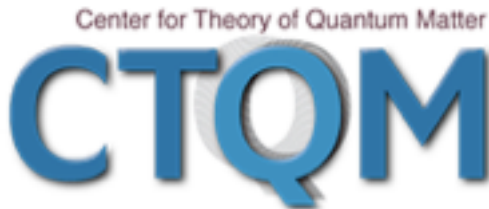


Simulations of BH Collisions in AdS Spacetimes

Paul Romatschke

CU Boulder & CTQM & JET Collaboration

Based on arXiv: 1410.4799 with Hans Bantilan



Outline

- Motivation
- Simulating BH Collisions
- Conclusions

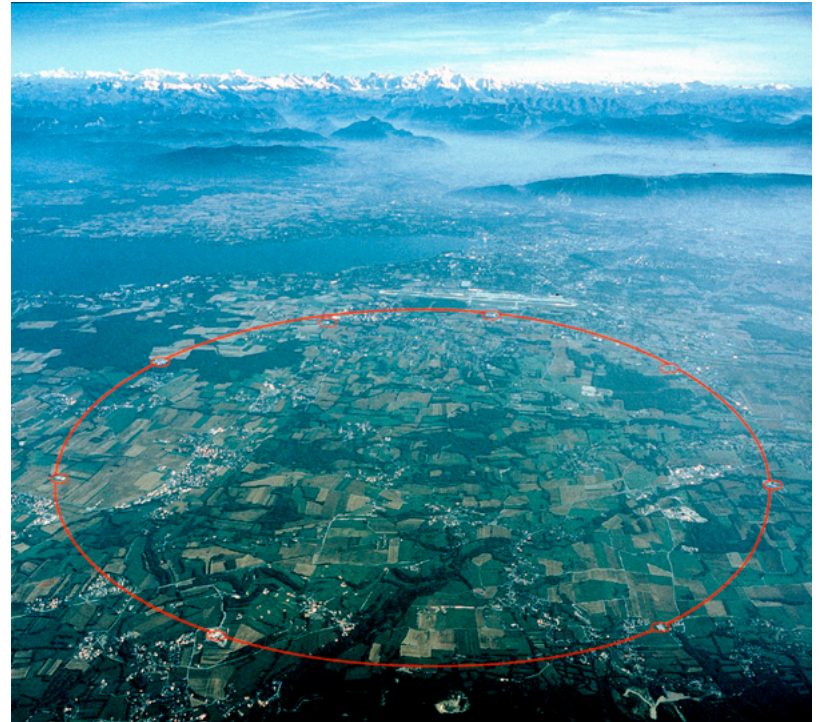
Motivation

Relativistic Ion Collisions



RHIC

LHC

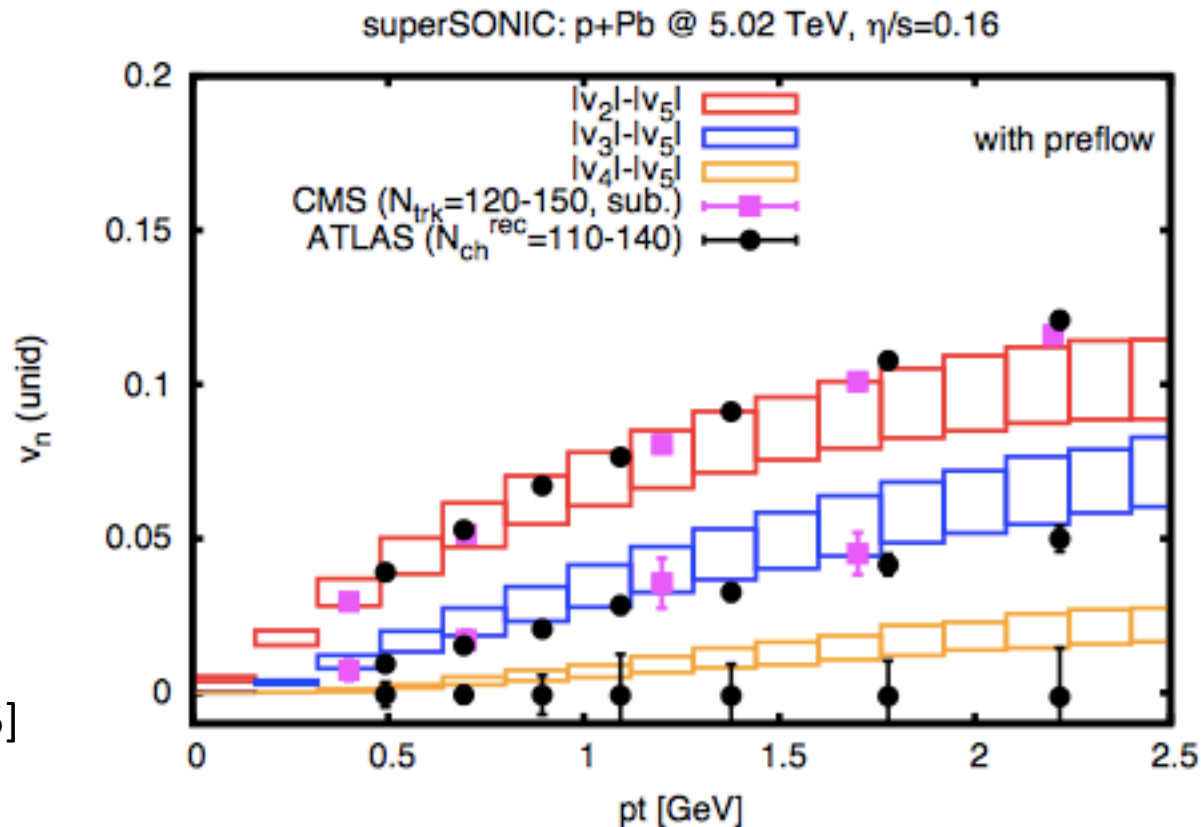


Heavy-Ion Collisions

- 2003-present: QCD matter behaves fluid-like, not gas-like (despite asymptotic freedom)
- Large “flow” signals, e.g. v_2 , v_3 , v_4 , v_5 in Pb+Pb collisions at LHC (correspond to $l=2..5$ in CMB Background)
- Hydrodynamic models correctly describe 99% of particles registered in experimental detectors

Light-on-Heavy-Ion Collisions

- Large “flow” signals, e.g. v_2 , v_3 also found in d+Au, p+Pb collisions



QGP: Matter @ 4 Trillion K

- Temperature: 4×10^{12} K
- Lifetime: 10^{-23} sec
- Size: 10^{-14} m

Gradients are large!

Why does hydrodynamics apply at all?

Even for small systems (p+Pb)?

Can we understand equilibration?

Motivation

Understand equilibration in relativistic ion collisions using AdS/CFT

Goal

Solve dynamical Einstein Equations (w/ or w/o additional fields) in asymptotic AdS for strong gravity situations (BH formation, BH collisions, etc.)

Want: general purpose tool to study far-from equilibrium strongly coupled systems!

Einstein Field Equations in GH

$$0 = R_{\mu\nu} + \frac{2\Lambda}{2-d}g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-2}T^\alpha{}_\alpha g_{\mu\nu} \right) - \kappa \left(2n_{(\mu}C_{\nu)} - (1+P)g_{\mu\nu}n^\alpha C_\alpha \right) - \nabla_{(\mu}C_{\nu)}$$

$$C^\mu \equiv H^\mu - \square x^\mu$$

(physical solutions satisfy $C^\mu = 0$)

Einstein Field Equations in GH

$$\begin{aligned}
 0 = & + \frac{2\Lambda}{2-d} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-2} T^\alpha{}_\alpha g_{\mu\nu} \right) \\
 & - \kappa \left(2n_{(\mu} C_{\nu)} - (1+P) g_{\mu\nu} n^\alpha C_\alpha \right) - \nabla_{(\mu} H_{\nu)} + \cancel{\nabla_{(\mu} \square x_{\nu)}} \\
 & - \frac{1}{2} g^{\alpha\beta} g_{\mu\nu, \alpha\beta} + \cancel{g^{\alpha\beta} g_{\beta(\mu, \nu)\alpha}} + \frac{1}{2} g^{\alpha\beta}{}_{, \alpha} (\cancel{g_{\alpha\beta, \nu}} - \cancel{g_{\nu\mu, \beta}} + g_{\beta\nu, \mu}) \\
 & - \cancel{(\log \sqrt{-g})_{, \mu\nu}} + \cancel{(\log \sqrt{-g})_{, \beta} \Gamma^\beta_{\mu\nu}} - \Gamma^\alpha{}_{\nu\beta} \Gamma^\beta{}_{\alpha\nu} - g^{\alpha\beta}{}_{, (\mu} g_{\nu)\alpha, \beta}
 \end{aligned}$$

$$C^\mu \equiv H^\mu - \square x^\mu$$

(physical solutions satisfy $C^\mu = 0$)

Einstein Field Equations in GH

$$\begin{aligned}
 0 = & + \frac{2\Lambda}{2-d} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-2} T^\alpha{}_\alpha g_{\mu\nu} \right) \\
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 & - \cancel{(\log \sqrt{-g})_{, \mu\nu}} + \cancel{(\log \sqrt{-g})_{, \beta}} \Gamma^\beta{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\beta} \Gamma^\beta{}_{\alpha\nu} - g^{\alpha\beta}{}_{, (\mu} g_{\nu)\alpha, \beta}
 \end{aligned}$$

+evolution eq's for H^μ

$$C^\mu \equiv H^\mu - \square x^\mu$$

(physical solutions satisfy $C^\mu = 0$)

We solve these numerically

Slides from Hans Bantilan, CU Boulder, Oct 2014

NUMERICAL RELATIVITY

Evolution Equations:

$$\begin{aligned} 0 = & -\frac{1}{2}g^{\alpha\beta}g_{\mu\nu,\alpha\beta} - g^{\alpha\beta}_{,(\mu}g_{\nu)\alpha,\beta} \\ & -H_{(\mu,\nu)} + H_{\alpha}\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu} \\ & -\kappa\left(2n_{(\mu}C_{\nu)} - (1+P)g_{\mu\nu}n^{\alpha}C_{\alpha}\right) \\ & -\frac{2}{3}\Lambda_5g_{\mu\nu} - 8\pi\left(T_{\mu\nu} - \frac{1}{3}T^{\alpha}_{\alpha}g_{\mu\nu}\right) \\ & \downarrow \\ 0 = & \mathcal{L}_f|_{ij}^n \quad (15 \text{ such equations, one for each } \mu\nu) \end{aligned}$$

Use second-order differencing to discretize these

COORDINATE CHOICE NEAR THE AdS BOUNDARY

- Coordinate choice in asymptotically AdS spacetimes
 - not enough to simply demand b.c.s for $\bar{g}_{\mu\nu}$, $\bar{H}_\mu$, $\bar{\phi}$
 $g_{\mu\nu} = g_{\mu\nu}^{AdS} + (1 - \rho)^\# \bar{g}_{\mu\nu}$
 $H_\mu = H_\mu^{AdS} + (1 - \rho)^\# \bar{H}_\mu$
 $\varphi = (1 - \rho)^\# \bar{\varphi}$
 - how to choose H_μ so that b.c.s are preserved by evolution?
- Example: tt component of field equations near $\rho = 1$

$$\tilde{\square} \bar{g}_{(1)tt} = (-8\bar{g}_{(1)\rho\rho} + 4\bar{H}_{(1)\rho})(1 - \rho)^{-2} + \dots$$

- regularity requires a delicate cancellation between terms in the near-boundary limit
- smart coordinate choice: $\bar{H}_{(1)\rho} = 2\bar{g}_{(1)\rho\rho}$

Numerics/Hardware

- C,C++, Fully parallel (openMPI)
- Hardware used:

Eridanus cluster (CU Boulder, 192 cores @ 2 GHz/core), Infiniband interconnections

Orbital cluster (Princeton, ~3700 cores @ 3.5 GHz/core), Infiniband interconnections



Simulations of BH Collisions in Global AdS₅

- Head-on Collisions
- Global AdS rather than Poincare patch
- Initial Data: pure AdS + massless scalar field (Coulomb branch), quickly collapsing to form BHs (non-planar horizon!)
- Use excision to simulate space-times with BHs

Heavy Ion Collisions as BH Collisions in AdS_5

‘nuclear collisions’ in 3+1d as shock wave
collisions in AdS_5 :

$$T_{++} \propto \rho(x_{\perp})\delta(x^+)$$

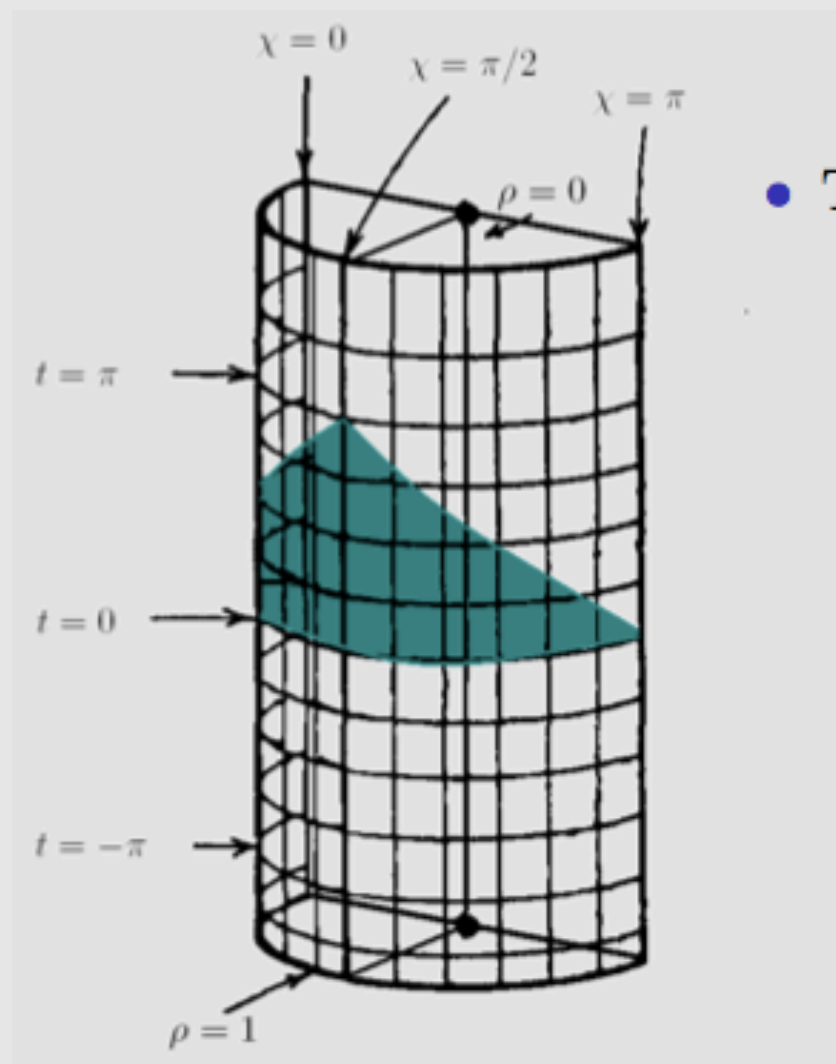
$$ds^2 = \frac{-2dx^+dx^- + dx_{\perp}^2 + dz^2 + dx^{+2}\Phi(x_{\perp}, z)\delta(x^+)}{z^2}$$

$$\lim_{z \rightarrow 0} \frac{\Phi(x_{\perp}, z)}{z^4} = \frac{\rho(x_{\perp})}{\kappa}$$

Heavy Ion Collisions as BH Collisions in AdS5

- Two infinitely boosted ‘nuclei’ superimposed become two shock waves in gravity
- Extremely high-energy analogue of black-hole collisions (Aichelburg-Sexl shock waves)
- Hard to treat collision process via numerical relativity
- Some insight may be gained analytically

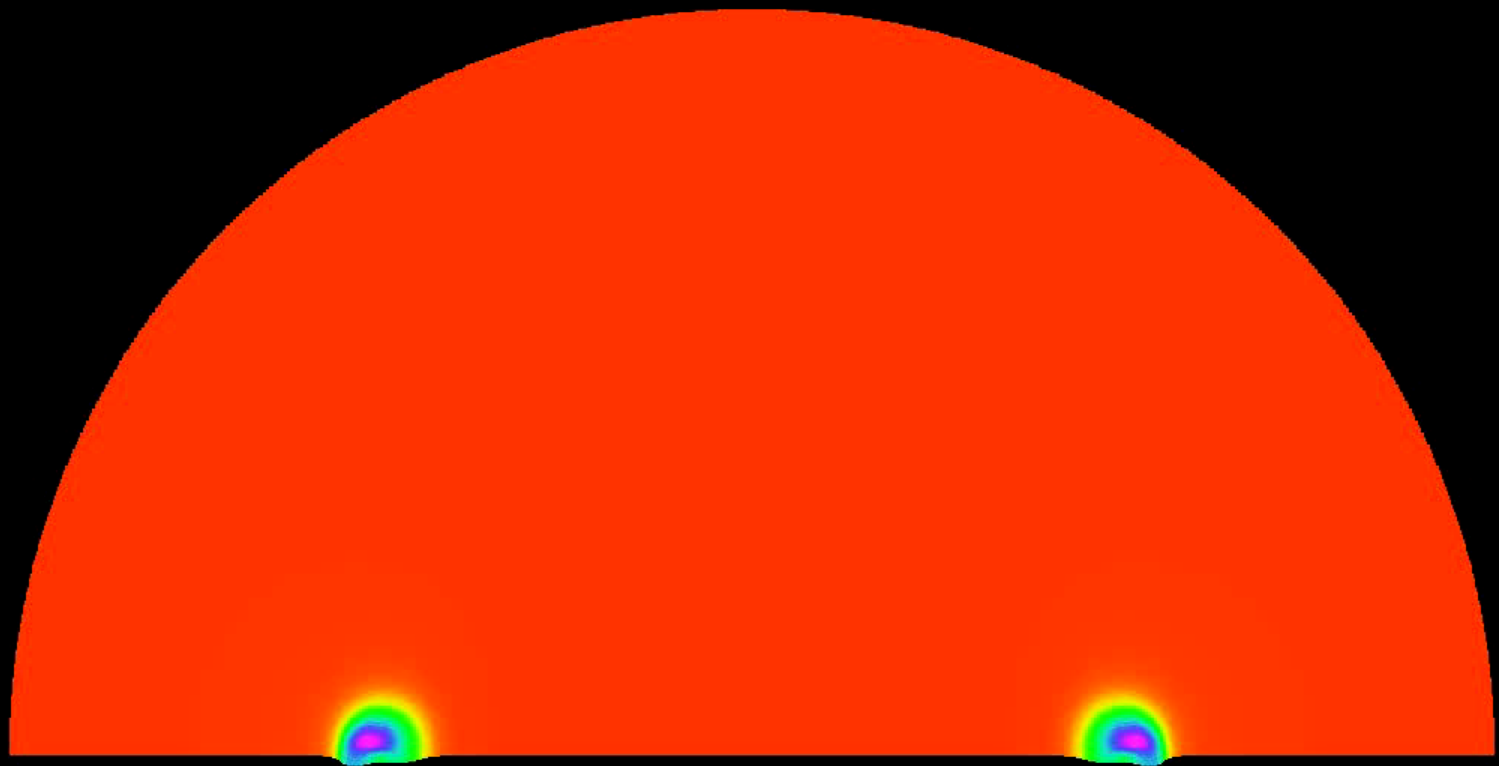
THE ANTI-DE SITTER SPACETIME



- The AdS_5 spacetime
 - its boundary is causally connected to its interior
 - spacelike infinity $\cup$ null infinity
 - forms a timelike 4-surface
 - identified with $\mathbb{R} \times S^3$

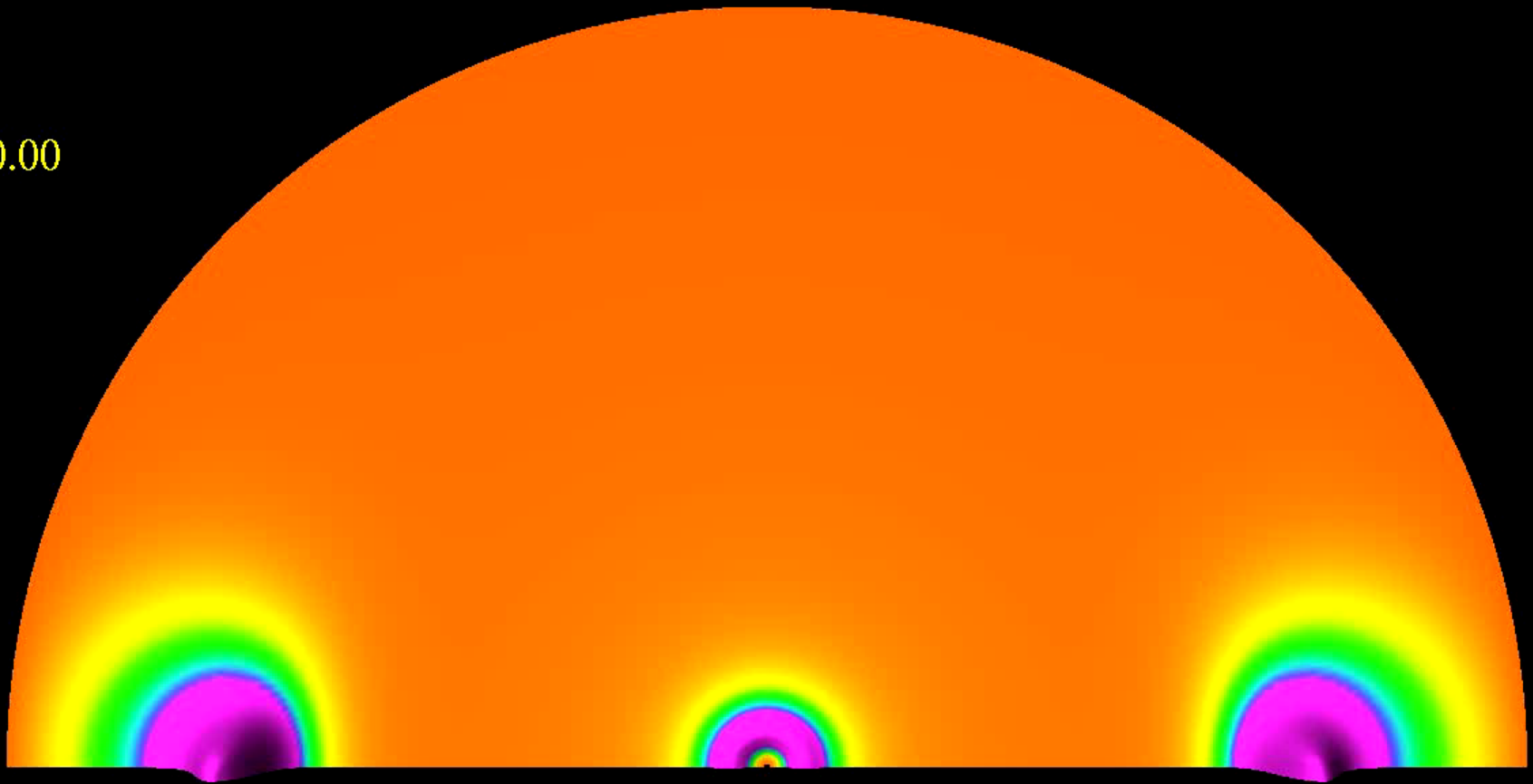
FIGURE: Conformal sketch of the AdS_5 spacetime; there is an internal 2-sphere geometry at each point of this sketch.

Metric, separation $D=0.5$

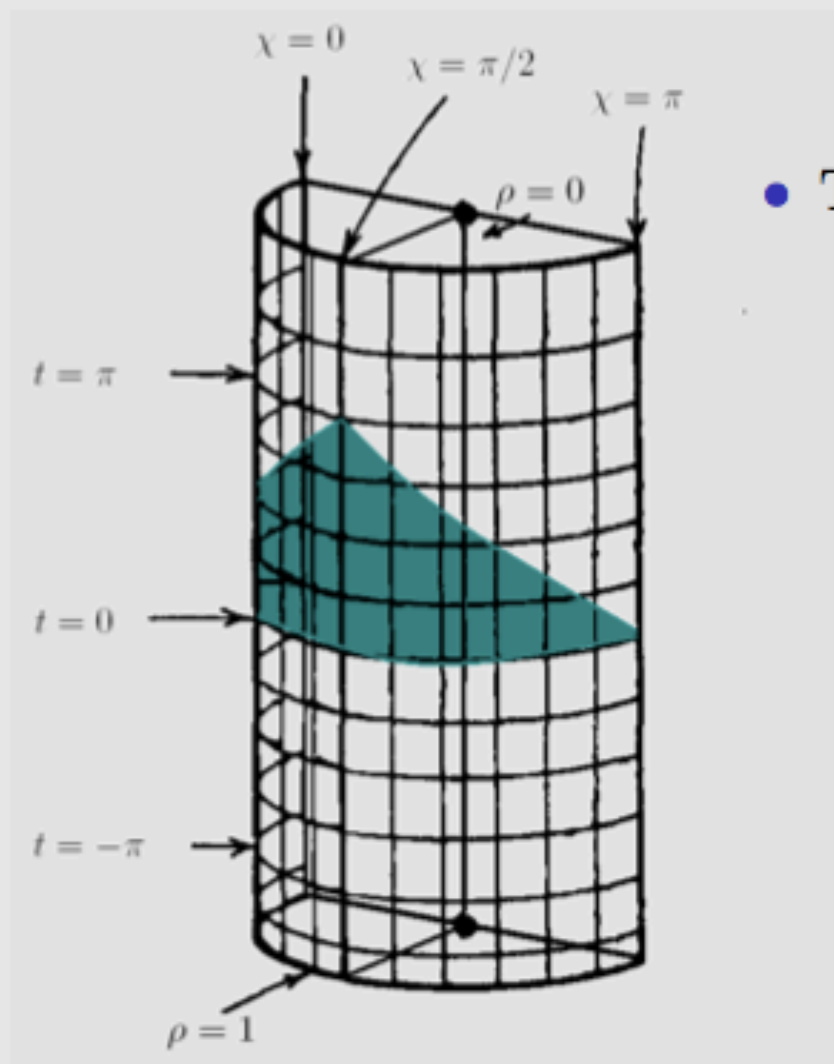


Metric, separation $D=0.7$

$t=0.00$



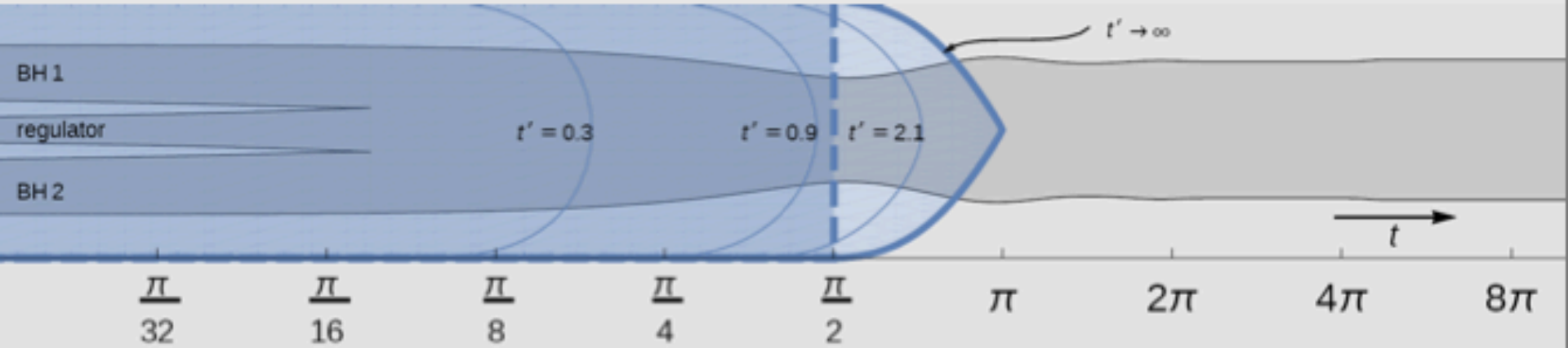
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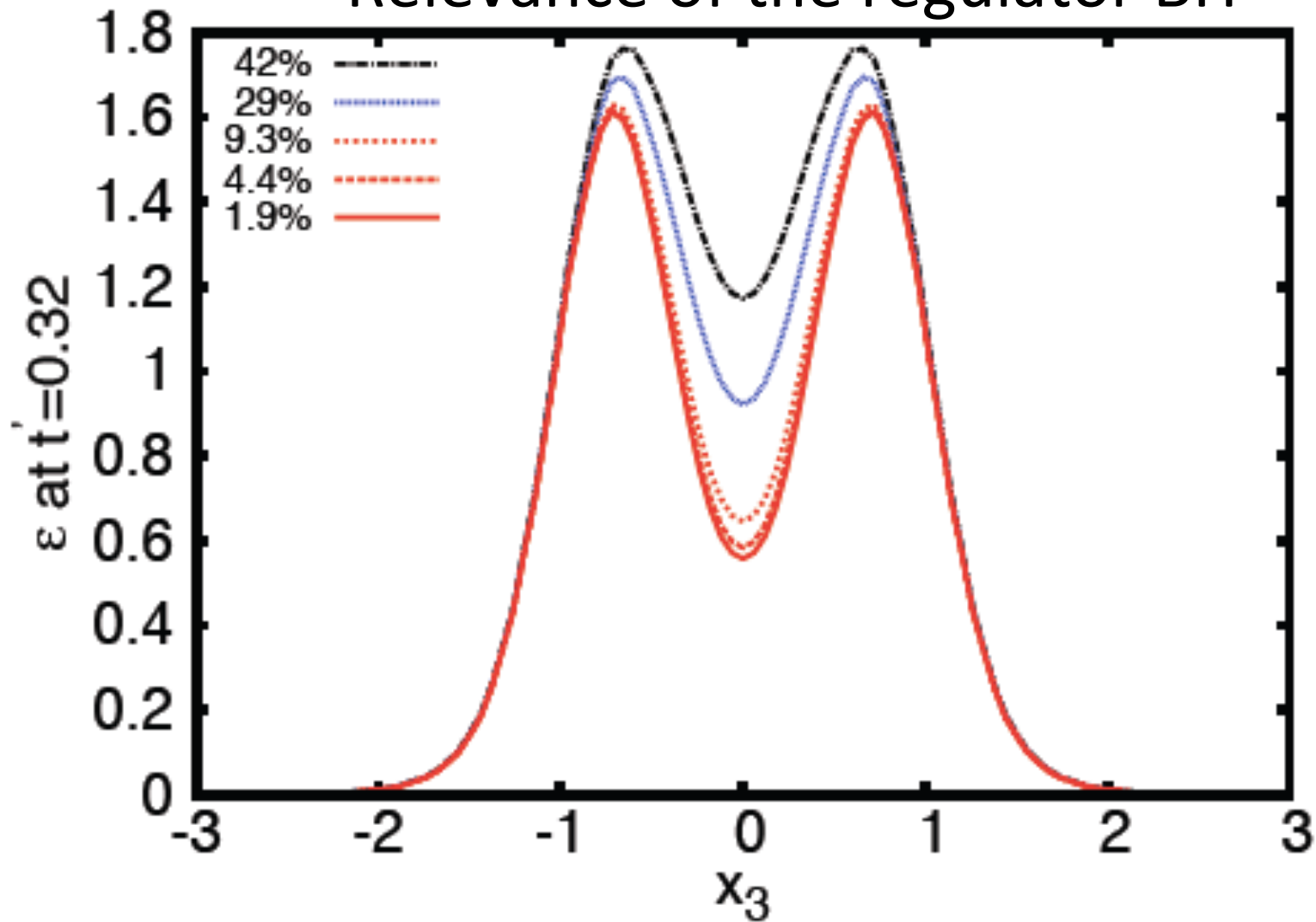
FIGURE: Conformal sketch of the AdS_5 spacetime; there is an internal 2-sphere geometry at each point of this sketch.

SPACE-TIME DIAGRAM OF THE BULK



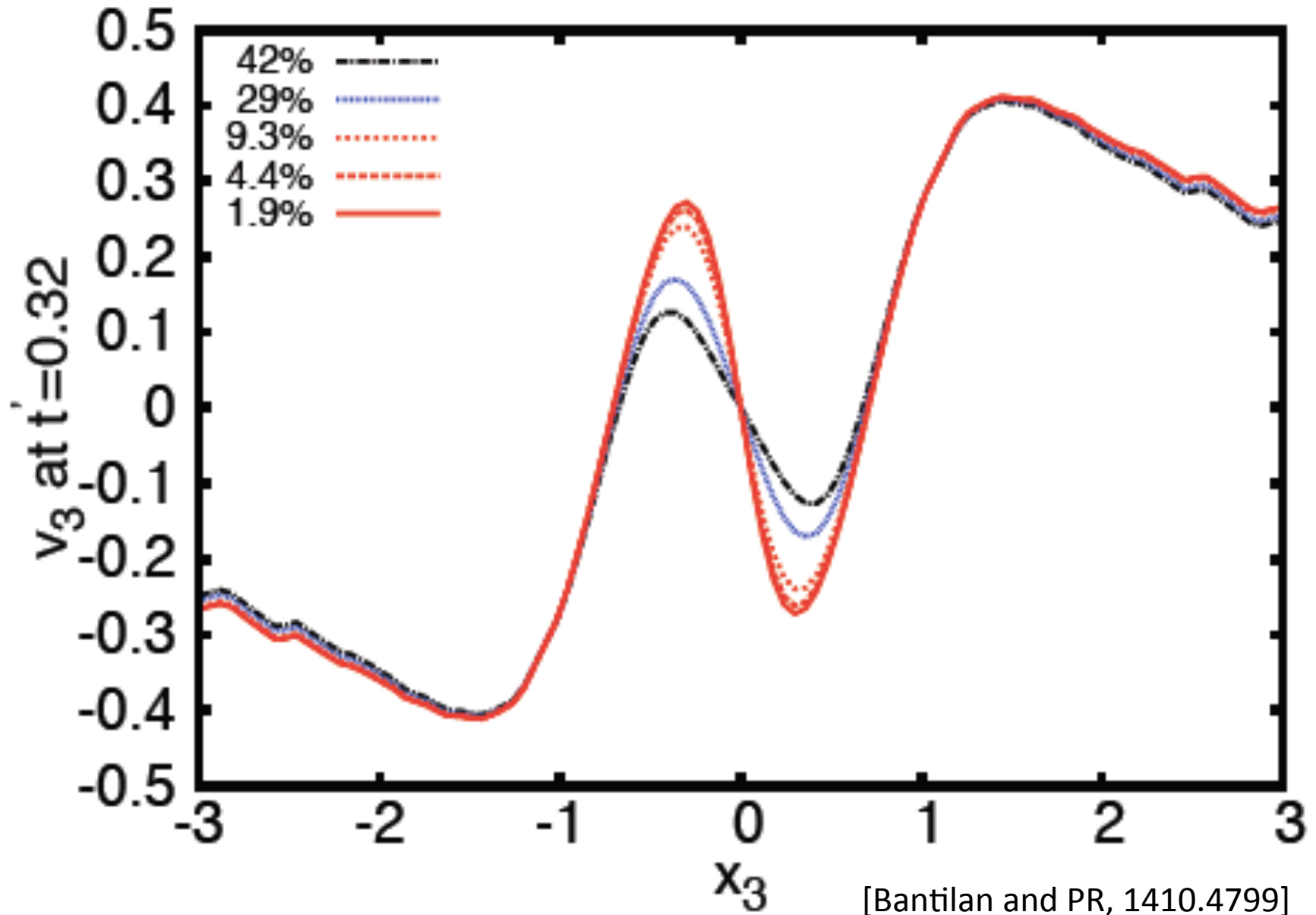
[Bantilan and PR, 1410.4799]

Relevance of the regulator BH

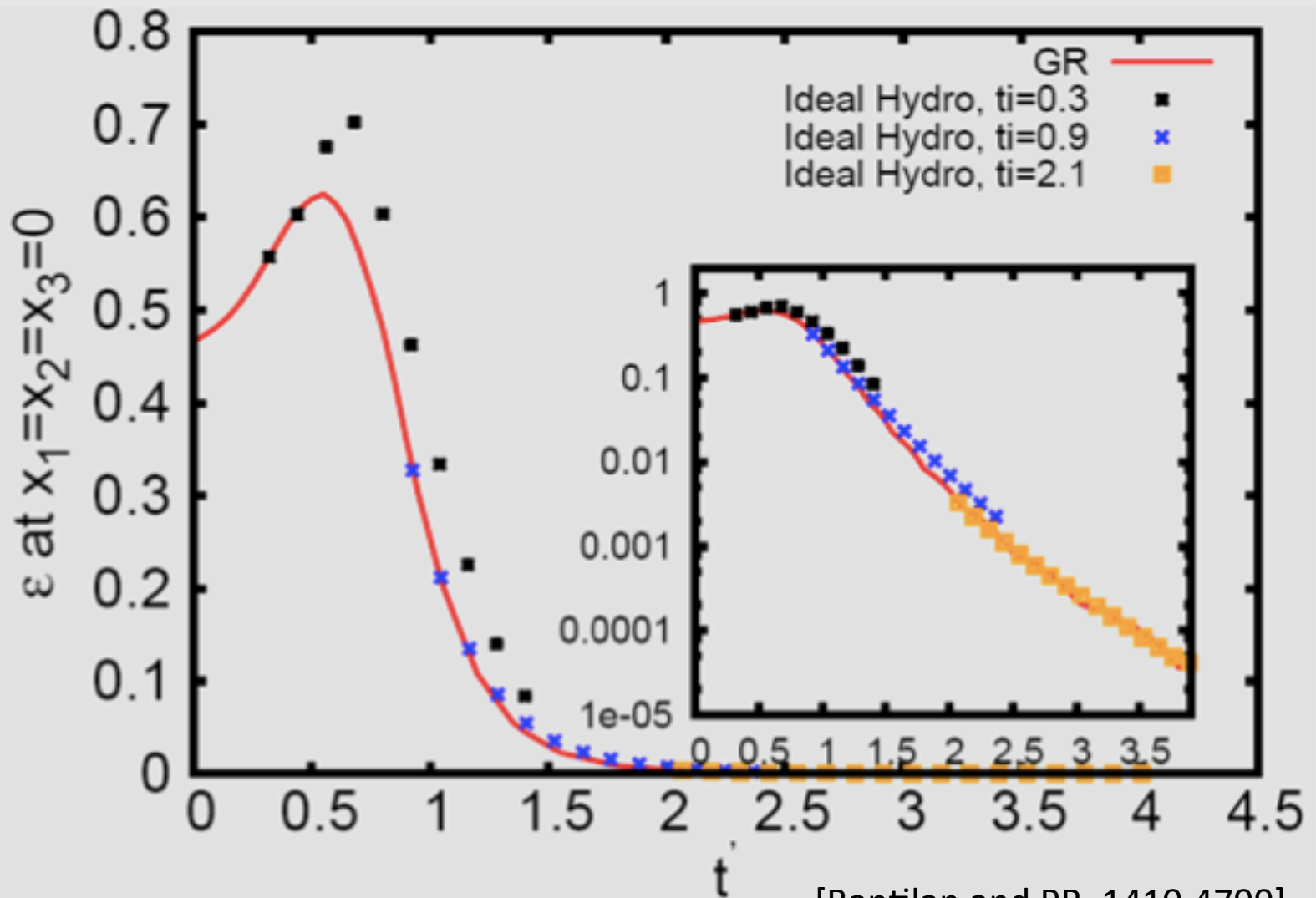


[Bantilan and PR, 1410.4799]

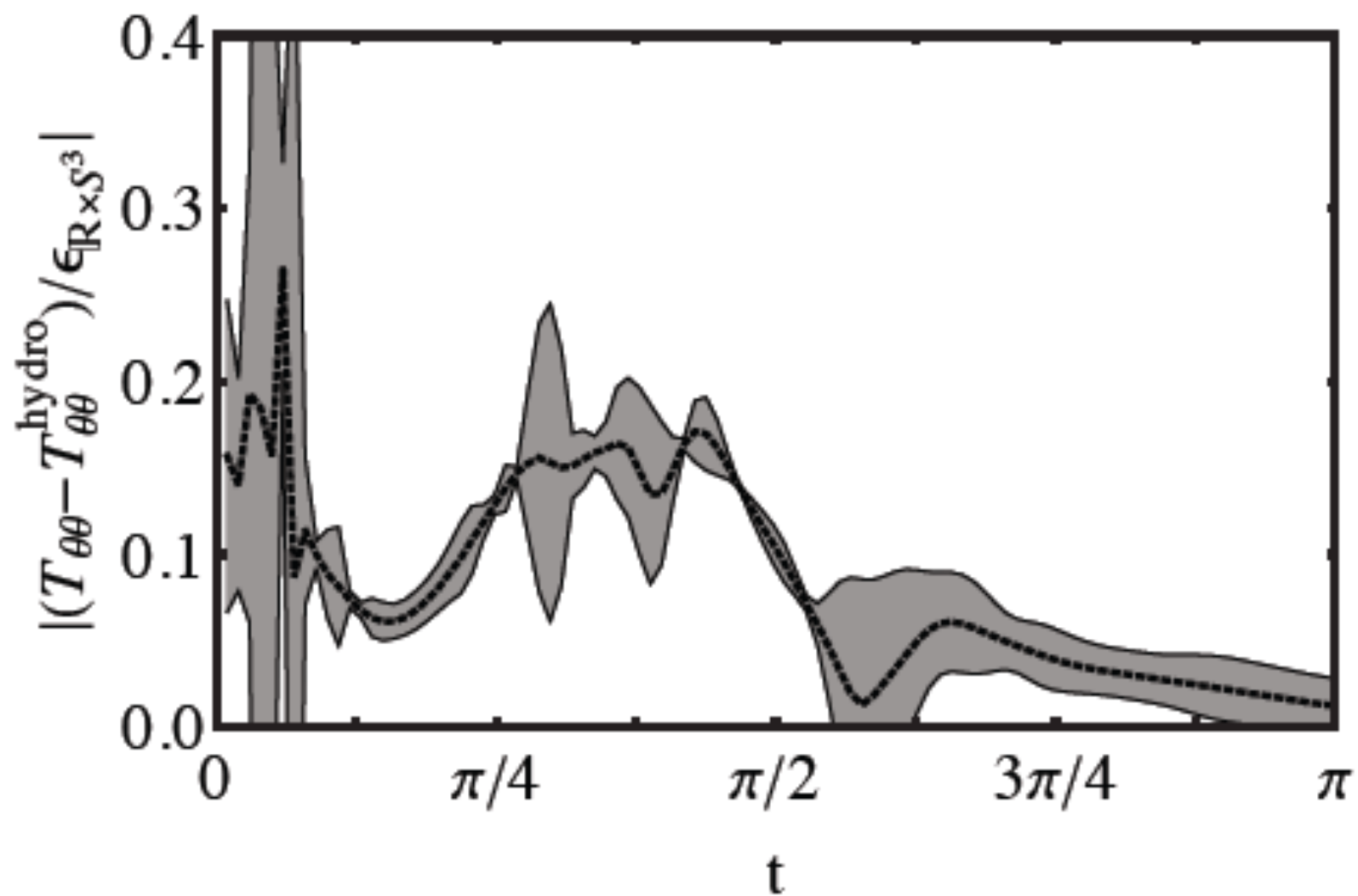
“Collision” velocities



COMPARISON TO HYDRODYNAMICS ON $\mathbb{R}^{3,1}$

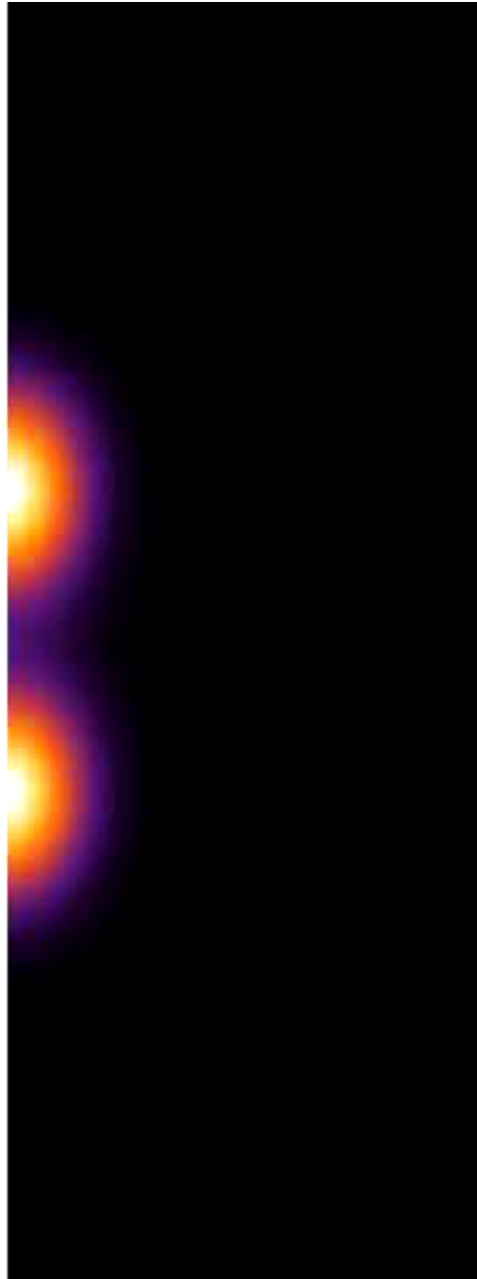


[Bantilan and PR, 1410.4799]



[Bantilan and PR, 1410.4799]

Boundary Energy Density



[Bantilan and PR, 1410.4799]

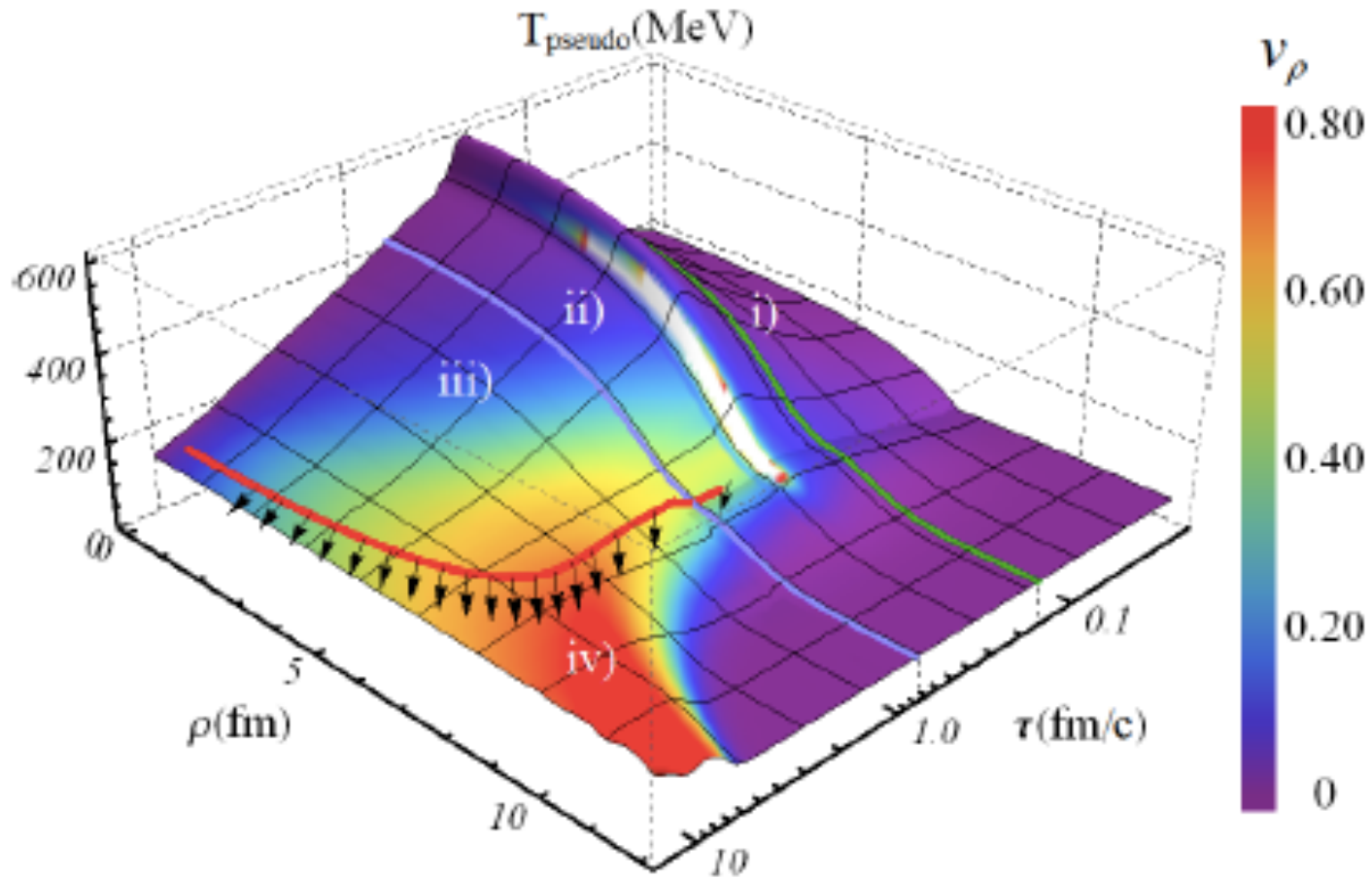
Simulations of BH collisions

- We have numerical results for metric in all of space-time, including boundary data
- We have metric data (a bit) inside trapped surfaces: could be useful for entanglement entropy calculations (?)
- Metric data/boundary data could be used for comparison to analytic results
- This data is publicly available. If you can't find something on our website, just ask!

AdS/CFT Phenomenology

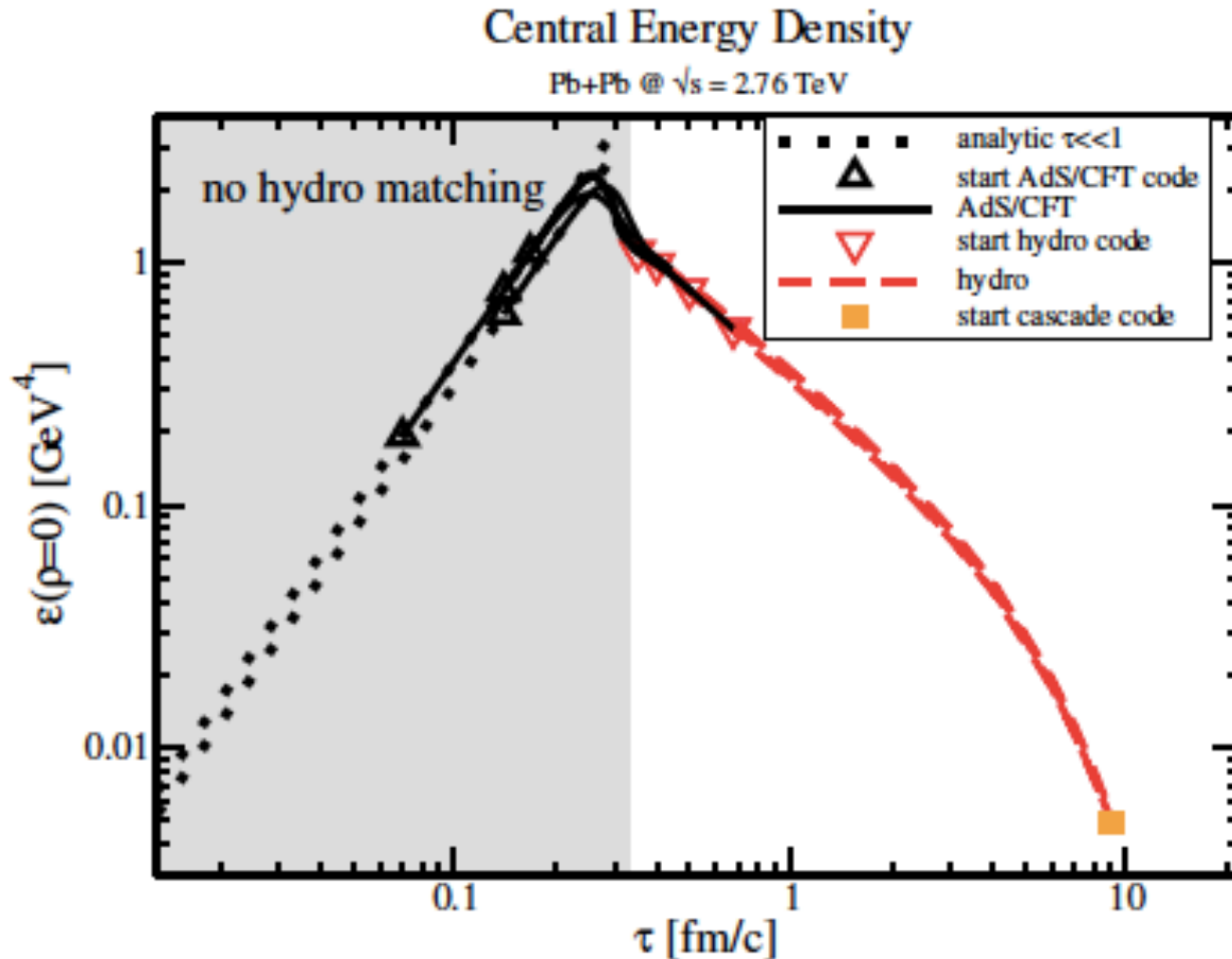
Can AdS/CFT dynamics be
experimentally probed in
relativistic ion collisions?

AdS+hydro+cascade (“SONIC”)



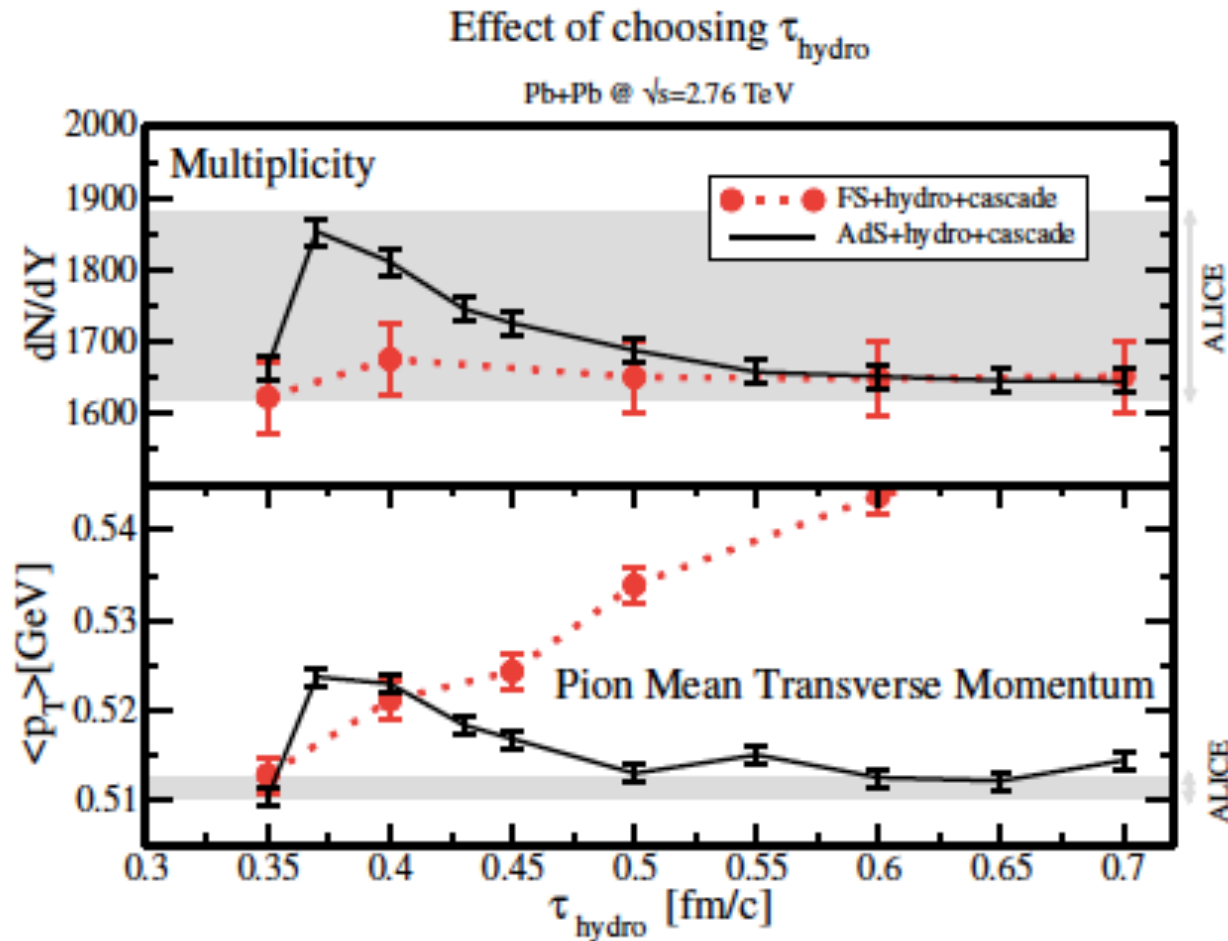
[van der Schee, PR & Pratt, PRL111 (2013)]

AdS+hydro+cascade



[van der Schee, PR & Pratt, 2013]

AdS+hydro+cascade



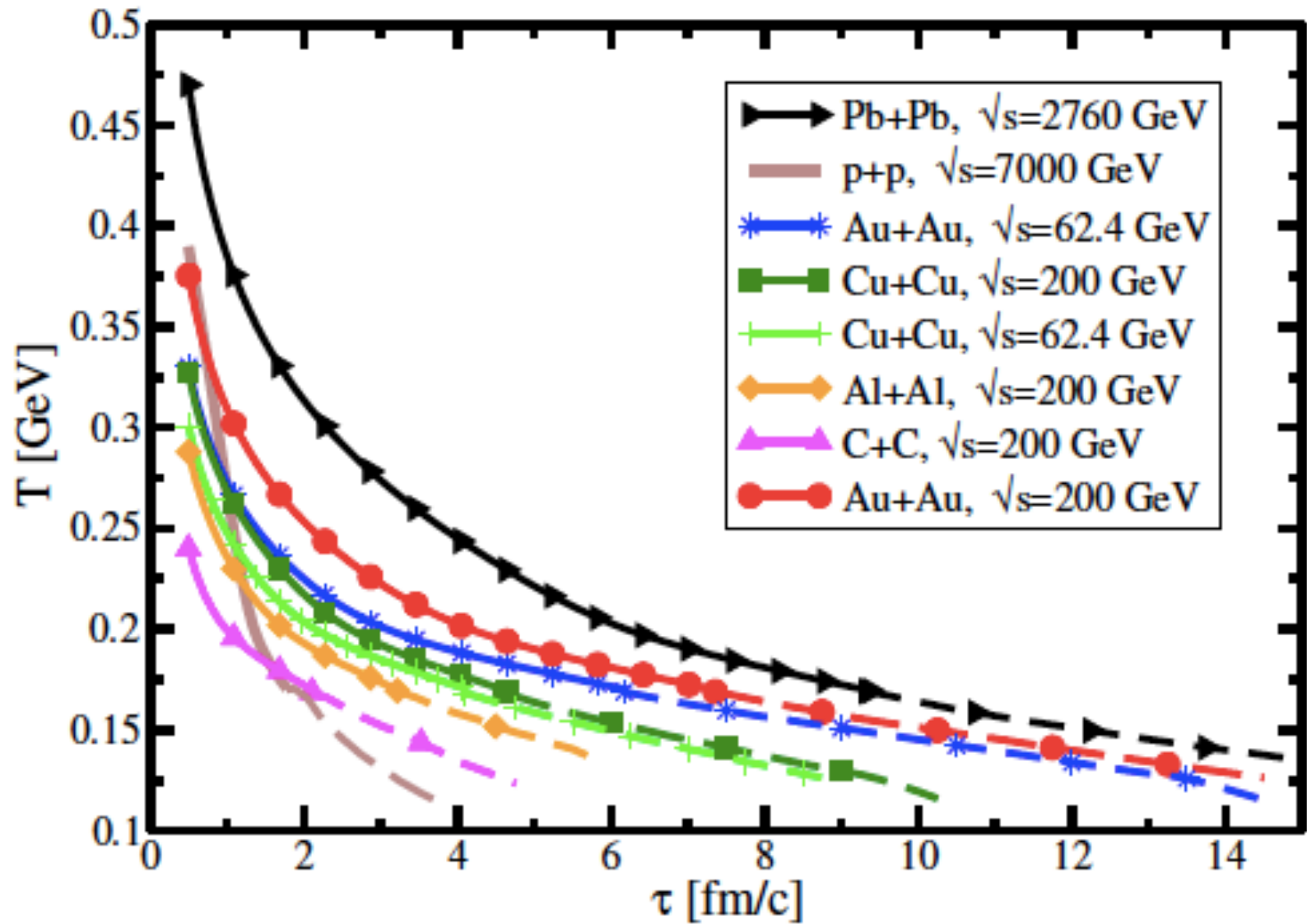
[van der Schee, PR & Pratt, 2013]

AdS+hydro+cascade

AdS/CFT+hydro results are
independent of choice of
switching time

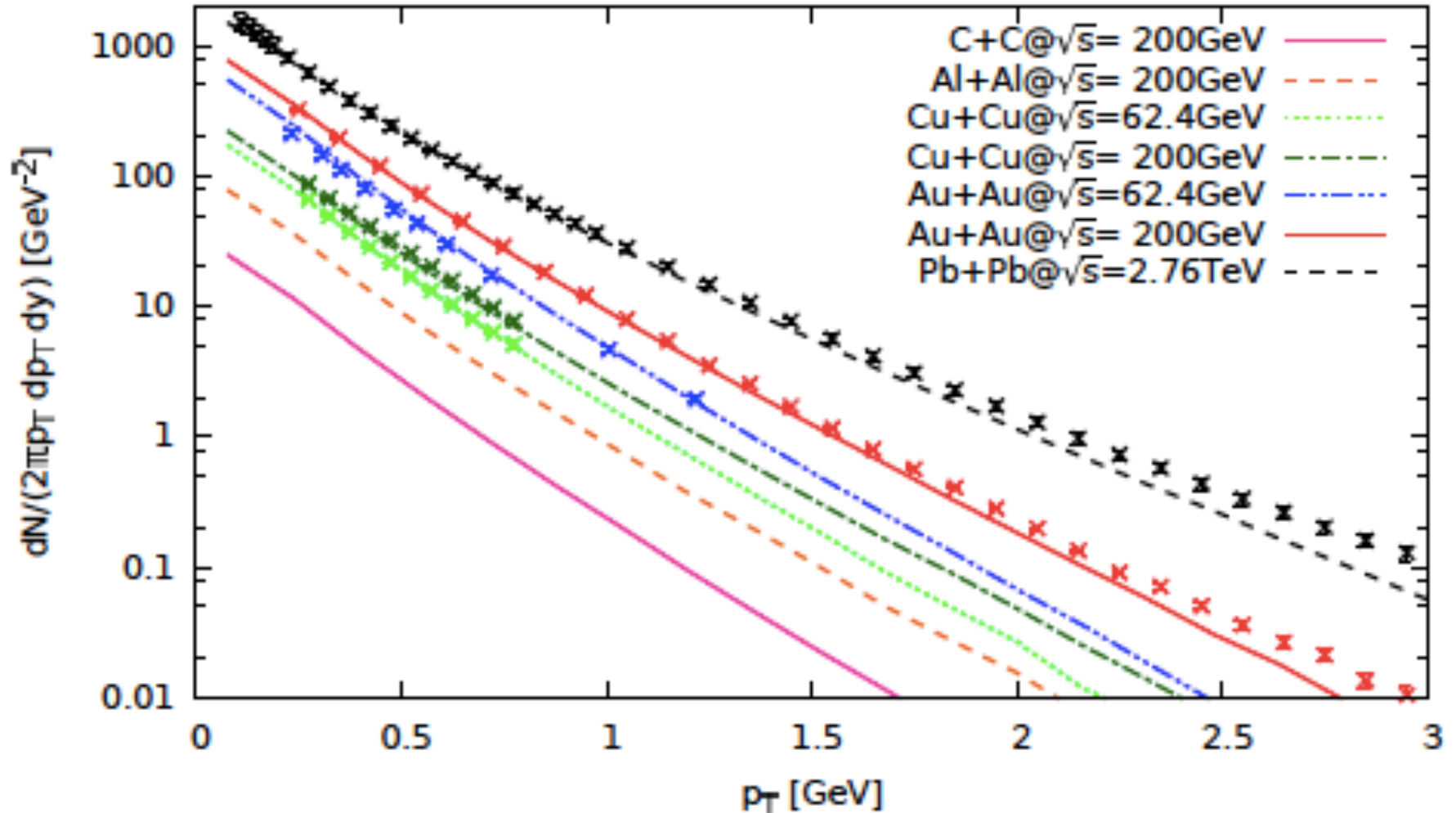
You get what you get.
No 'tuning'

Central Temperature Evolution



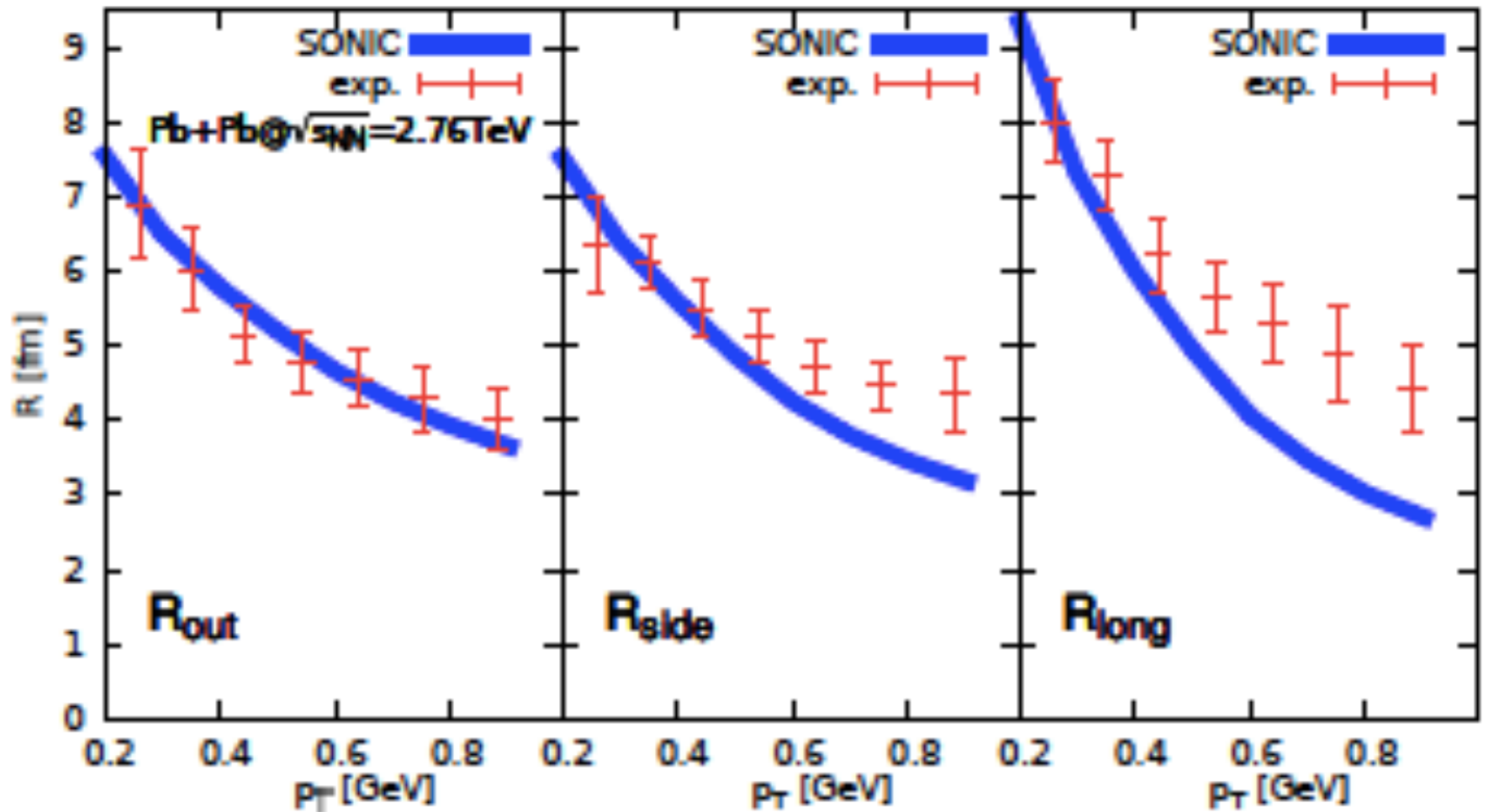
[Habich, Nagle and PR, 1409.0040]

SONIC works well for describing exp' data in AA



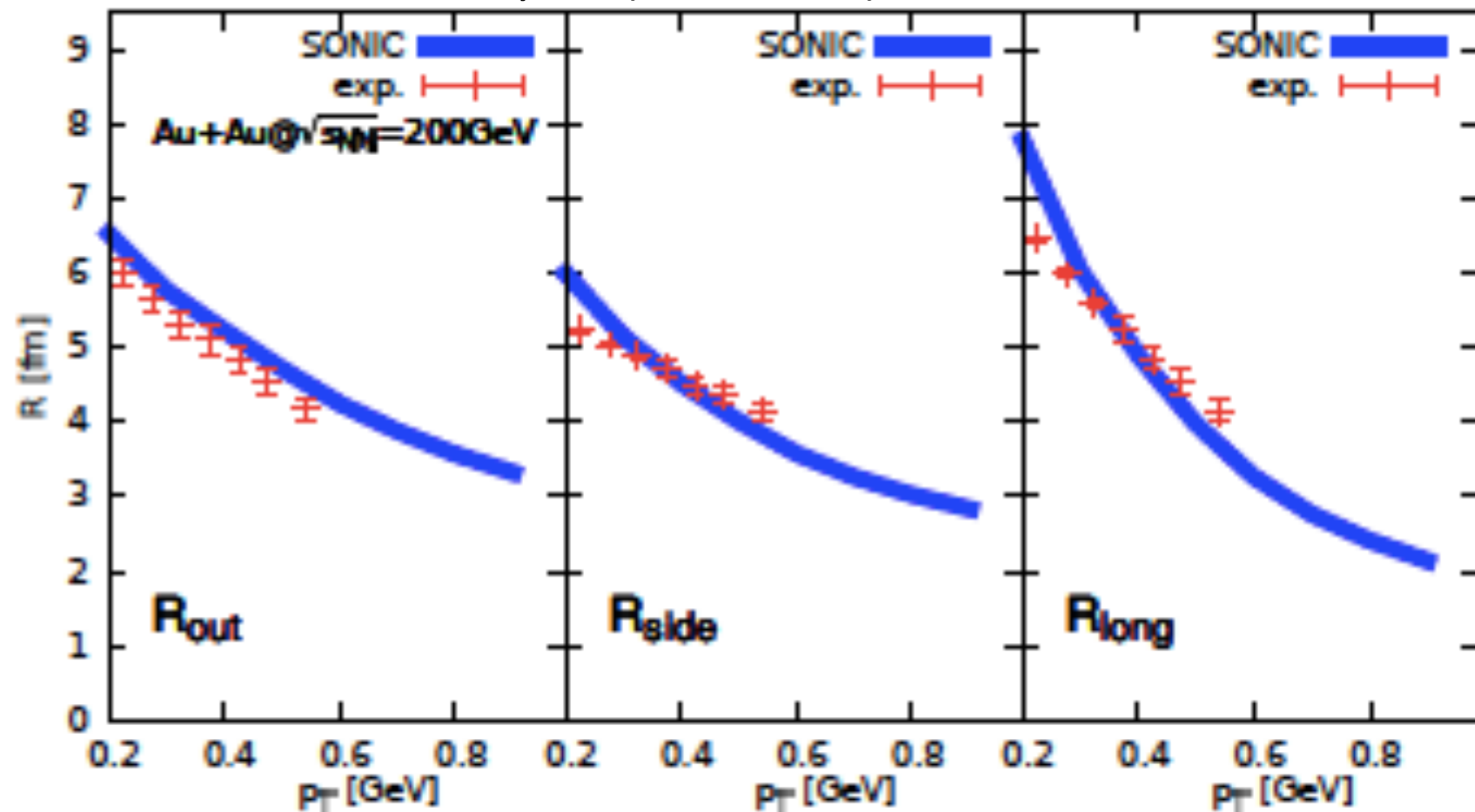
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro ($\eta/s=0.08$)+Hadron Cascade



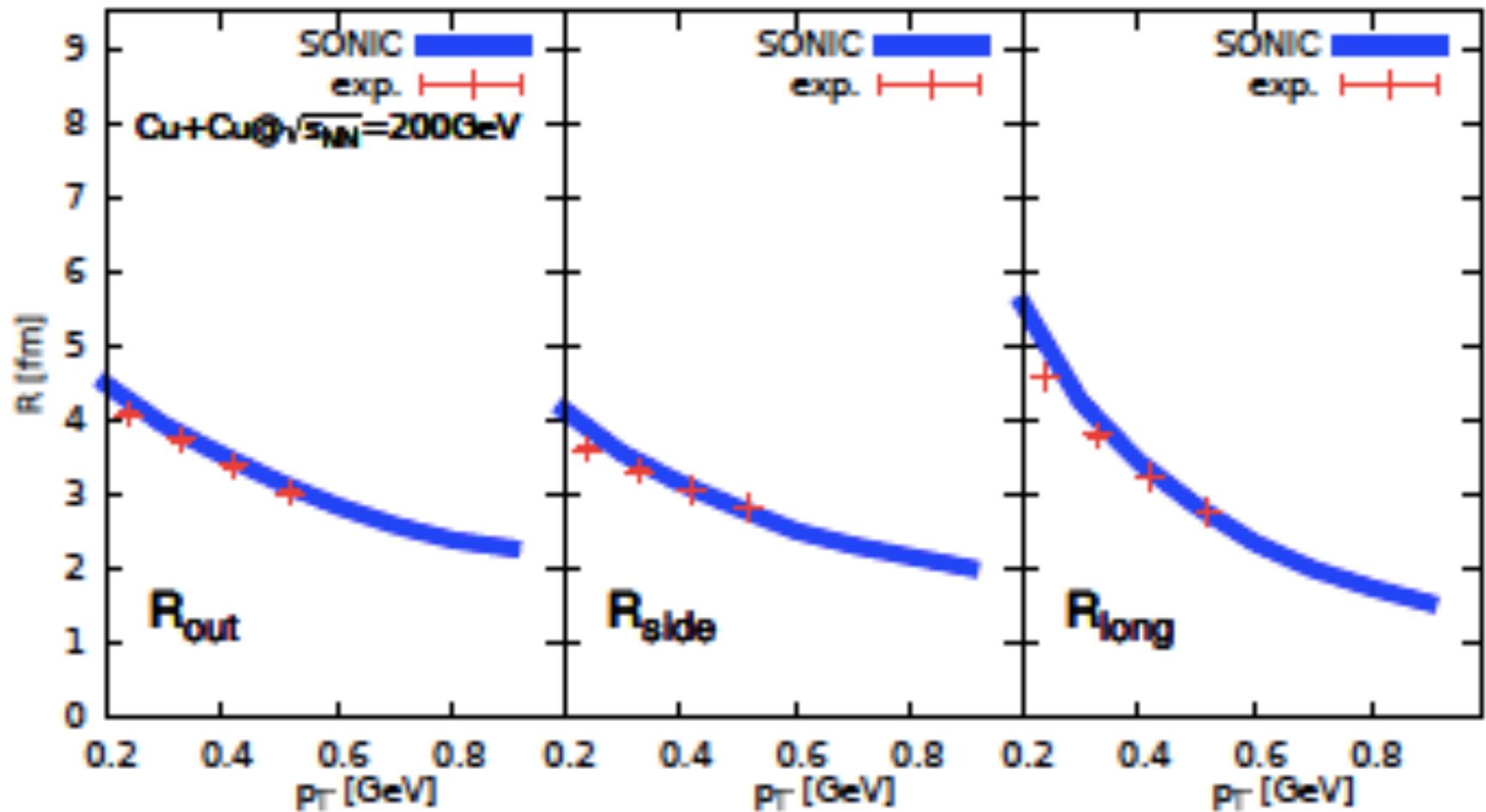
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro (eta/s=0.08)+Hadron Cascade



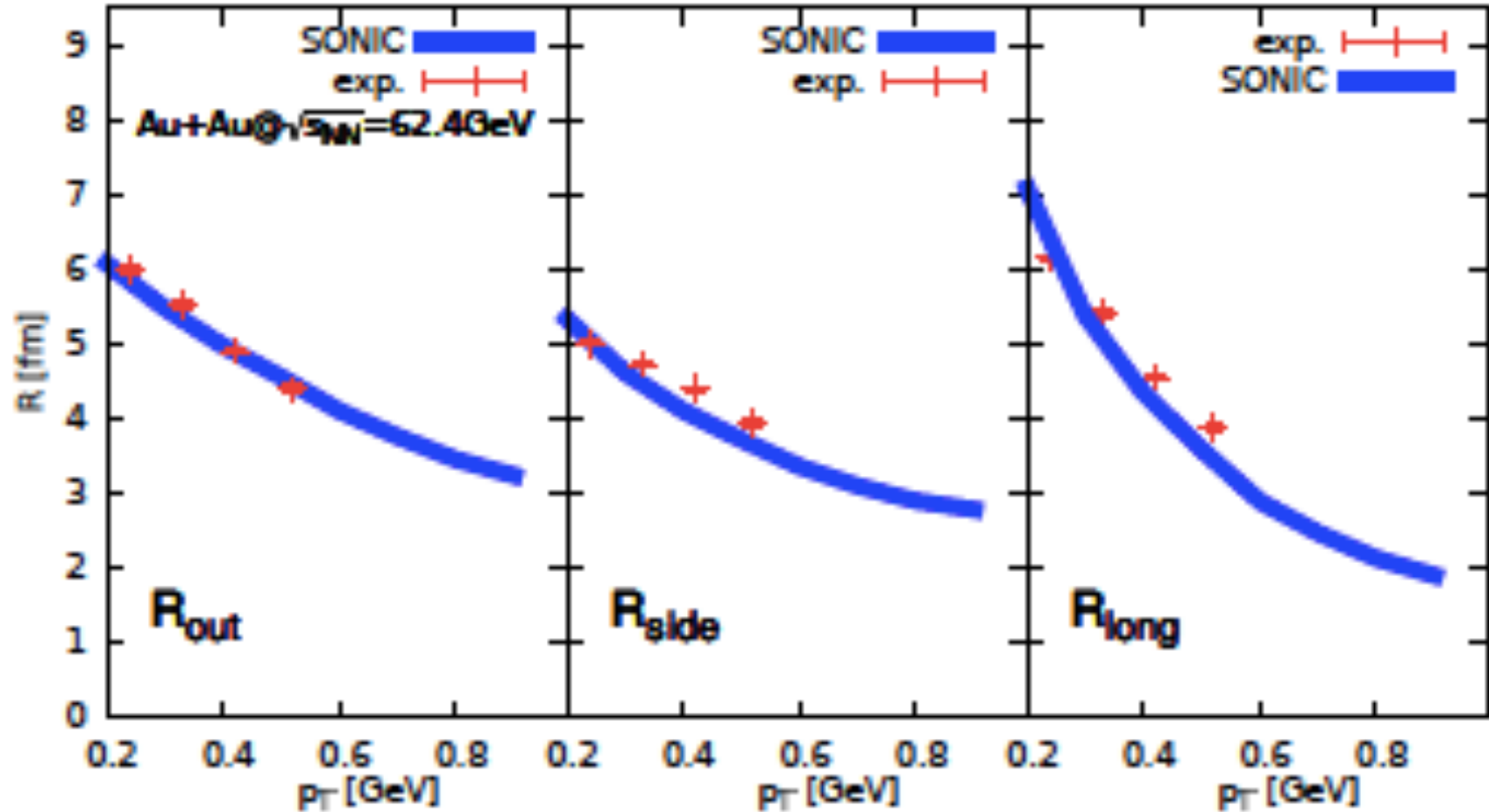
[Habich, Nagle and PR, 1409.0040]

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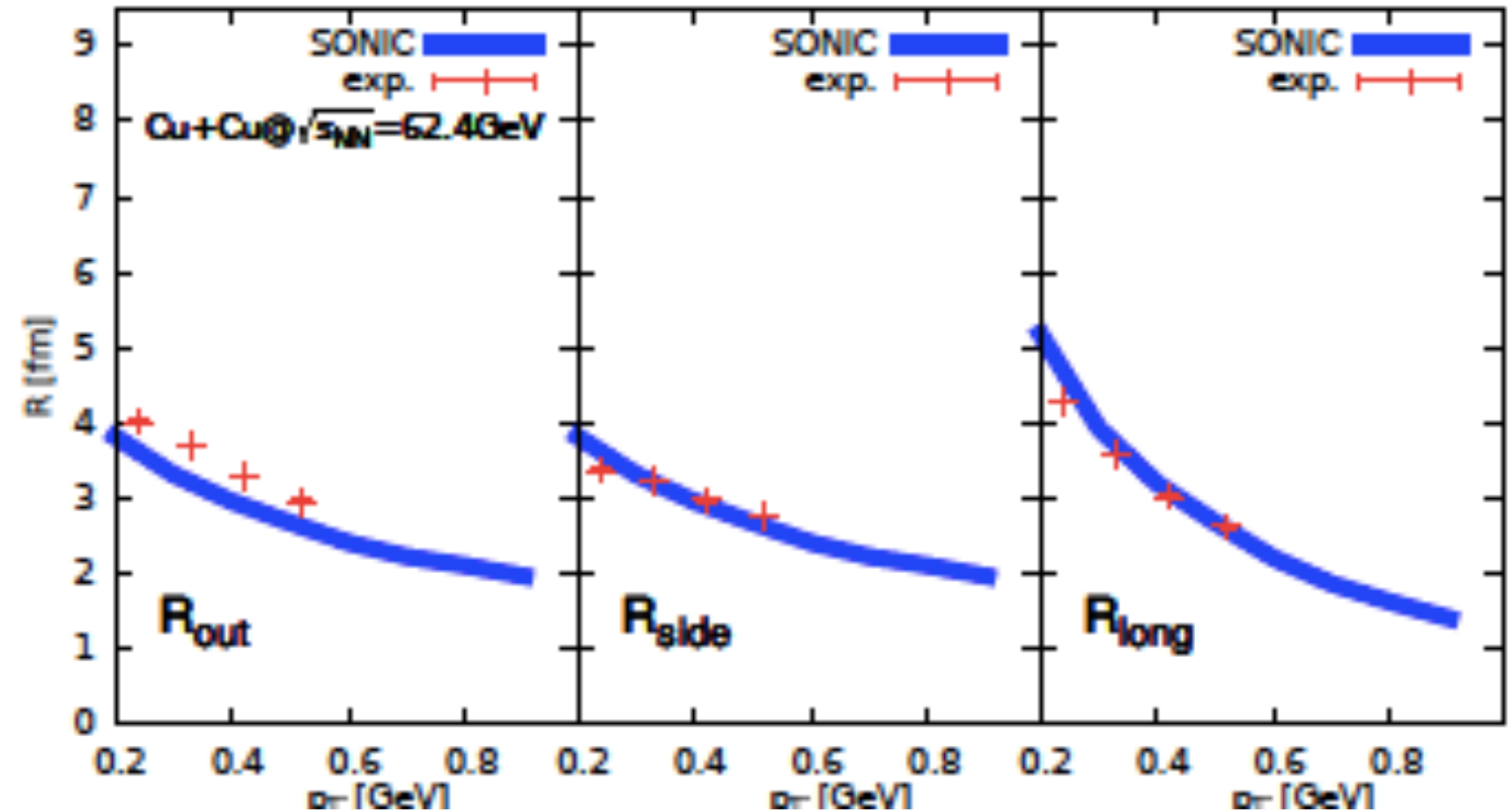
[Habich, Nagle and PR, 1409.0040]

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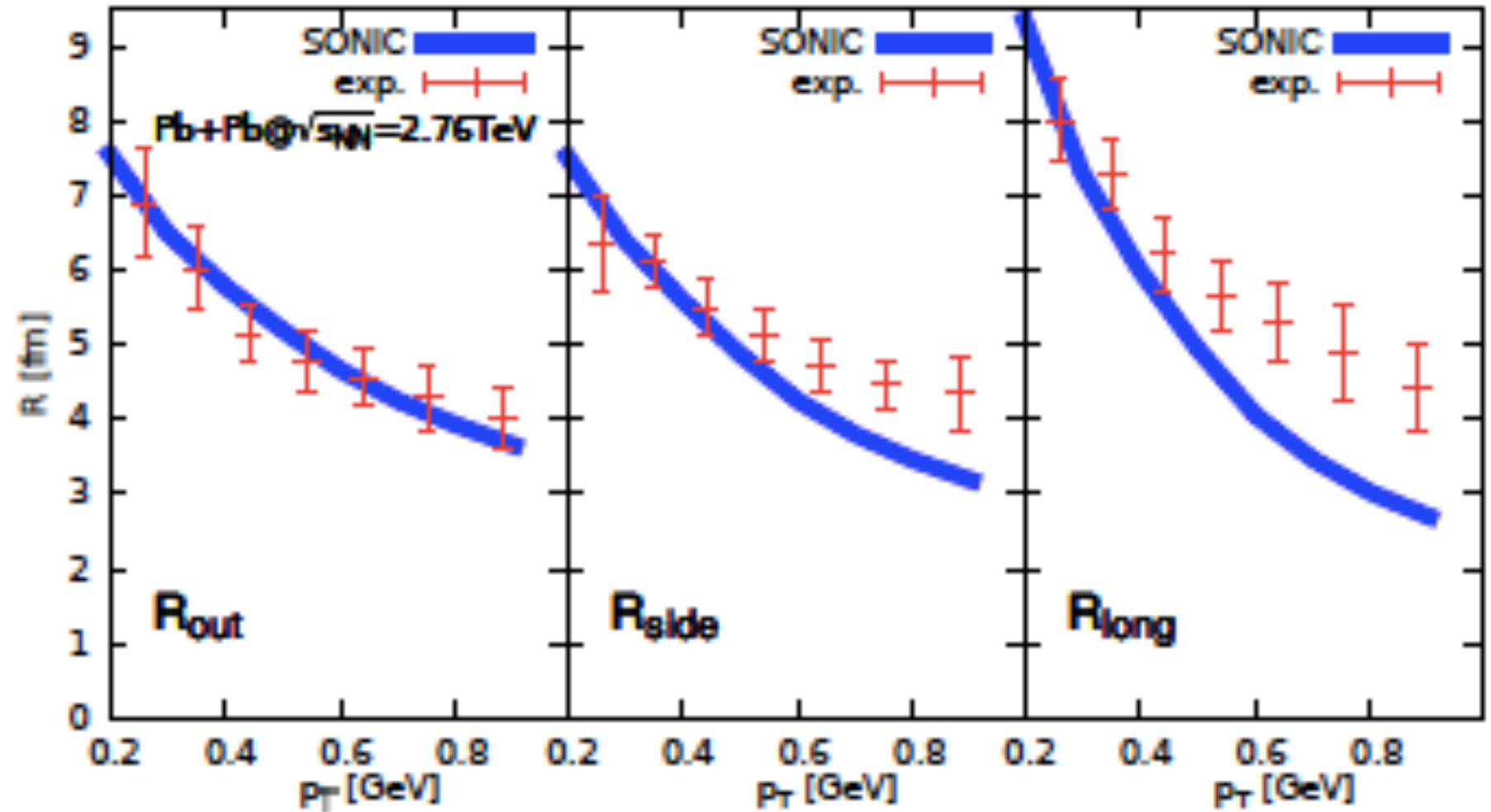
[Habich, Nagle and PR, 1409.0040]

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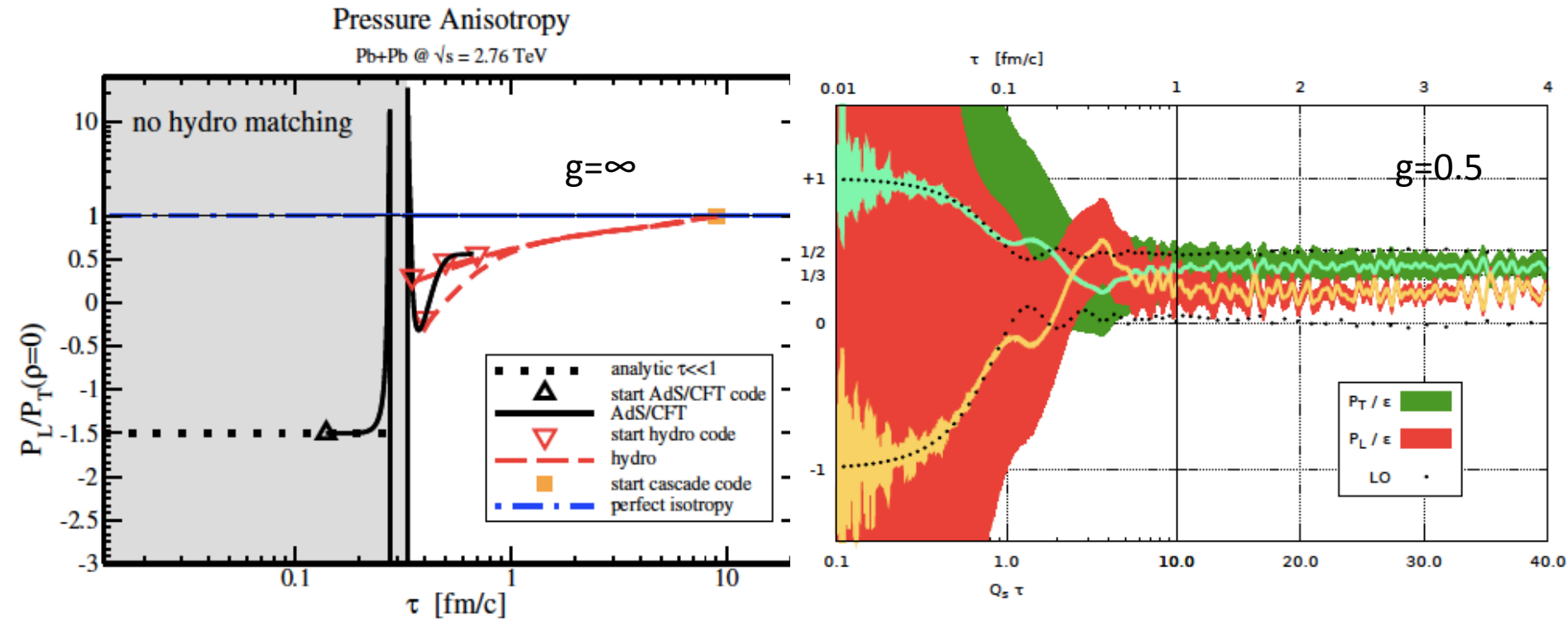
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro (eta/s=0.08)+Hadron Cascade



[Habich, Nagle and PR, 1409.0040]

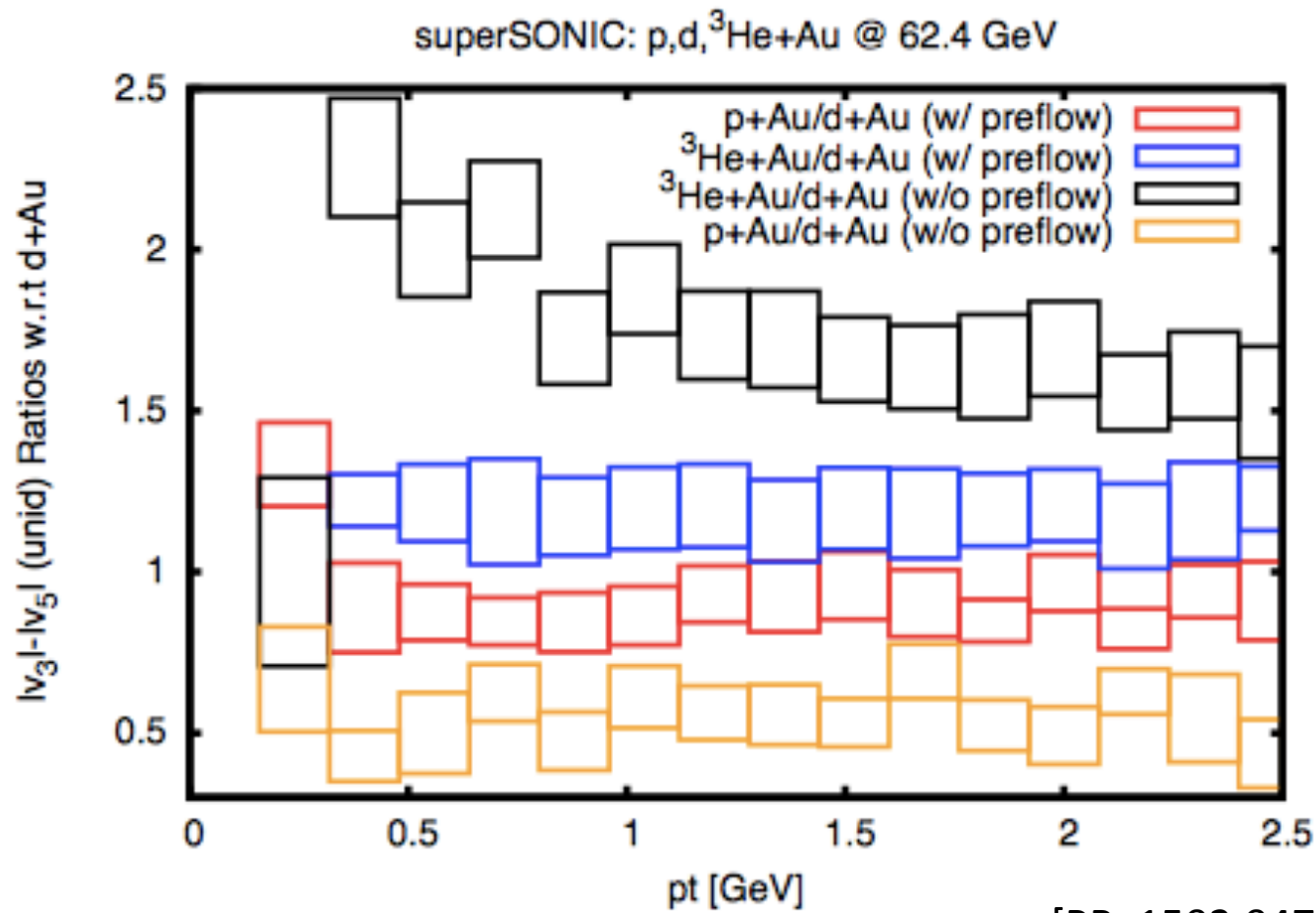
Strong vs. Weak Coupling



[van der Schee, PR & Pratt, 2013] [Epelbaum, Gelis, 2013]

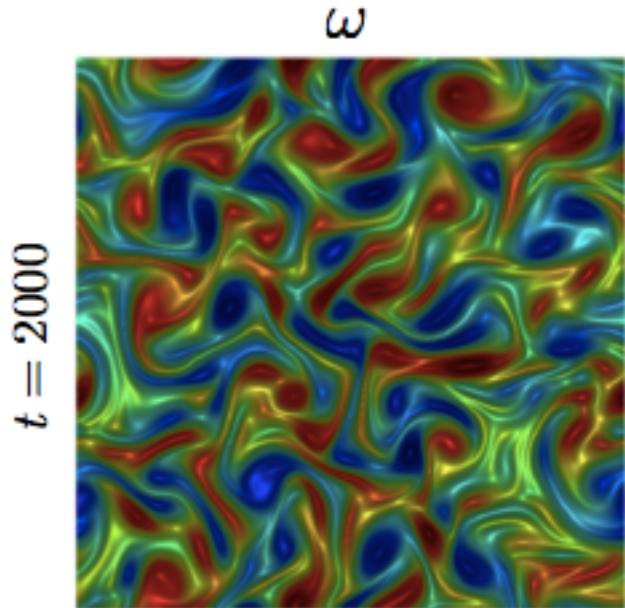
How about making more comparisons like these???

A way to probe QCD pre-eq flow?



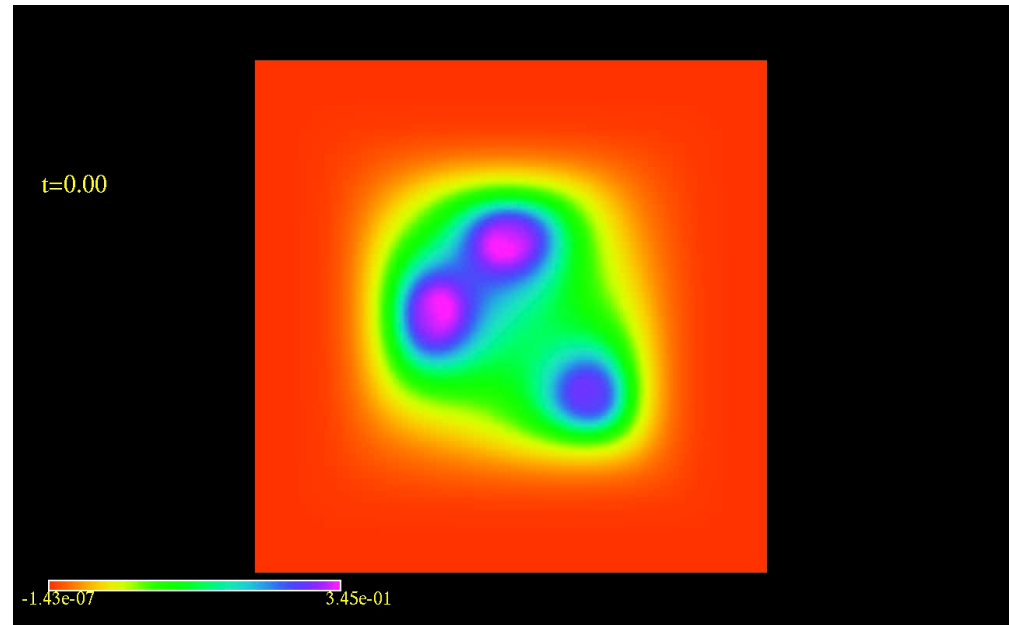
[PR, 1502.04745]

Turbulent Gravity



[Adams, Chesler, Liu 2013]

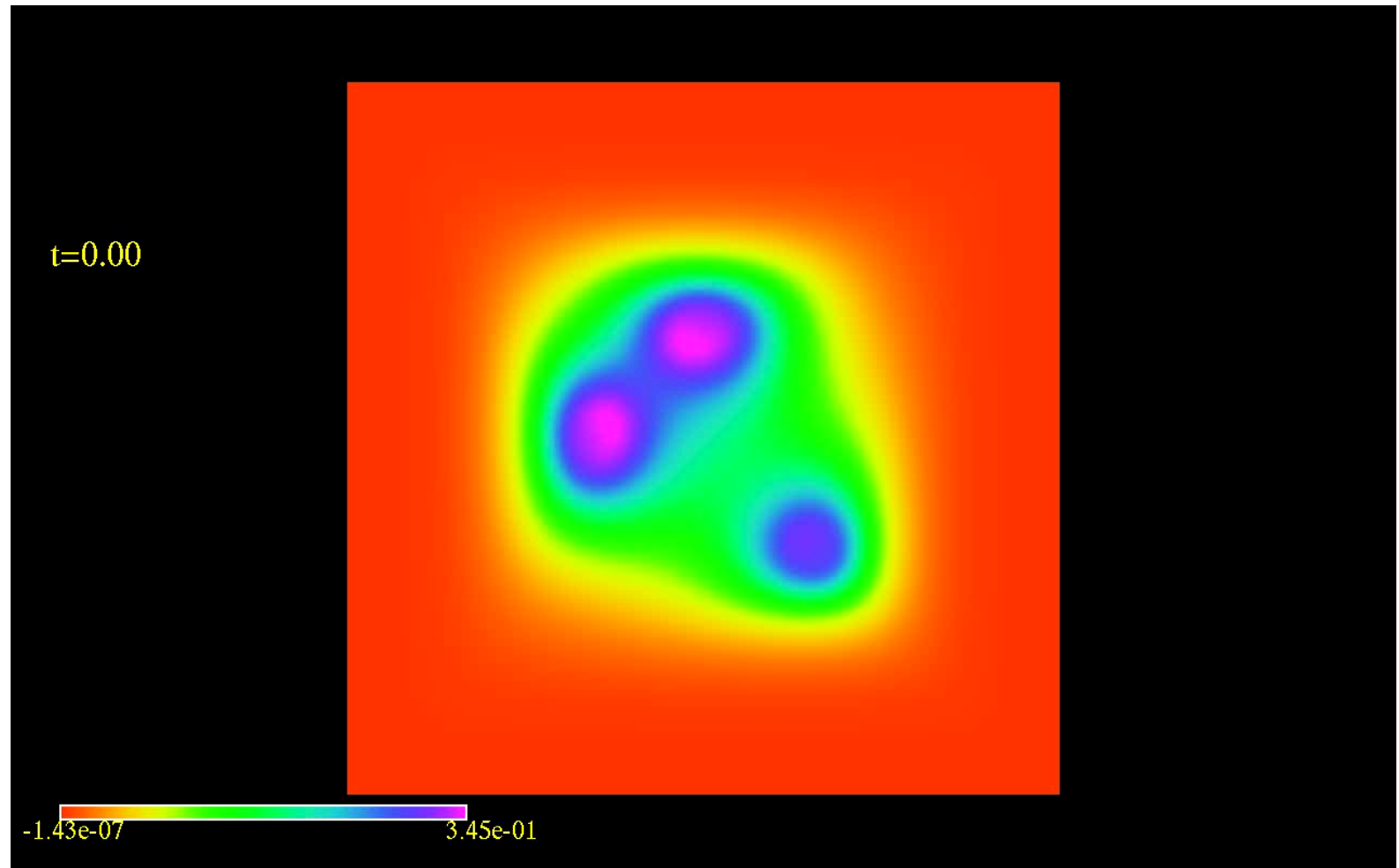
Planar Horizon (Black Brane)



[Gorda & Bantilan, unpublished]

No/Poincare Horizon

Turbulent Gravity



[Gorda & Bantilan, unpublished]

Summary/Conclusions

- General Purpose numeric solutions to Einstein Equations in asymptotic AdS using GH
- Successful numerical solutions for BH collisions in global AdS₅
- AdS (preeq)+Hydro(eq)+Hadron Cascade works surprisingly well in describing experimental data
- More results (e.g. AdS stability) coming soon!

Bonus Material

NUMERICAL RELATIVITY

Evolution Equations:

$$\begin{aligned} 0 = & -\frac{1}{2}g^{\alpha\beta}g_{\mu\nu,\alpha\beta} - g^{\alpha\beta}_{,(\mu}g_{\nu)\alpha,\beta} \\ & -H_{(\mu,\nu)} + H_\alpha\Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu} \\ & -\kappa\left(2n_{(\mu}C_{\nu)} - (1+P)g_{\mu\nu}n^\alpha C_\alpha\right) \\ & -\frac{2}{3}\Lambda_5 g_{\mu\nu} - 8\pi\left(T_{\mu\nu} - \frac{1}{3}T^\alpha_{\alpha}g_{\mu\nu}\right) \\ & \downarrow \\ 0 = & \mathcal{L}_f|_{ij}^n \quad (15 \text{ such equations, one for each } \mu\nu) \end{aligned}$$

Use second-order differencing to discretize these

NUMERICAL RELATIVITY

Evolution Equations:

$$0 = \mathcal{L}_f|_{ij}^n \quad (15 \text{ such equations, one for each } \mu\nu)$$

Solve by a Newton-Gauss-Seidel iterative scheme:

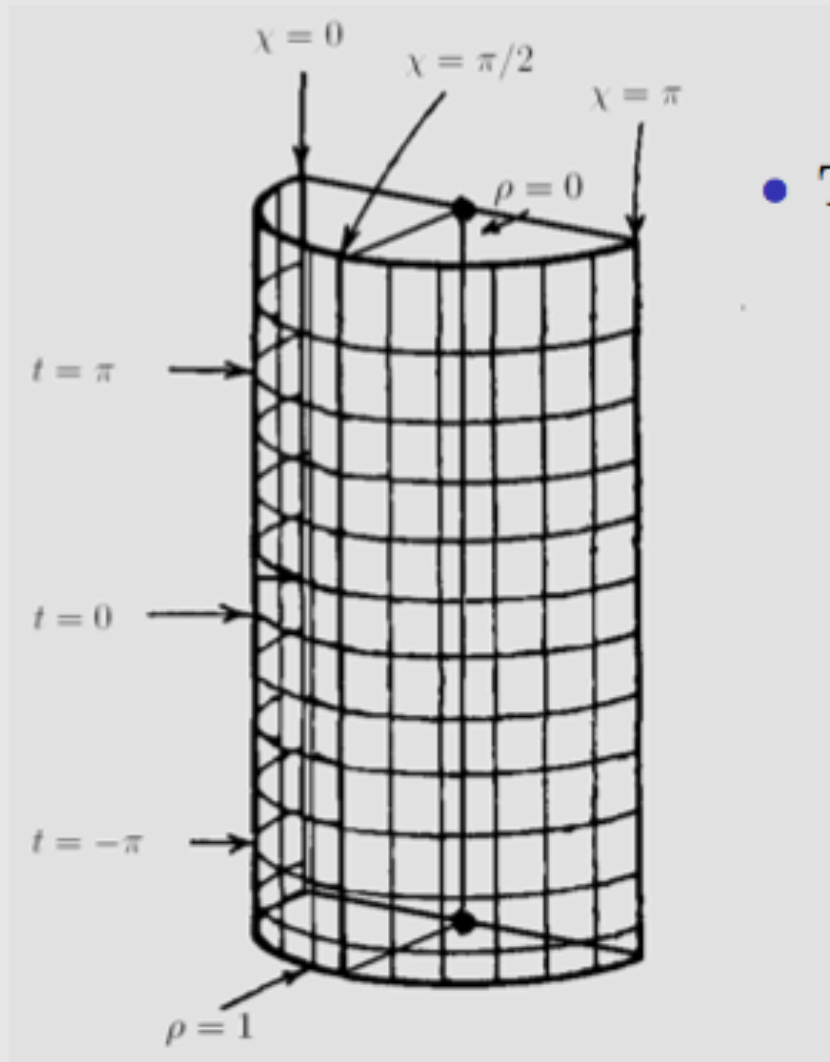
- three-level scheme at time levels t^{n+1} , t^n , t^{n-1}
- knowns: f_{ij}^n , f_{ij}^{n-1} , unknowns: f_{ij}^{n+1}
- the f_{ij}^n are used as an initial guess for the f_{ij}^{n+1}
- one iteration step:

$$\tilde{f}_{ij}^{n+1} \rightarrow \tilde{f}_{ij}^{n+1} - \frac{\mathcal{R}_f|_{ij}^n}{\mathcal{J}_f|_{ij}^n} \quad (\mathcal{R}_f|_{ij}^n = \mathcal{L}_{\tilde{f}}|_{ij}^n \text{ and } \mathcal{J}_f|_{ij}^n = \frac{\partial \mathcal{L}_f|_{ij}^n}{\partial f_{ij}^{n+1}})$$

(letting $\tilde{f}_{ij}^{n+1}$ be an approximate solution)

- iterate until $\mathcal{R}_f|_{ij}^n$ becomes sufficiently small

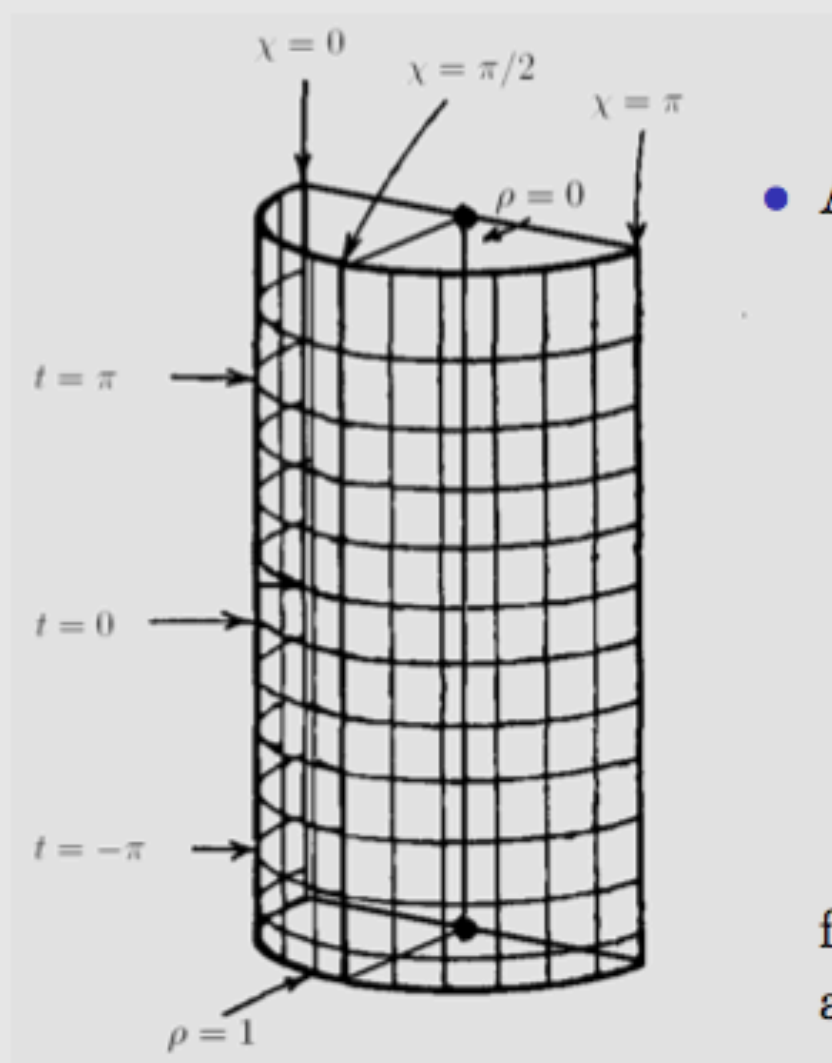
THE ANTI-DE SITTER SPACETIME



- The AdS_5 spacetime
 - its boundary is causally connected to its interior
 - spacelike infinity $\cup$ null infinity
 - forms a timelike 4-surface
 - identified with $\mathbb{R} \times S^3$

FIGURE: Conformal sketch of the AdS_5 spacetime; there is an internal 2-sphere geometry at each point of this sketch.

ASYMPTOTICALLY ANTI-DE SITTER SPACETIMES

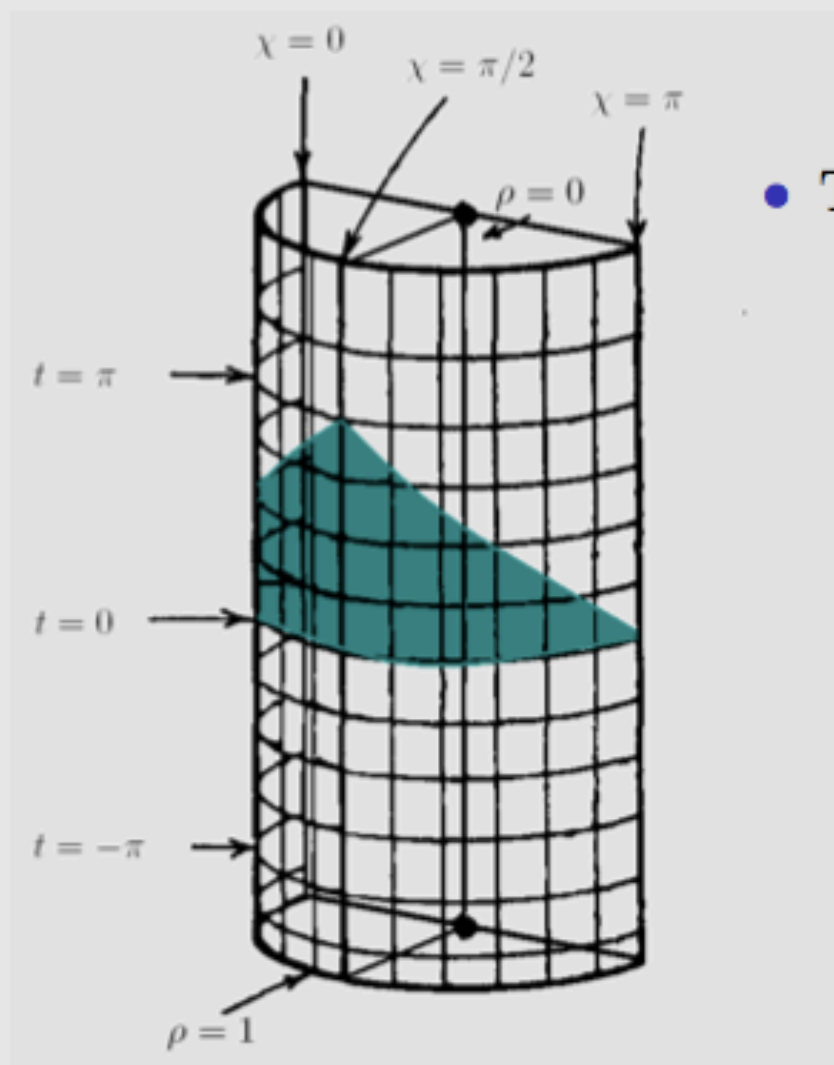


- An asymptotically AdS_5 spacetime
 - its boundary is causally connected to its interior
 - spacelike infinity $\cup$ null infinity
 - forms a timelike 4-surface
 - identified with $\mathbb{R} \times S^3$
 - metric in local coordinates:
$$\begin{cases} g_{mn} - g_{mn}^{\text{AdS}} \sim (1 - \rho)^2 \\ g_{\rho\rho} - g_{\rho\rho}^{\text{AdS}} \sim (1 - \rho)^2 \\ g_{\rho m} - g_{\rho m}^{\text{AdS}} \sim (1 - \rho)^3 \end{cases} \quad \text{as } \rho \rightarrow 1$$

for $x^m = (t, \chi, \theta, \phi)$ boundary coordinates
and ρ AdS radial coordinate

FIGURE: Conformal sketch of an asymptotically AdS_5 spacetime that preserves the $\text{SO}(3)$ symmetry that rotates a 2-sphere at each point.

THE ANTI-DE SITTER SPACETIME



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FIGURE: Conformal sketch of the AdS_5 spacetime; there is an internal 2-sphere geometry at each point of this sketch.

NUMERICAL RELATIVITY

Initial Data:

$$0 = \mathcal{L}_\zeta|_{ij} \quad (5 \text{ such equations, one for each } \mu)$$

Solve by a multigrid method:

1. Compute residual on fine grid
 $\mathcal{R}_{\zeta_h} = \mathcal{L}_{\tilde{\zeta}_h}.$
2. Inject fine grid residual $\mathcal{R}_{\zeta_h}$ and approx soln $\tilde{\zeta}_h$ onto coarse grid
 $\mathcal{R}_{\zeta_h} \rightarrow \mathcal{R}_{\zeta_{inj}}, \tilde{\zeta}_h \rightarrow \tilde{\zeta}_{inj}.$
3. Find approx soln $\tilde{\zeta}_{2h}$ on coarse grid by solving difference equation
 $\mathcal{L}_{\zeta_{2h}} = d_{2h}$ for $d_{2h} = \mathcal{L}_{\zeta_{inj}} - \mathcal{R}_{\zeta_{inj}}.$
4. Compute correction on coarse grid
 $\tilde{v}_{2h} = \tilde{\zeta}_{2h} - \tilde{\zeta}_{inj}.$
5. Interpolate correction from coarse grid to fine grid
 $\tilde{v}_{2h} \rightarrow \tilde{v}_h.$
6. Generate next approx on fine grid using coarse-grid correction
 $\tilde{\zeta}_h^{\text{new}} = \tilde{\zeta}_h + \tilde{v}_h.$

NUMERICAL RELATIVITY

Other ingredients:

- Kreiss-Oliger style numerical dissipation
- Apparent horizon finder and excision
- Lapse damping
- Coordinate choice near the AdS boundary

COORDINATE CHOICE NEAR THE AdS BOUNDARY

- Coordinate choice in asymptotically AdS spacetimes
 - not enough to simply demand b.c.s for $\bar{g}_{\mu\nu}$, $\bar{H}_\mu$, $\bar{\phi}$
$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + (1 - \rho)^{\#} \bar{g}_{\mu\nu}$$
$$H_\mu = H_\mu^{AdS} + (1 - \rho)^{\#} \bar{H}_\mu$$
$$\varphi = (1 - \rho)^{\#} \bar{\varphi}$$
 - how to choose H_μ so that b.c.s are preserved by evolution?
- Example: tt component of field equations near $\rho = 1$

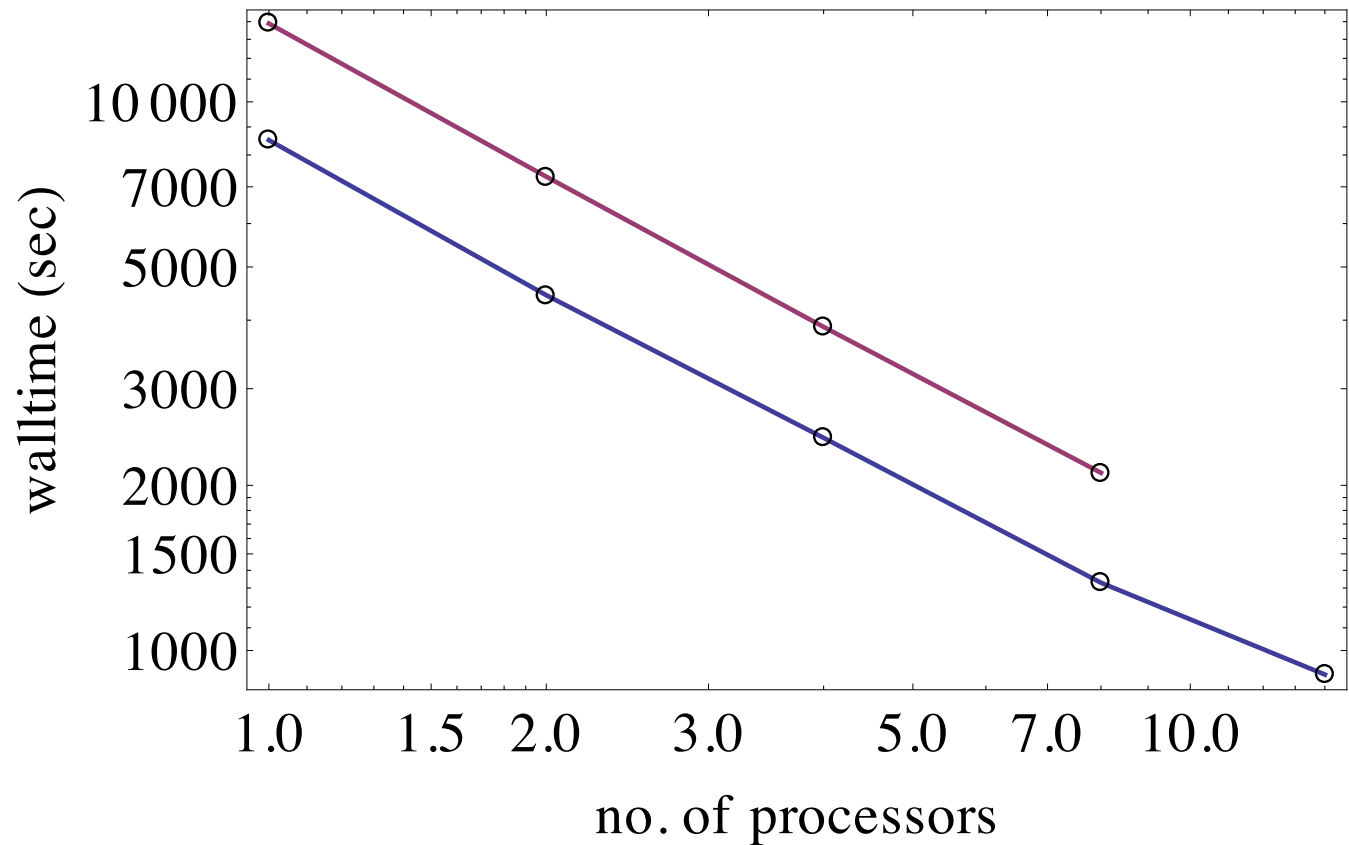
$$\tilde{\square} \bar{g}_{(1)tt} = (-8\bar{g}_{(1)\rho\rho} + 4\bar{H}_{(1)\rho})(1 - \rho)^{-2} + \dots$$

- regularity requires a delicate cancellation between terms in the near-boundary limit
- smart coordinate choice: $\bar{H}_{(1)\rho} = 2\bar{g}_{(1)\rho\rho}$

Infiniband connections essential

Lnxfarm: single node,
multiple cores used

Eridanus: single core
on different nodes
used



From Bulk to Boundary

$$t' = W \sin t$$

$$x_1 = W \sin \chi \sin \theta \cos \phi$$

$$x_2 = W \sin \chi \sin \theta \sin \phi$$

$$x_3 = W \cos \chi$$

$$W \equiv 1/(\cos t + \sin \chi \cos \theta)$$

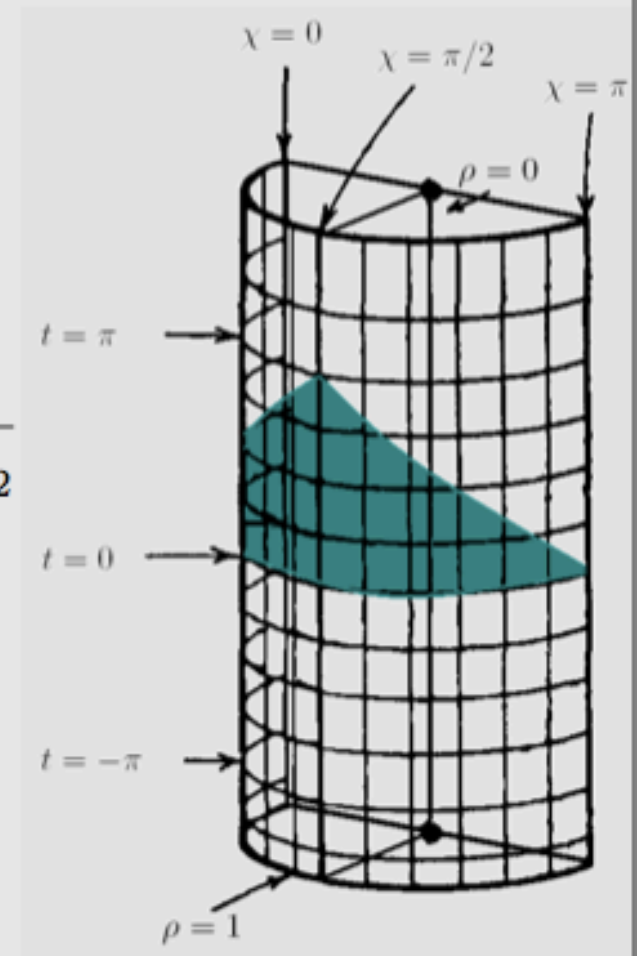
$$= \sqrt{(t')^2 + \frac{1}{4}(1 + x_1^2 + x_2^2 + x_3^2 - (t')^2)^2}$$

$$g_{\mathbb{R}^{3,1}}^{ab} = W^{-2} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} g_{\mathbb{R} \times S^3}^{\mu\nu}$$

$$\epsilon_{\mathbb{R}^{3,1}} = W^{-4} \epsilon_{\mathbb{R} \times S^3}$$

$$u_{\mathbb{R}^{3,1}}^a = W^{-1} \frac{\partial x^a}{\partial x^\mu} u_{\mathbb{R} \times S^3}^\mu$$

$$T_{\mathbb{R}^{3,1}}^{ab} = W^{(-d-2)} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} T_{\mathbb{R} \times S^3}^{\mu\nu}$$



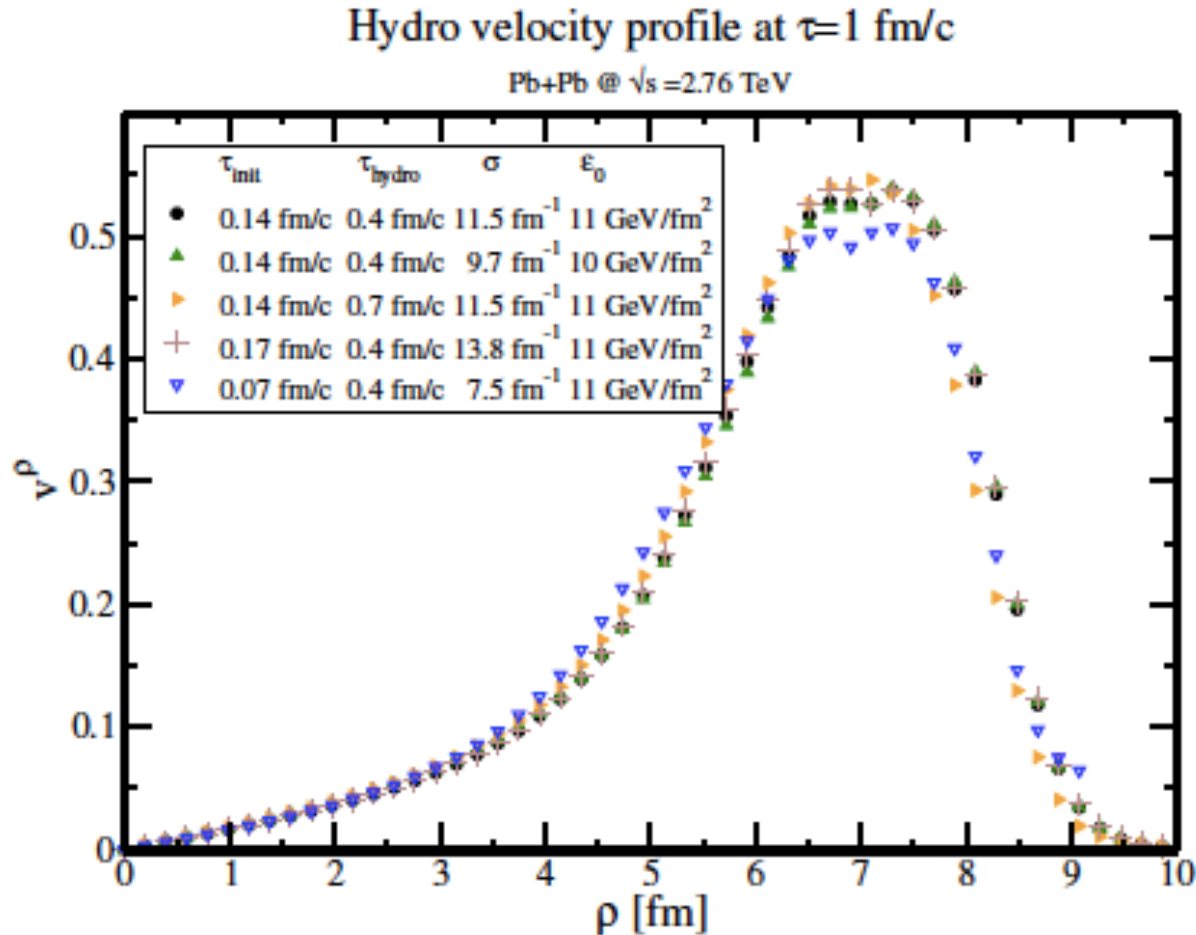
CONFORMAL FIELD THEORY DUAL

- Bulk field / CFT operator
 - e.g. metric dynamics / CFT stress tensor one-point function
$$\bar{g}_{\mu\nu}(x^m, \rho) \qquad \langle T_{\mu\nu}(x^m) \rangle_{\text{CFT}}$$
- How do bulk fields encode boundary CFT operators?

$$\langle T_{\mu\nu} \rangle_{\text{CFT}} = \lim_{\rho \rightarrow 1} \left[\frac{1}{8\pi} \left({}^{(\rho)}\Theta_{\mu\nu} - {}^{(\rho)}\Theta \Sigma_{\mu\nu} - \frac{3}{L} \Sigma_{\mu\nu} + {}^{(\rho)}G_{\mu\nu} \frac{L}{2} \right) - t_{\mu\nu} \right]$$

Given a $\rho = \text{const.}$ time-like hypersurface ∂M_ρ , ${}^{(\rho)}\Theta_{\mu\nu} = -\Sigma^\alpha{}_\mu \Sigma^\beta{}_\nu \nabla_{(\alpha} S_{\beta)}$ is the extrinsic curvature of ∂M_ρ , S^μ is a space-like, outward pointing unit vector normal to the surface ∂M_ρ , $\Sigma_{\mu\nu} \equiv g_{\mu\nu} - S_\mu S_\nu$ is the induced 4-metric on ∂M_ρ , ∇_α is the covariant derivative operator, and ${}^{(\rho)}G_{\mu\nu}$ is the Einstein tensor associated with $\Sigma_{\mu\nu}$. Setting $L = 1$, the non-zero components of the (non-dynamical) Casimir contribution $t_{\mu\nu}$ that we have explicitly subtracted above are $t_{tt} = 3(1 - \rho)^2 / (64\pi)$, $t_{\chi\chi} = (1 - \rho)^2 / (64\pi)$, $t_{\theta\theta} = (1 - \rho)^2 \sin^2 \chi / (64\pi)$, and $t_{\phi\phi} = t_{\theta\theta} \sin^2 \theta$.

Shocks with Transverse Profiles



[van der Schee, PR & Pratt, 2013]

Use AdS/CFT initial conditions and compare to data

- Idea: pre-equilibrium flow for smooth, central collisions is simple
- Parametrize as

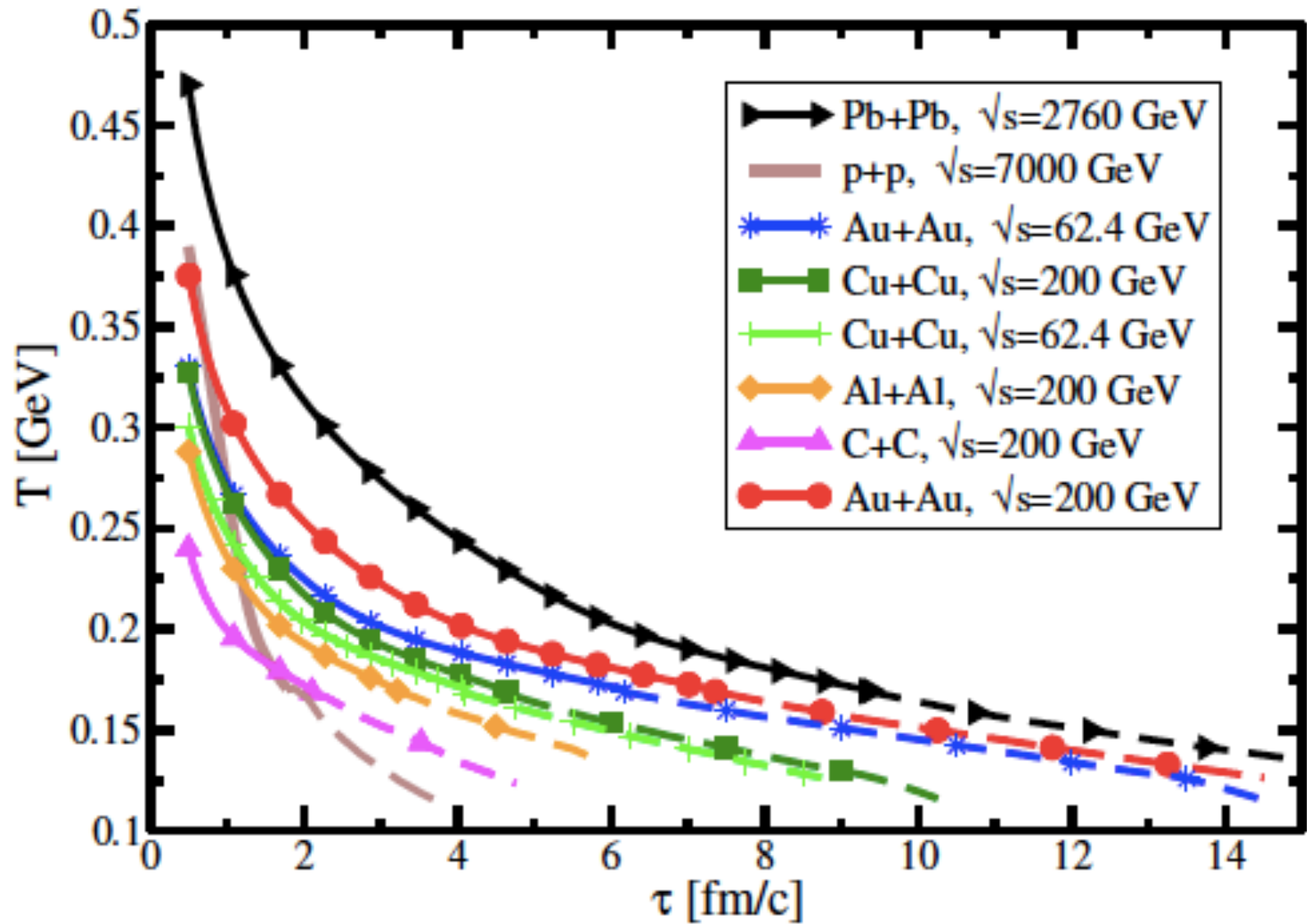
$$v^r(\tau, r) = -\frac{\tau}{3} \partial_r \ln T_A^2(r).$$

[Habich, Nagle and PR, 1409.0040]

and use for different collision systems (Pb+Pb, Au+Au, Cu+Cu, Al+Al, C+C, p+p)

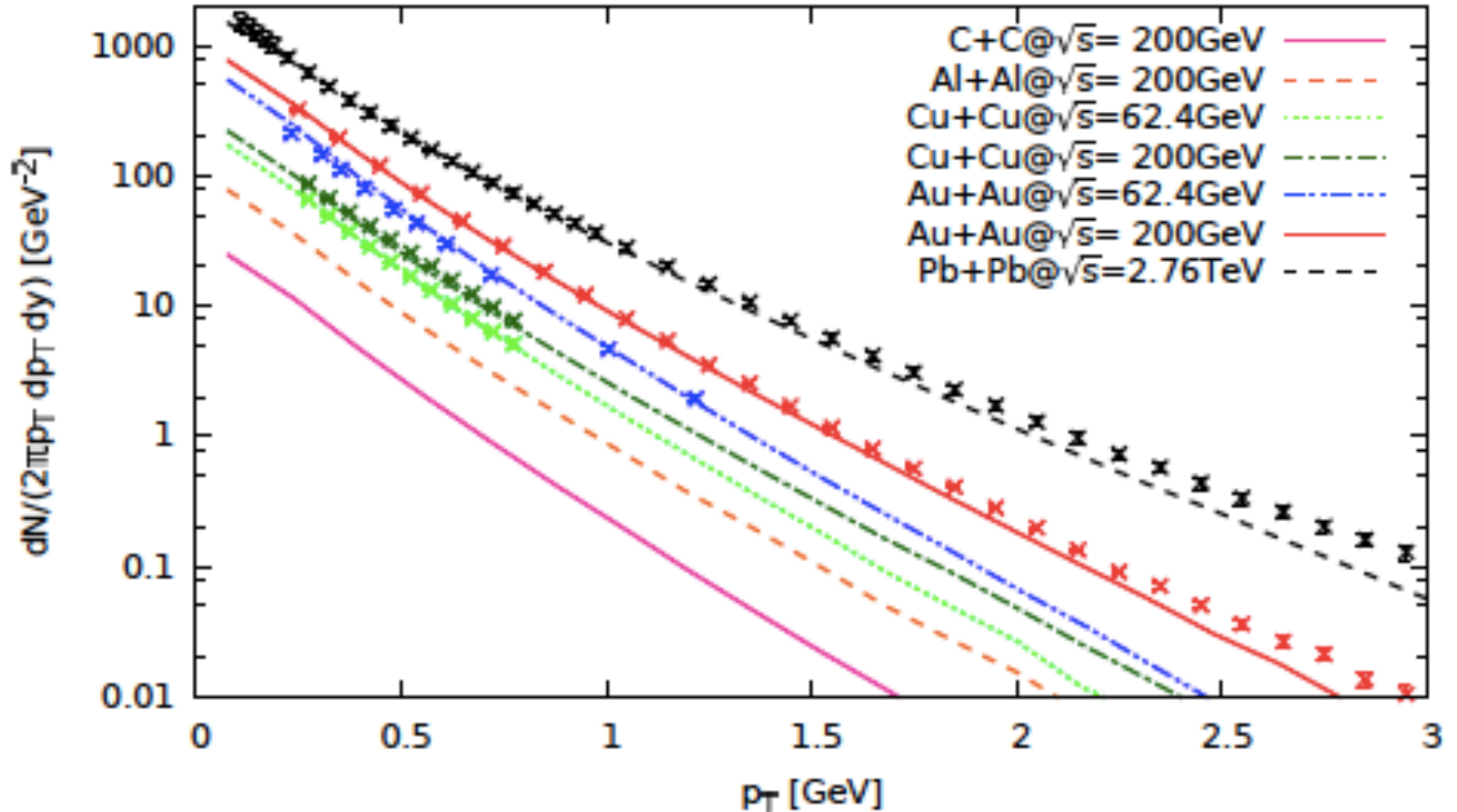
Compare: $\frac{T_{0x}}{T_{00}} \approx -\frac{\partial_x T_{00}}{2T_{00}} t,$ [Scott & Vredevoogd, PRC79 2009]

Central Temperature Evolution



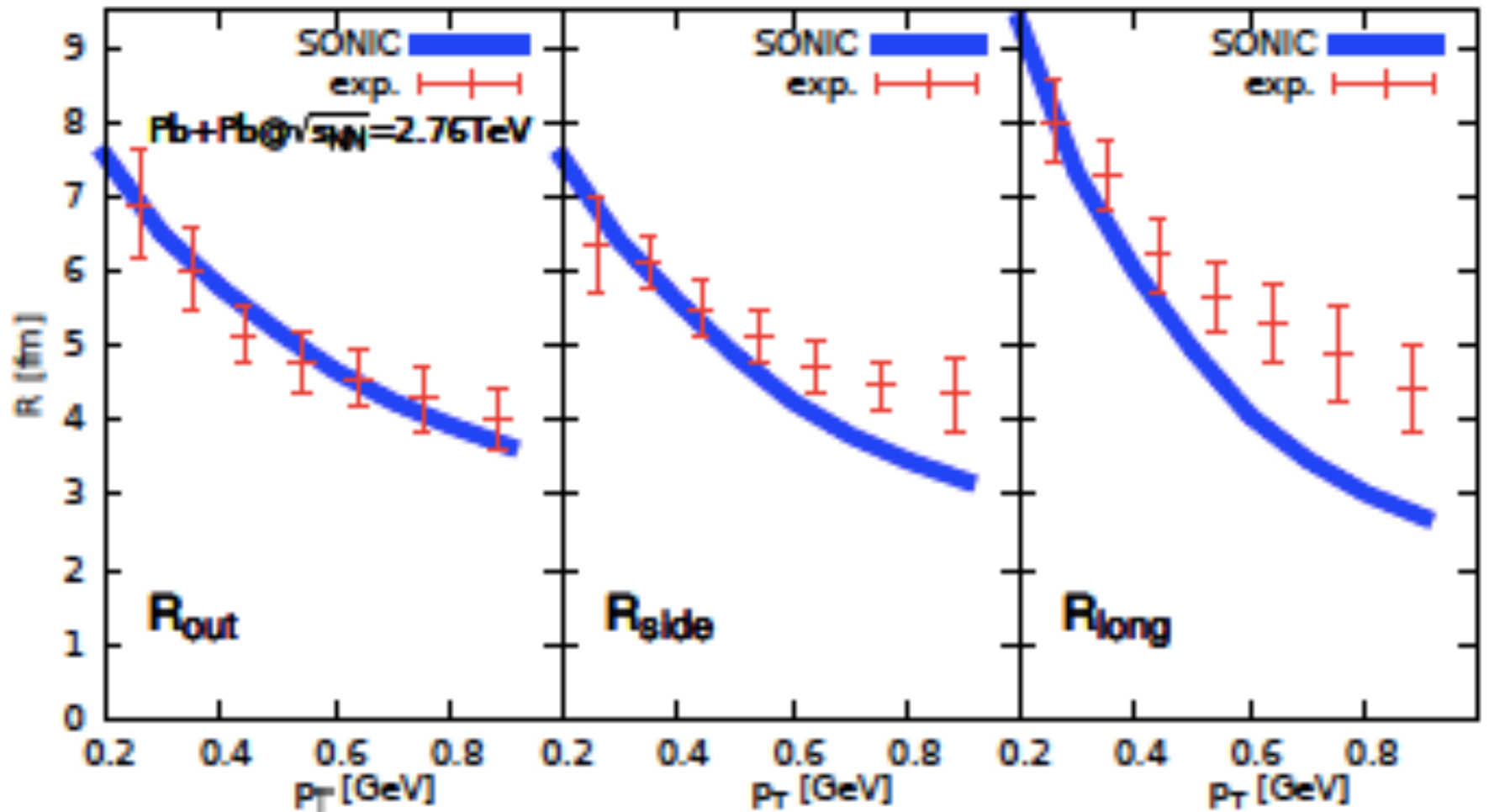
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro ($\eta/s=0.08$)+Hadron Cascade



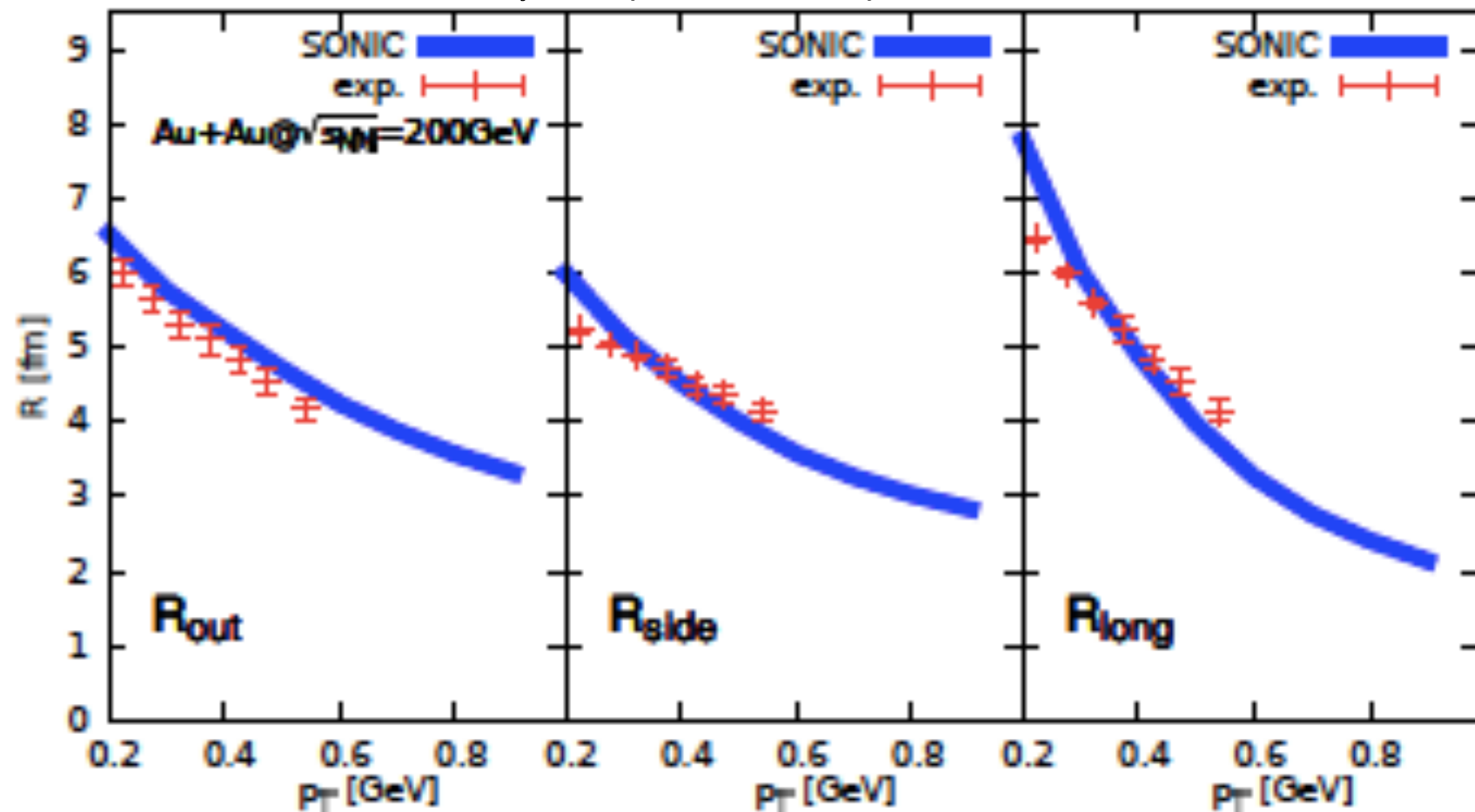
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro (eta/s=0.08)+Hadron Cascade



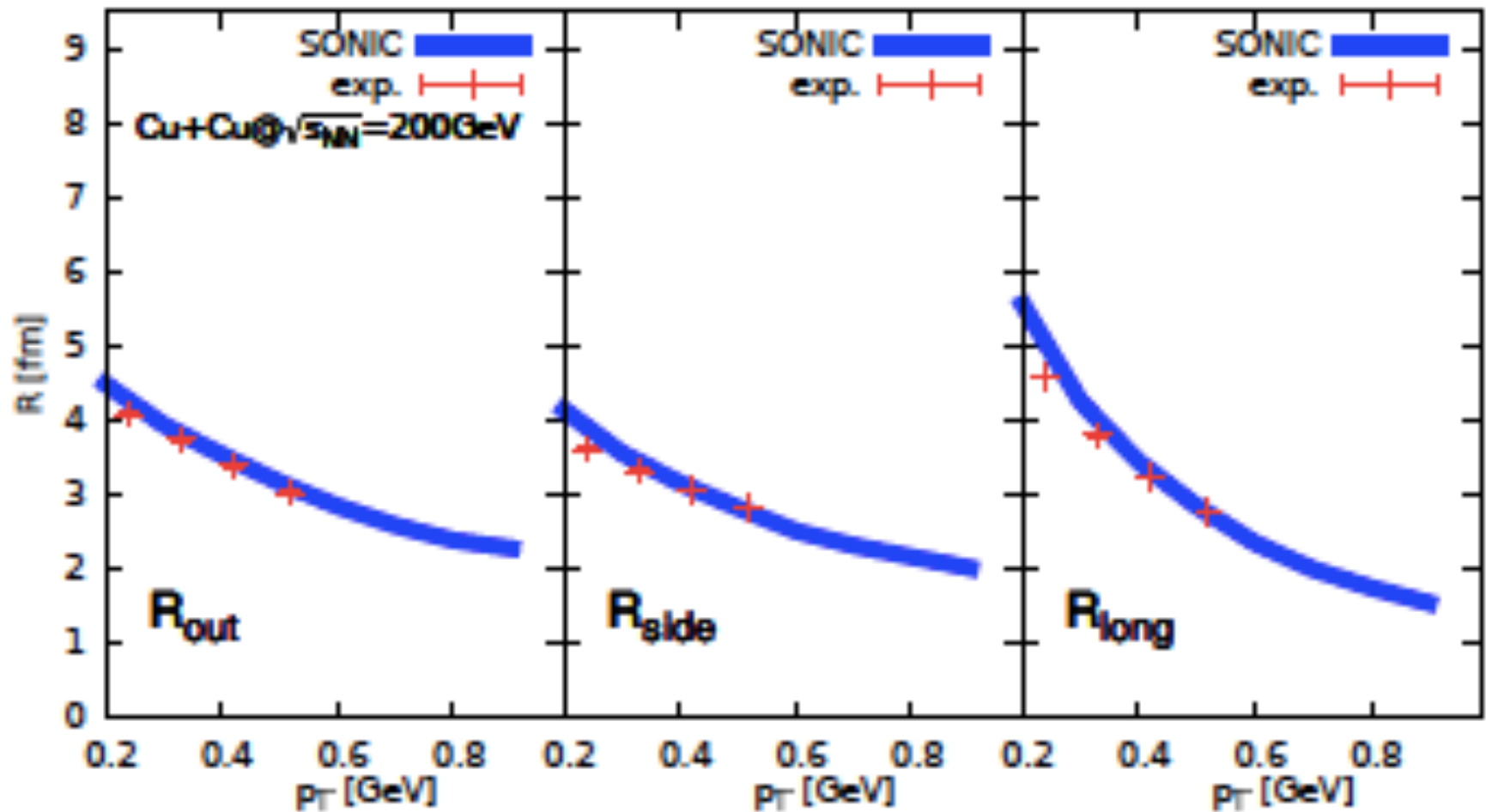
[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro (eta/s=0.08)+Hadron Cascade



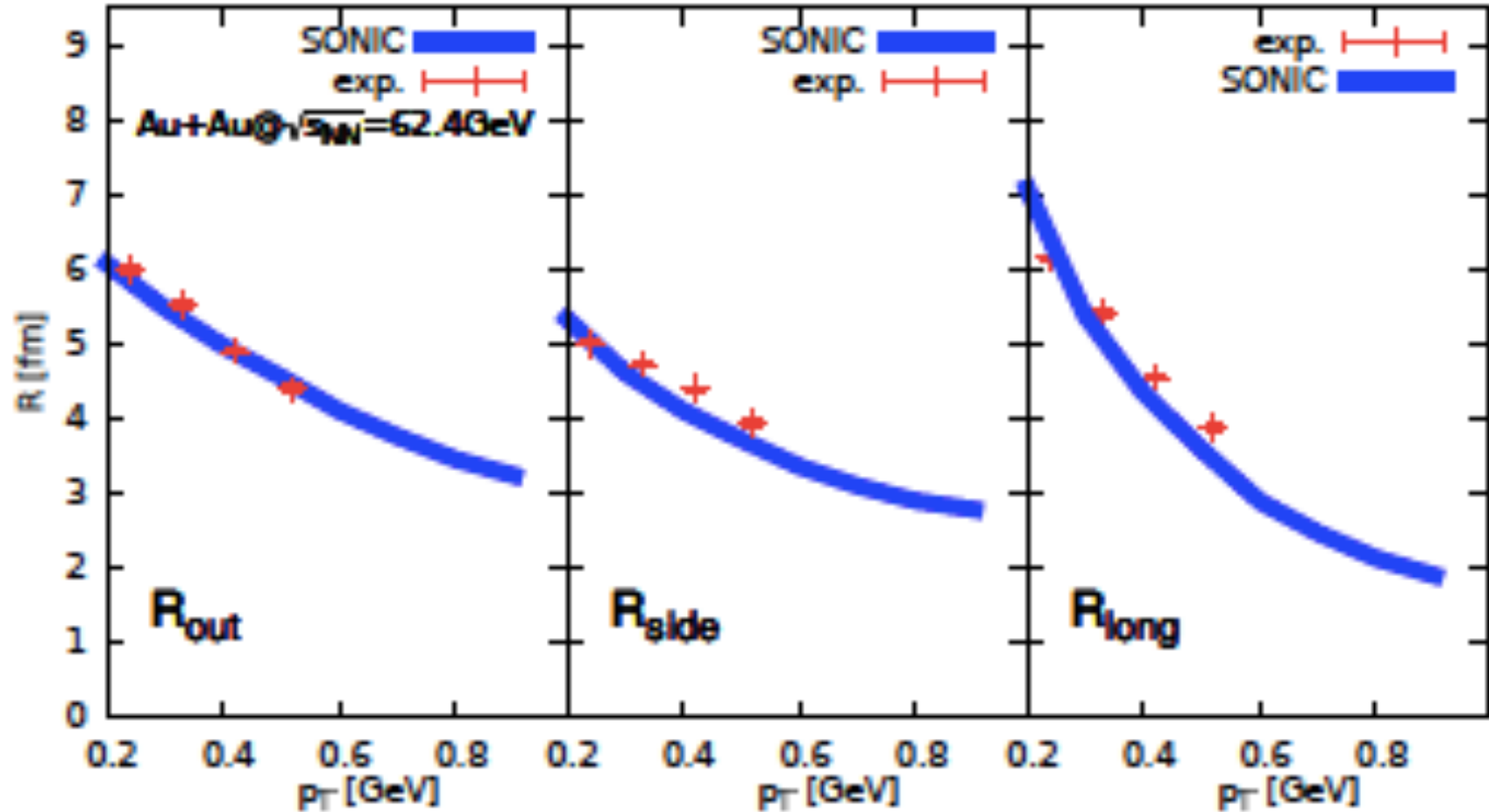
[Habich, Nagle and PR, 1409.0040]

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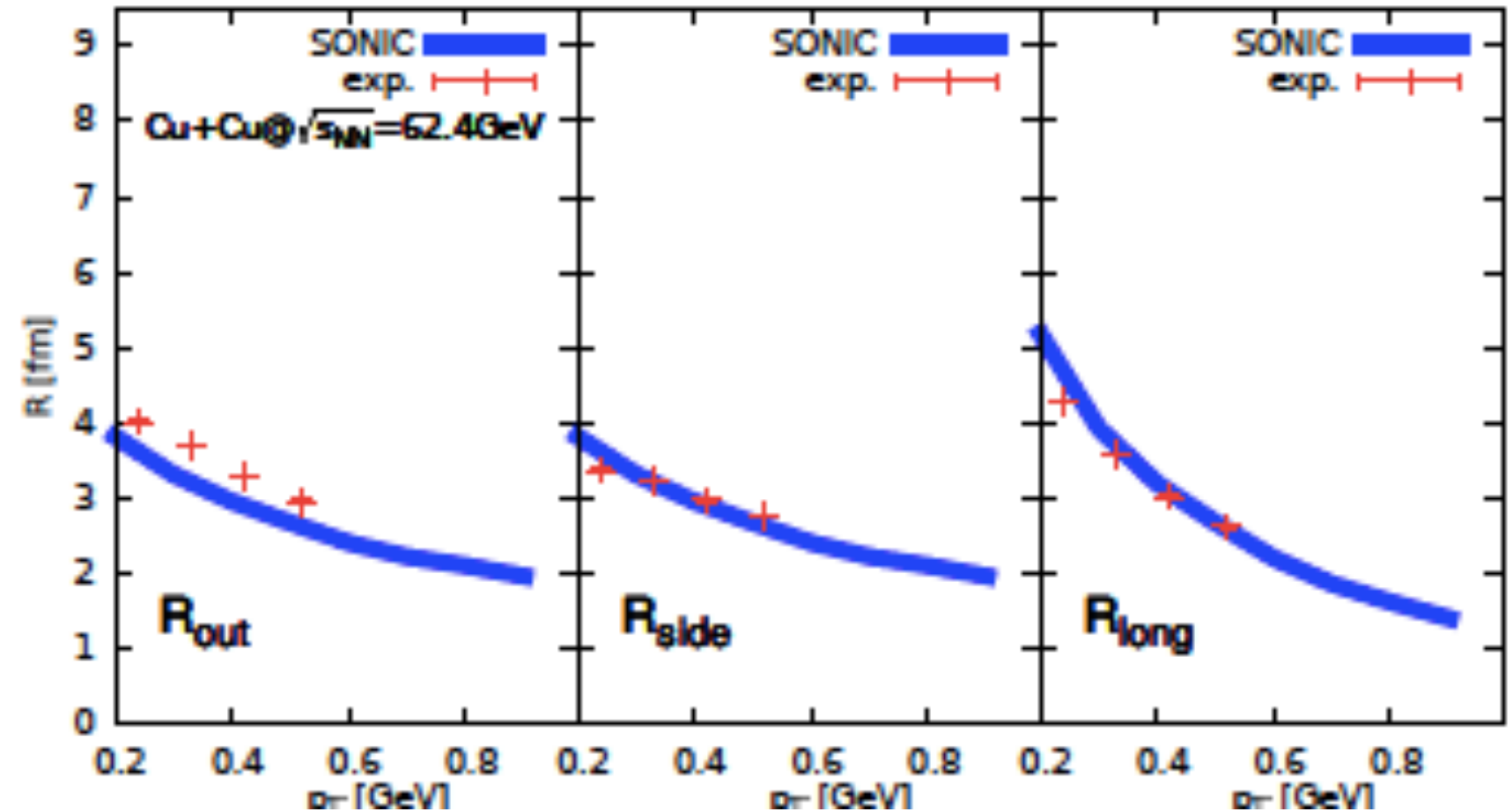
[Habich, Nagle and PR, 1409.0040]

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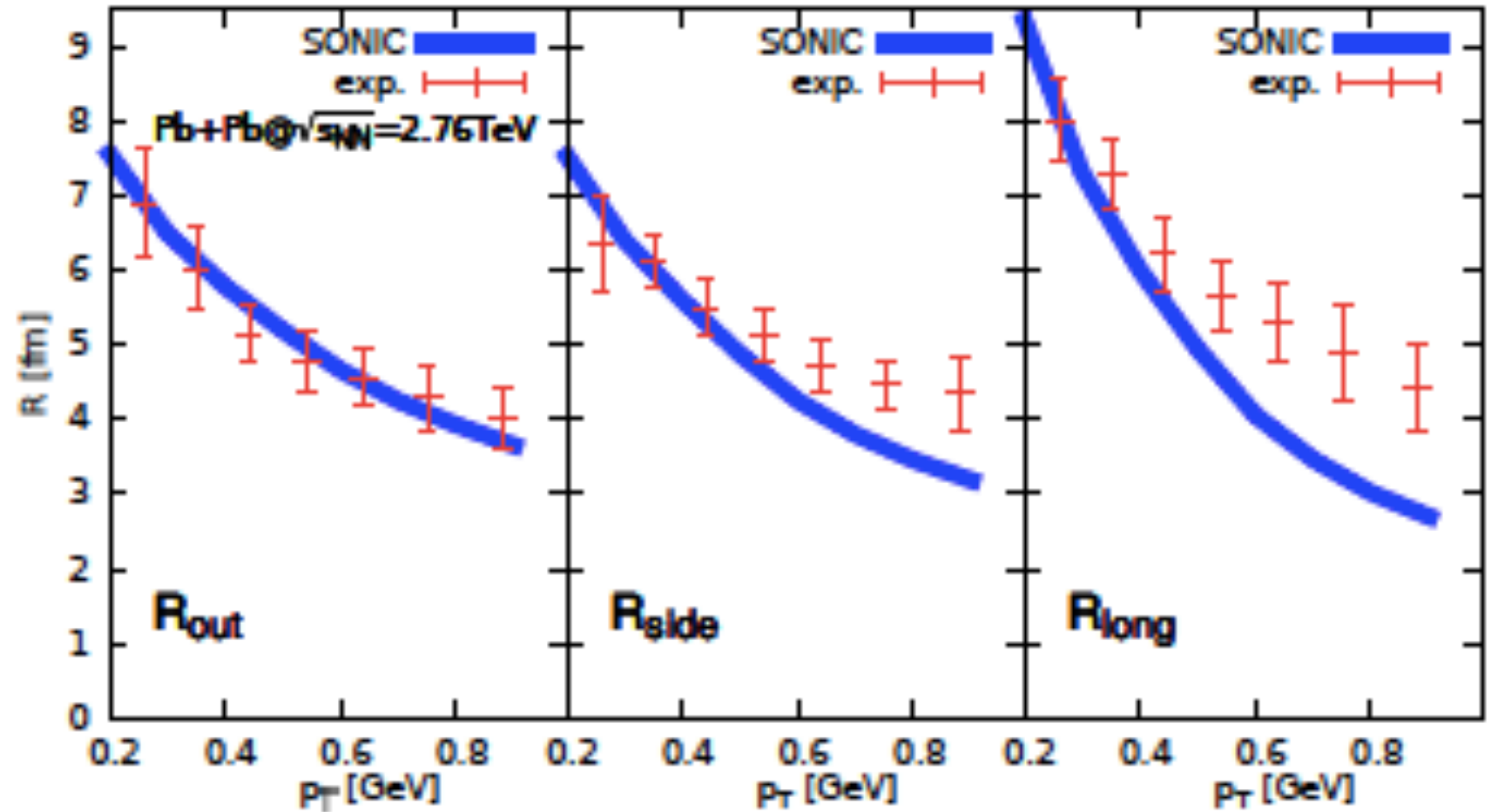
[Habich, Nagle and PR, 1409.0040]

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[Habich, Nagle and PR, 1409.0040]

SONIC: AdS+Hydro (eta/s=0.08)+Hadron Cascade



[Habich, Nagle and PR, 1409.0040]