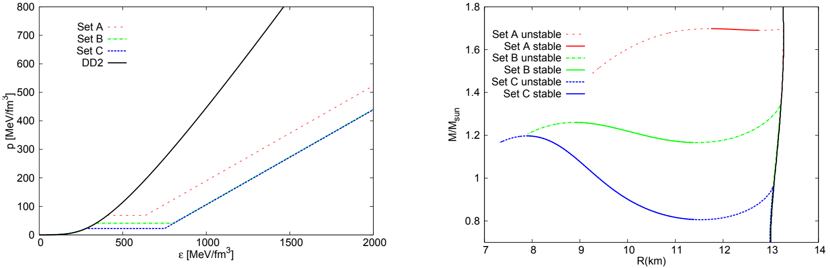
SU(3)-EoS for the ultradense core and DD2-EoS for the stars outer layer





The other papameters:
$m_q = 300$ MeV, $m_\sigma = 600$ MeV and $g_\omega = 0$

Hybrid Stars → Twin Stars



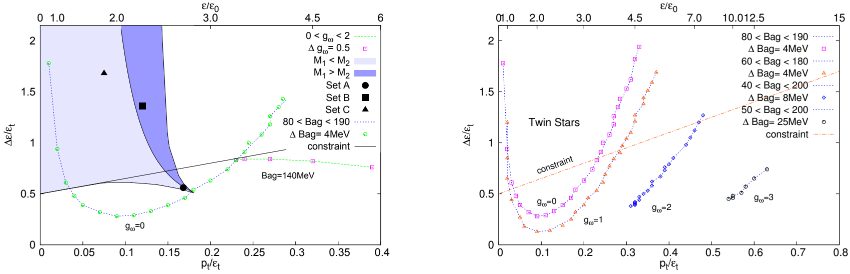
Parametrizied quark matter EoS yields Twin Stars

$$p(\epsilon) = c_s^2 (\epsilon - \epsilon_*) \ , \quad \text{with: } \epsilon_* := \epsilon_t + \Delta\epsilon - \frac{1}{c_s^2} p_t \ ,$$

Transition pressure p_t and jump in energy density $\Delta\epsilon$ under direct influence ($c_s^2 = \frac{1}{3}$)

Twin stars hard to find in microscopic modelling

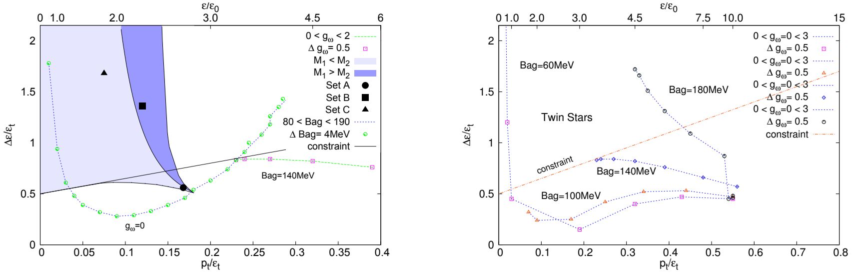
SU(3)-EoS for the ultadense core and DD2-EoS for the stars crust do not yield (reasonable) Twin Star solutions



Blue shaded area: Twin Star region

Twin stars hard to find in microscopic modelling

SU(3)-EoS for the ultadense core and DD2-EoS for the stars crust do not yield (reasonable) Twin Star solutions



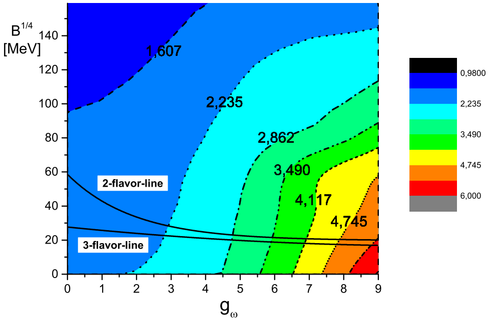
Blue shaded area: Twin Star region

Quark Stars: Stable?

Contour plot of vacuum pressure $B^{1/4}$ vs. repulsive coupling g_ω

Bodmer Witten hypothesis

1. Above the 2-flavour-line: Iron, i.e. hadronic matter more stable $\frac{E}{A}\Big|_{u,d} \geq 311$ MeV
2. Below the 3-flavour-line: Pure quark matter more stable $\frac{E}{A}\Big|_{u,d} \leq 311$ MeV



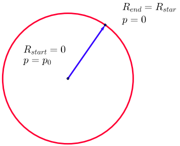
The other papameters:
$m_q = 300$ MeV and $m_\sigma = 600$ MeV

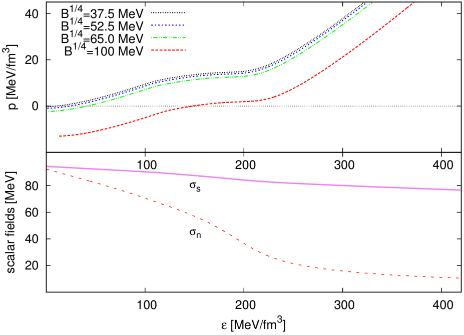
Nontrivialities in the SU(3)-EoS $\rightarrow$ Twin Stars?

Different values of the vacuum energy term $B^{1/4}$...

1. ...shifts the EoS in positive pressure range
2. ...crossover chiral phase transition

Small B:
Nontriviality in positive pressure range!

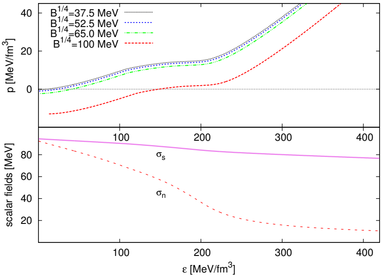


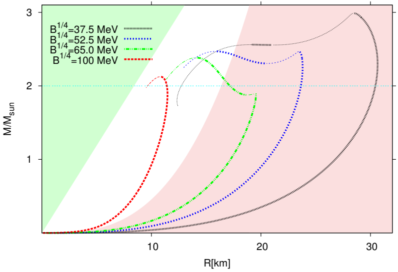


The other papameters:
$m_\sigma = 600$ MeV and $g_\omega = 4.0$

SU(3) Twin stars and constraints

Shaded areas are excluded

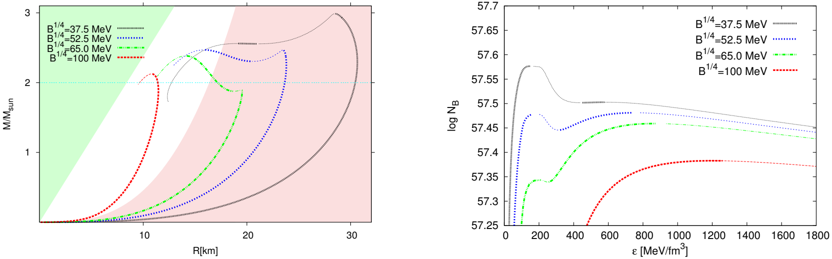




small $B \rightarrow$ large and heavy stars (but excluded by rotation)
($m_\sigma = 600$ MeV and $g_\omega = 4$)

Baryon number conservation

Perturbation might cause collapse from one to the other branch
($2M_\odot$ constraint)

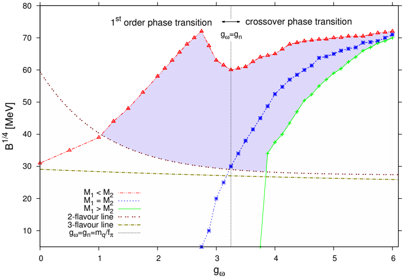


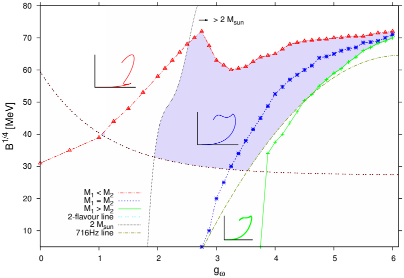
$B^{1/4} = 52.5$ MeV has a sibling at same baryon number
($m_\sigma = 600$ MeV and $g_\omega = 4$)

Contour lines of Twin stars

Shaded areas are allowed

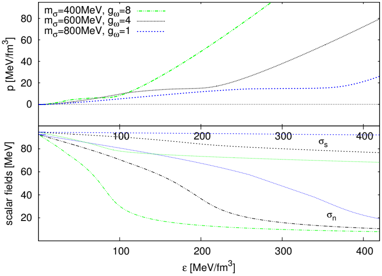
$m_\sigma = 600$ MeV $g_\omega = 4$

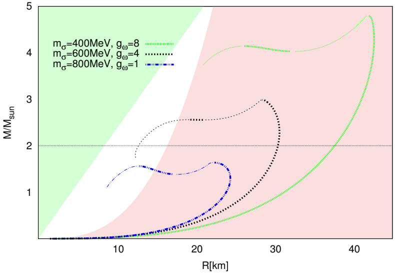




Relatively large parameter space where all constraints are fulfilled

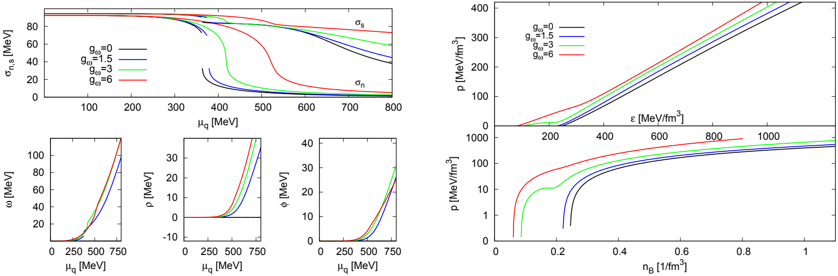
Varying m_σ and g_ω at $B^{1/4} = 37.5$ MeV





small m_σ needs relatively large g_ω $\rightarrow$ EoS stiff $\rightarrow$ large M and R
large m_σ needs relatively small g_ω $\rightarrow$ EoS soft $\rightarrow$ small M and R

The EoS at $T = 20$ MeV for different repulsive coupling



From $T = 0 \rightarrow T = 10$ MeV the radius increases $\sim 10 - 20\%$,
the mass is nearly unaffected - work in progress...

Neutron star merger, both stars with 1.4 $M_\odot$

Animation by Matthias Hanauske

Summary and Outlook

Summary

1. SU(3) Quark Matter EoS
   → Hybrid Star solutions
   → ...find Twin Star solutions..
2. Twins hard to find in microscopic modelling
3. Do these stars fulfill the 716Hz constraint?
4. Stability of quark matter?
5. Relatively large parameter space allowed for $m_\sigma = 600$ MeV
6. Interplay of m_σ and g_ω determine "nonlinearity" in the EoS
7. Twin Stars-from one EoS only - crossover chiral phase transition

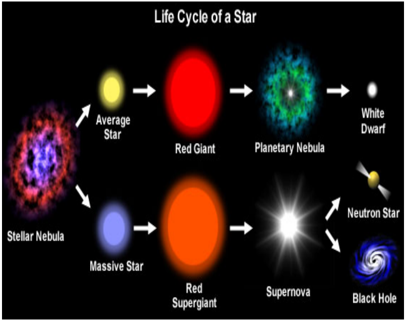
Outlook

1. NICER experiment (Radius measurement of compact stars)
2. Simulation of a collaps from first stable branch to second stable branch
3. Finite T calculation (Supernova EoS) - nontriviality already located - work in progress
4. Cooling process via neutrino emission
5. Compact star merger and gravitational wave emission (LISA)

back up slides

Death of a star

- If the nuclear fuel is exhausted any stars life will end differently
- Stars with $M \leq 8M_{\odot}$ eject outer layers in a planetary nebula $\rightarrow$ White dwarf
- Stars with $M \geq 8M_{\odot}$ end in a Supernova explosion $\rightarrow$ Compact star
- Stars with $M \geq 20M_{\odot}$ end in a $\rightarrow$ Black hole



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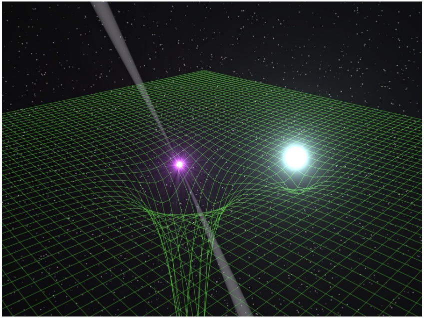
New pulsar mass measurements

Recent measurements

1. Demorest et. al; 2010
   PSR J1614-2230 with $M = 1.97 \pm 0.04 M_{\odot}$
2. Antoniadis et. al; 2013
   PSR J0348+0432 with $M = 2.01 \pm 0.04 M_{\odot}$

set new constraints on thermodynamic quantities. Until 2010 the Hulse Taylor Pulsar with $M = 1.4411 \pm 0.00035 M_{\odot}$ was the heaviest.



Credit: Science Magazine

Neutron star merger, both stars with 1.4 $M_\odot$

Animation by Filippo Galeazzi

Macro: How to compute compact stars

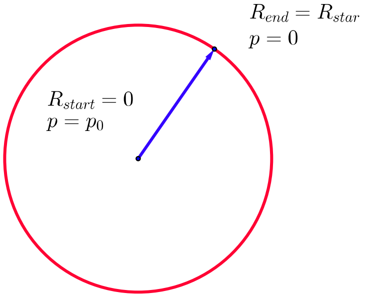
The Tolman-Oppenheimer-Volkoff equations (TOV) are:

- general relativistic equations to determine the mass-radius relations of compact stars
- Input is an Equation of State (EoS): $p(\epsilon)$ where $\epsilon(r)$

$$
\begin{aligned}
\frac{dm}{dr} &= 4\pi r^2 \epsilon(r) \\
\frac{dp}{dr} &= -\frac{G\epsilon(r)m(r)}{r^2}\left(1+\frac{p(r)}{\epsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}
\end{aligned}
$$

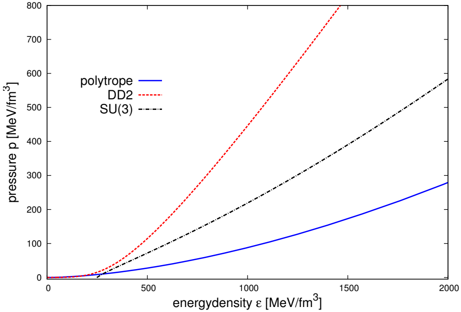
TOV equations: Boundary conditions

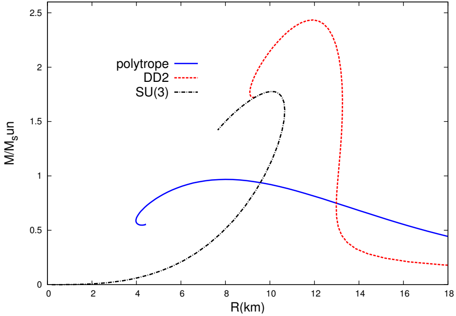
1. Start integration of the TOV equations at the center with boundary conditions $m(R_{start} = 0) = 0$
2. End integration of the TOV equations at the stars surface where the pressure vanishes $m(R_{end}) = M_{star}$



Combining Micro and Macro

From an EoS $\rightarrow$ TOV - equations $\rightarrow$ Mass-Radius Relations



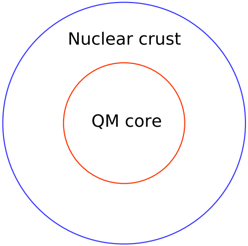


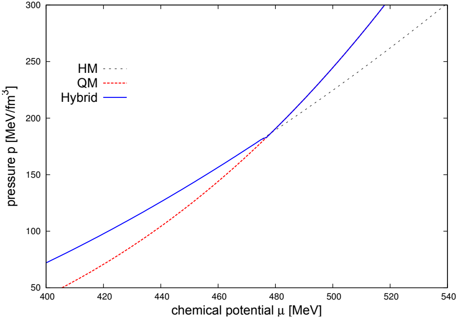
Each EoS predicts a specific mass vs. radius line

- Quark stars: Selfbounded objects
- Neutron stars: Bounded by gravity

Hybrid stars: Maxwell construction

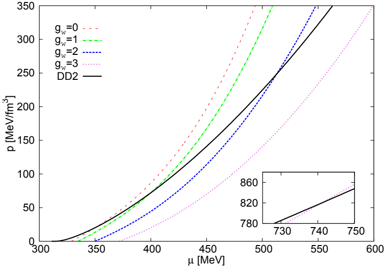
Combining a nuclear matter EoS and a Quark matter EoS:
Pressure p has to be dominant vs. chem. Potential μ

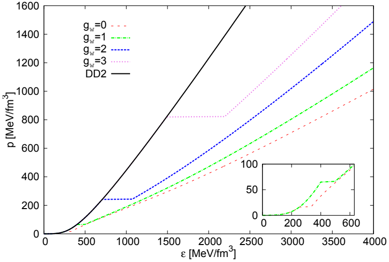




Results for $0 \leq g_\omega \leq 3$

From the intersecting point in the $p - \mu$ plane the EoS changes from the HM EoS to the QM EoS

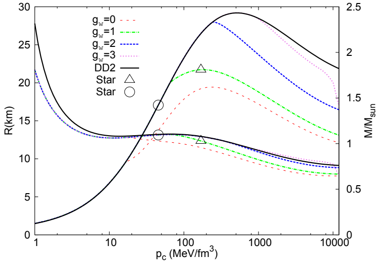


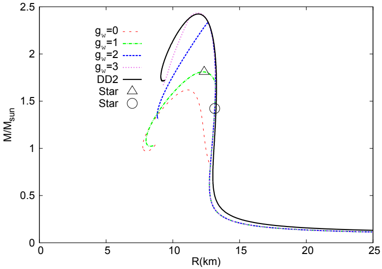


$$m_q = 300 \text{ MeV},\ m_\sigma = 600 \text{ MeV},\ B = 100 \text{ MeV}$$

Results for $0 \leq g_\omega \leq 3$

At a certain central pressure the star configurations get unstable

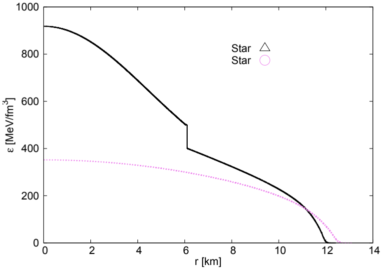


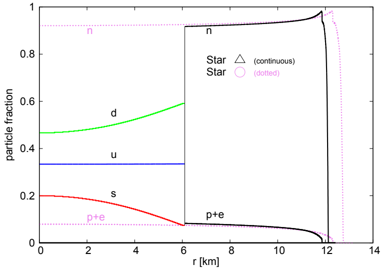


$$m_q = 300 \text{ MeV}, \; m_\sigma = 600 \text{ MeV}, \; B = 100 \text{ MeV}$$

Results for $0 \leq g_\omega \leq 3$

Particle composition of two individual stars:
△(hybrid $1.8M_\odot$) and ◯(hadronic $1.4M_\odot$)

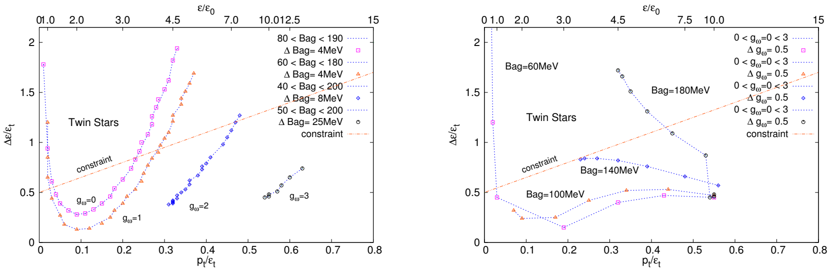




$$m_q = 300 \text{ MeV}, \; m_\sigma = 600 \text{ MeV}, \; B = 100 \text{ MeV}$$

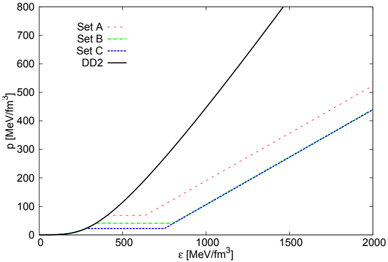
$$\boxed{\frac{\Delta\epsilon_{crit}}{\Delta\epsilon} = \frac{1}{2} + \frac{3}{2}\frac{p_{trans}}{\epsilon_{trans}}}$$

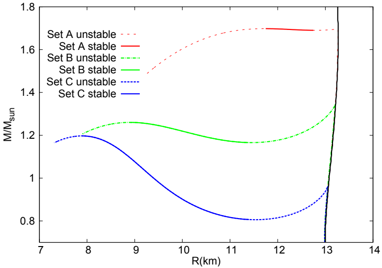
Twin stars: Hard to find



$$m_q = 300 \text{ MeV and } m_\sigma = 600 \text{ MeV}$$

The influence of the speed of sound



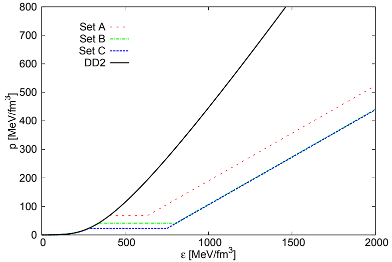


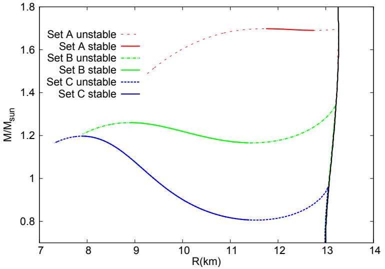
Twin Star Area scanned with SoS dependent EoS

$$p(\epsilon) = c_s^2 \left(\epsilon - \epsilon_*\right) \ , \quad \text{with: } \epsilon_* := \epsilon_t + \Delta\epsilon - \frac{1}{c_s^2} p_t \ ,$$

p_t/ϵ_t and $\Delta\epsilon/\epsilon_t$ under direct influence, $c_s^2 = \frac{1}{3}$

The influence of the speed of sound





Twin Star Area scanned with SoS dependent EoS

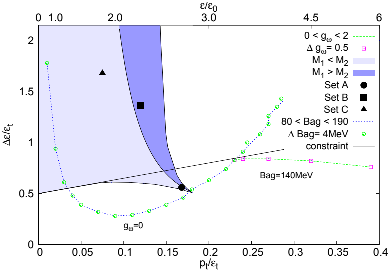
$$p(\epsilon) = c_s^2 \left(\epsilon - \epsilon_*\right) \;, \quad \text{with: } \epsilon_* := \epsilon_t + \Delta\epsilon - \frac{1}{c_s^2} p_t \,,$$

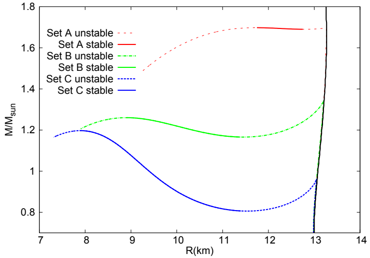
p_t/ϵ_t and $\Delta\epsilon/\epsilon_t$ under direct influence, $c_s^2 = \frac{1}{3}$

Looking for Twin star solutions

General analysis and parametrization via transition pressure p_t, transition energy density ϵ_t and jump in the energy density $\Delta\epsilon$:

Alford, Han et al. arXiv:1302.4732 and arXiv:1501.07902





Blue shaded area: Twin Star region

First step: Construction of meson matrices

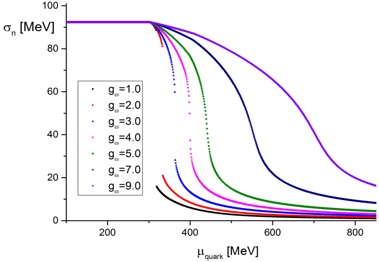
$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_n}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\sigma_n}{\sqrt{2}} & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}$$

1. σ_n and σ_s: scalar mesons

$$\mathbf{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_n+\rho^0}{\sqrt{2}} & \rho^+ & 0 \\ \rho^- & \frac{\omega_n-\rho^0}{\sqrt{2}} & 0 \\ 0 & 0 & \omega_s \end{pmatrix}$$

2. ω, ρ and $\omega_s = \phi$: vector mesons

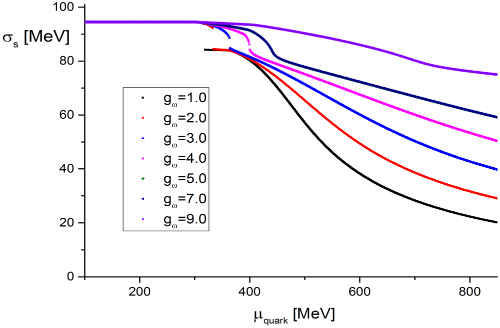
σ_n as a function of μ_q while varying g_ω



Smaller vector coupling leads to 1.order phase transition $\mu \sim 300\text{MeV}$

fixed $m_q = 300\text{MeV}$ and $m_\sigma = 600\text{MeV}$

σ_s as a function of μ_q while varying g_ω



The Equation of State: Solve the Lagrangian

With

$$\mathcal{Z} = \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{\left(\int_0^\beta d\tau \int_V d^3\vec{r}(\mathcal{L}+\bar{\Psi}\gamma^0\mu\Psi)\right)}$$

and

$$\begin{aligned} p &= \frac{ln\mathcal{Z}}{\beta} = -\Omega \\ \epsilon &= -p + \sum_i \mu_i n_i \end{aligned}$$

we have given a relation for the necessary values.

EoS - Grancanonical Potential Ω

Having performed the $T \to 0$ approximation
the resulting grandcanonical potential
is

$$\Omega = \mathcal{V} + \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^{k_F^f} dk \cdot k^2 \left(\sqrt{k_{n,s}^2 + \tilde{m}^2} - \tilde{\mu}_f \right)$$

with

$$\begin{aligned} \mathcal{V} &= -\frac{1}{2} \left(m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\varphi^2 \varphi^2 \right) + \frac{\lambda_1}{4} (\sigma_n^2 + \sigma_s^2)^2 + \frac{\lambda_2}{4} (\sigma_n^4 + \sigma_s^4) \\ &+ \frac{m_0^2}{2} (\sigma_n^2 + \sigma_s^2) - \frac{2\sigma_n^2 \sigma_s}{\sqrt{2}} \cdot c - h_n \sigma_n - h_s \sigma_s + B \end{aligned}$$

The energy density and the pressure are then determined to

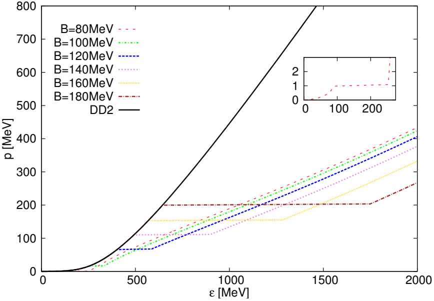
$$
\begin{aligned}
\epsilon &= \epsilon_e + \frac{\lambda_1}{4}(\sigma_n^2+\sigma_s^2)^2 + \frac{\lambda_2}{4}(\sigma_n^4+\sigma_s^4) + \frac{m_0^2}{2}(\sigma_n^2+\sigma_s^2) \\
&- \frac{2\sigma_n^2\sigma_s}{\sqrt{2}} \cdot c - h_n\sigma_n - h_s\sigma_s + B + \frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right) \\
&+ \frac{3}{\pi^2}\sum_{f=u,d,s}\int_0^{k_F^f} dk \cdot k^2 \left(\sqrt{k_{n,s}^2+\tilde{m}^2}\right)
\end{aligned}
$$

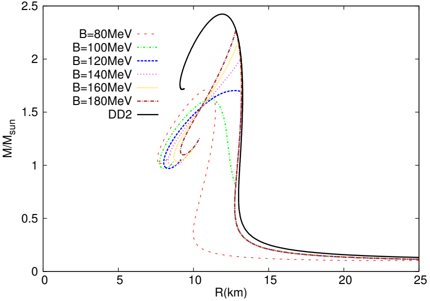
and

$$
\begin{aligned}
p &= -\frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right) + \frac{\lambda_1}{4}(\sigma_n^2+\sigma_s^2)^2 + \frac{\lambda_2}{4}(\sigma_n^4+\sigma_s^4) \\
&+ \frac{m_0^2}{2}(\sigma_n^2+\sigma_s^2) - \frac{2\sigma_n^2\sigma_s}{\sqrt{2}} \cdot c - h_n\sigma_n - h_s\sigma_s + B \\
&+ \frac{3}{\pi^2}\sum_{f=u,d,s}\int_0^{k_F^f} dk \cdot k^2 \left(\sqrt{k_{n,s}^2+\tilde{m}^2} - \tilde{\mu}_f\right)
\end{aligned}
$$

Hybrid stars: Maxwell construction

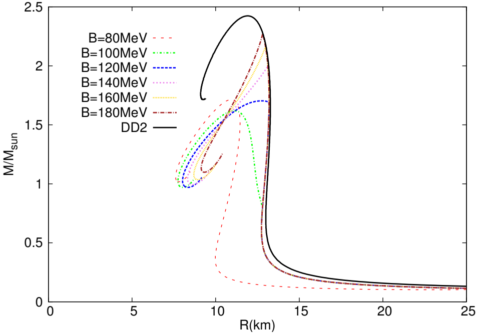
Different EoS and corresponding Mass-Radius Relations





Hybrid stars: Maxwell construction

The resulting mass radius relations while varying the vacuum pressure B



- $2M_{\odot}$ can be reached
- The appearence of a QM core destabilizes the star under certain circumstances