Stability of Differentially Rotating Neutron Stars

Lukas R. Weih

30. Oct 2017

AstroCoffee Seminar, Goethe-University, Frankfurt a.M.

Why differentially rotating neutron stars?
Motivation

Why differentially rotating neutron stars?

NS-NS binary merger

↓

differentially rotating NS

prompt collapse to BH?

stable NS?
Motivation

Why differentially rotating neutron stars?

NS-NS binary merger

\[ \downarrow \]

differentially rotating NS

\[ \leftarrow \]

prompt collapse to BH?  

\[ \rightarrow \]

stable NS?

Why equilibrium solutions?

Full general-relativistic simulation \[ \approx \] 200000 core hours  
Sequence of equilibrium solutions \[ \lesssim \] 1 core hour
Motivation

Why differentially rotating neutron stars?

NS-NS binary merger

↓

differentially rotating NS

prompt collapse to BH? stable NS?

Why equilibrium solutions?

Full general-relativistic simulation \( \approx 200000 \) core hours

Sequence of equilibrium solutions \( \lesssim 1 \) core hour

⇒ Use equilibrium models of dif. rot. NS to determine stability of merger remnant
Overview

- Short review of equilibrium solutions of neutron stars (NS)
  - How to include differential rotation
  - Stability of non- and uniformly rotating models
- Stability of Differentially Rotating NSs
- Universal Relation for Determining the Turning Point
- Maximum Mass of Differentially Rotating Neutron Stars
- Validity of approximating the merger remnant
- Summary and Outlook
General Relativity and the Metric of NSs

Axisymmetric, stationary metric:

\[ ds^2 = -e^{2\nu} dt^2 + e^{2\psi} (d\Phi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2) \]

Solve Einstein eq. together with eq. of hydrostationary equilibrium

\[ \ln h - \ln u^t + \int_{\Omega_{pole}}^{\Omega} F(\Omega') d\Omega' = \nu_{pole} \quad (\text{with } h = \frac{e + P}{\rho}) \]

and zero-temperature EOS

\[ P = P(\rho) \quad (1) \]

iteratively until convergence is reached.

⇒ Specify \( \rho_c, r_{\text{ratio}}, F(\Omega) \) and RNS-code yields \( \approx 10 \text{ stars/min} \)
General Relativity and the Metric of NSs

Axisymmetric, stationary metric:

\[ ds^2 = -e^{2\nu} dt^2 + e^{2\psi} (d\Phi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2) \]

Solve Einstein eq. together with eq. of hydrostationary equilibrium

\[ \ln h - \ln u^t + \int_{\Omega_{pole}}^{\Omega} F(\Omega') d\Omega' = \nu_{pole} \quad \text{(with } h = \frac{e + P}{\rho}) \]

and zero-temperature EOS

\[ P = P(\rho) \quad \text{(1)} \]

iteratively until convergence is reached.

⇒ Specify \( \rho_c, r_{\text{ratio}}, F(\Omega) \) and RNS-code yields \( \approx 10 \text{ stars/min} \)
The Rotation Law

\[ F(\Omega) = A^2(\Omega_c - \Omega) \] (j-constant law) determines the rotation profile.

**Figure:** *Left:* Cross section of rest-mass density in x-z-plane.
*Right:* Rotation profiles depending on degree of differential rotation, \( A \).

Lukas R. Weih (ITP, Goethe-Uni)
Equilibrium Solutions

Figure: Left: Rest mass over central rest-mass density. Non-rotating sequence and mass-shedding limit for uniformly and dif. rotating models. Right: Sequences of constant angular momentum for high degree of differential rotation.
The Turning Point Criterion

- Along a sequence (parameterized $\rho_0$) of constant angular momentum, the point of maximum mass marks the onset of secular instability (collapse to BH).
- For rotating (uniformly) NSs neutral-stability line (F-mode=0) shifts to smaller $\rho_0$.
- What about differentially rotating NSs?

![Graph showing turning-point line (blue) and neutral-stability line (red) for uniformly rot. models.](image)

**Figure:** Turning-point line (blue) and neutral-stability line (red) for uniformly rot. models.
Stability of Differentially Rotating NSs

Evolve selected models numerically in time:

Figure: Lines of const. ang. mom. for moderate (left) and high (right) degree of dif. rot. Selected models are marked with open circles.
Stability of Differentially Rotating NSs

Evolve selected models in numerically time:

![Graph showing time evolution of central rest-mass density for models A-G.](image)

**Figure:** Time evolution of central rest-mass density for models A-G.

![Graph showing models A-G in M - \( \rho_0 \) plane.](image)

**Figure:** Models A-G in \( M - \rho_0 \) plane.
Stability of Differentially Rotating NSs

Evolve selected models numerically in time:

![Graph showing lines of constant angular momentum for moderate and high degree of differential rotation. Selected models are marked with open circles.]

Figure: Lines of constant angular momentum for moderate (left) and high (right) degree of differential rotation. Selected models are marked with open circles.
Stability of Differentially Rotating NSs

Evolve selected models numerically in time (○ stable, □ unstable)

Figure: Lines of const. ang. mom. for moderate (left) and high (right) degree of dif. rot. Selected models are marked with open circles.
Universal Relation for Determining the Turning Point

Determine turning-point mass, $M_{\text{max,dr}}$, as function of dimensionless angular momentum $j = J/M^2$ for different EOSs:

Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

Determine turning-point mass, $M_{\text{max,dr}}$, as function of dimensionless angular momentum $j = J/M^2$ for different EOSs:

Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

Determine turning-point mass, $M_{\text{max,dr}}$, as function of dimensionless angular momentum $j = J/M^2$ for different EOSs:

Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

Determine turning-point mass, $M_{\text{max,dr}}$, as function of dimensionless angular momentum $j = J/M^2$ for different EOSs:

Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

For uniform rotation (lower bundle) from Breu & Rezzolla (2016):

\[ M_{\text{max}} = \left( 1 + a_1 \left( \frac{j}{j_{\text{max}}} \right)^2 + a_2 \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]

**Figure:** Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

For differential rotation (upper bundles):

\[ M_{\text{max,dr}} = \left( 1 + a_1(A) \left( \frac{j}{j_{\text{max}}} \right)^2 + a_2(A) \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]

**Figure:** Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.
Universal Relation for Determining the Turning Point

For differential rotation (upper bundles):

\[ M_{\text{max,dr}} = \left( 1 + a_1(A) \left( \frac{j}{j_{\text{max}}} \right)^2 + a_2(A) \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]

\[ = \left( 1 + a_1 \left( \frac{j}{j_{\text{max}}} \right)^2 + (b_0 + b_1 \tilde{A}^2 + b_2 \tilde{A}^4) \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]
Universal Relation for Determining the Turning Point

For differential rotation (upper bundles):

\[ M_{\text{max,dr}} = \left( 1 + a_1(A) \left( \frac{j}{j_{\text{max}}} \right)^2 + a_2(A) \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]

\[ = \left( 1 + a_1 \left( \frac{j}{j_{\text{max}}} \right)^2 + (b_0 + b_1 \tilde{A}^2 + b_2 \tilde{A}^4) \left( \frac{j}{j_{\text{max}}} \right)^4 \right) M_{\text{TOV}} \]

Figure: Normalised turning-point mass over dimensionless angular momentum and degree of differential rotation.

Lukas R. Weih (ITP, Goethe-Uni)

AstroCoffee Seminar

30.Oct.2017 19 / 26
Maximum mass of differentially rotating neutron stars

For $j = j_{\text{max}}$ the maximum mass for given $\tilde{A}$ is obtained:

$$M_{\text{max,dr}}(j_{\text{max}}, \tilde{A}) / M_{\text{TOV}} = 1.2 + c_1 \left( \frac{\tilde{A}}{\tilde{A}_{\text{max}}} \right)^2 + c_2 \left( \frac{\tilde{A}}{\tilde{A}_{\text{max}}} \right)^4$$

Figure: Maximum mass as function of degree of differential rotation.
Maximum mass of differentially rotating neutron stars

For \( j = j_{\text{max}} \) the maximum mass for given \( \tilde{A} \) is obtained:

\[
M_{\text{max,dr}} \left( j_{\text{max}}, \tilde{A} \right) / M_{\text{TOV}} = 1.2 + c_1 \left( \frac{\tilde{A}}{\tilde{A}_{\text{max}}} \right)^2 + c_2 \left( \frac{\tilde{A}}{\tilde{A}_{\text{max}}} \right)^4
\]

\( \Rightarrow \) For \( \tilde{A} = \tilde{A}_{\text{max}} \): \( M_{\text{max,dr}} = (1.54 \pm 0.05) M_{\text{TOV}} \).
Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing

Figure: Rotation profile of BNS merger remnant (Hanauske et al. (2017)).
Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing
- But, which $F(\Omega)$ to choose?

![Rotation profile of BNS merger remnant](image)

**Figure:** Rotation profile of BNS merger remnant (Hanauske et al. (2017)).
Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing
- But, which $F(\Omega)$ to choose?
- Approximate actual rotation profile with a fit, $\Omega(r)$

Figure: Rotation profile from BNS merger remnant *Hanauske et al (2016).*
Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing
- But, which $F(\Omega)$ to chose?

- Approximate actual rotation profile with a fit, $\Omega(r)$
- Supply RNS with $\Omega(r)$ instead of $F(\Omega)$

Figure: Rotation profile from BNS merger remnant *Hanauske et al (2016).*
Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing
- But, which $F(\Omega)$ to chose?

- Approximate actual rotation profile with a fit, $\Omega(r)$
- Supply RNS with $\Omega(r)$ instead of $F(\Omega)$
- Solve iteratively until convergence of $F(\Omega) = u^t u_\phi$ is reached

Figure: Rotation profile from BNS merger remnant *Hanauske et al (2016)*.
Comparison of equilibrium model to merger remnant

Figure: Equilibrium model with realistic rotation profile compared to actual data of a remnant from a BNS merger simulation.
Stability criterium for uniformly NS can be extended also to differential rotation.
Stability criterion for uniformly NS can be extended also to differential rotation.

Turning point as valid approximation for stability limit. HMNS are not unconditionally unstable.
Summary

- Stability criterium for uniformly NS can be extended also to differential rotation.
- Turning point as valid approximation for stability limit. HMNS are not unconditionally unstable.
- Calculate turning-point mass for given $j$ and $A$ simply in terms of $M_{\text{TOV}}$.
  \[\Rightarrow\] Maximum mass possible with dif. rot.: \((1.54 \pm 0.05)M_{\text{TOV}}\).
Recently, $F(\Omega)$ proposed to compute more realistic equilibrium solutions 
*Uryu et al (2017).*

⇒ Sequences of const. ang. mom. have a turning point.

⇒ Stability criterion also true for realistic equilibrium models?