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Particle acceleration in explosive reconnection

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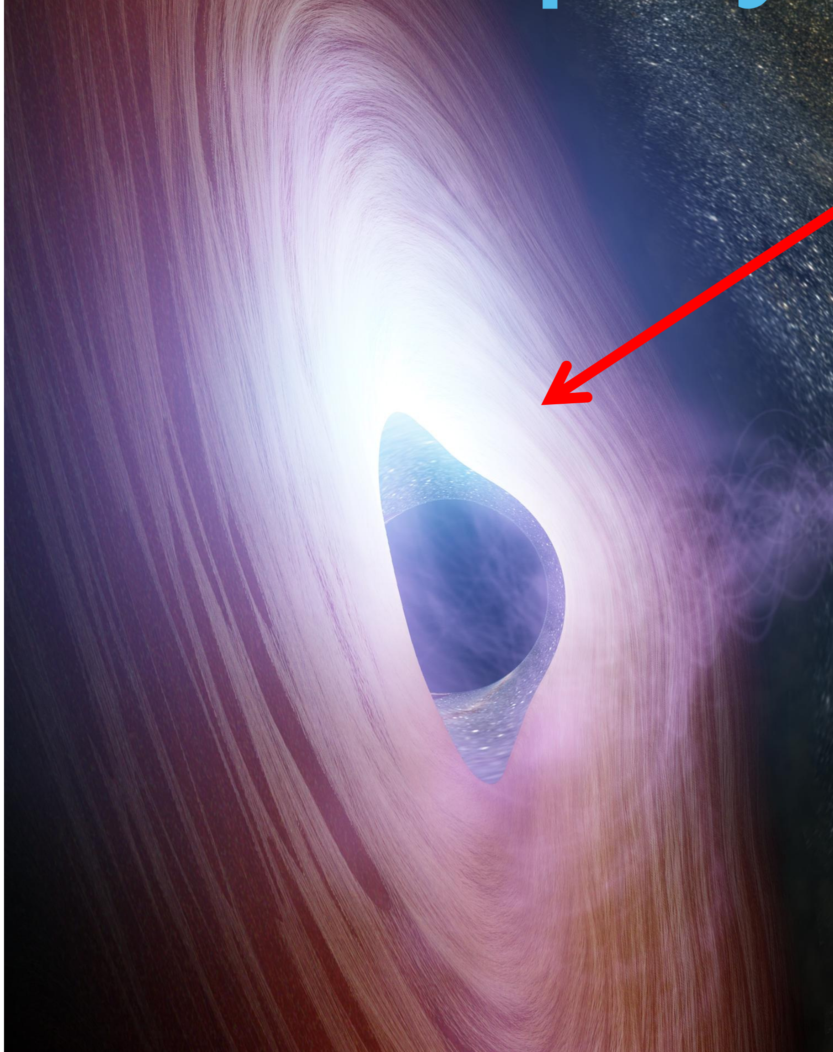
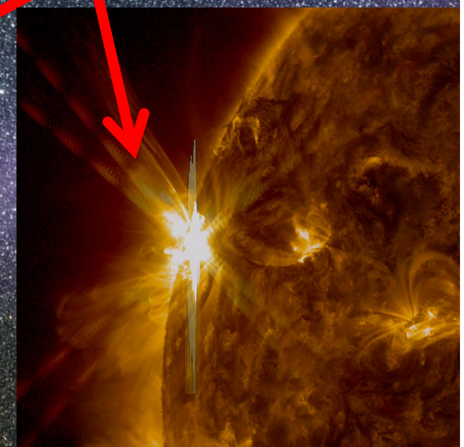
Department of Mathematics, KU Leuven

Astrophysical outflows

Stars and black holes can launch flares from their corona





These outflows are a source of extremely energetic particles guided by magnetic fields

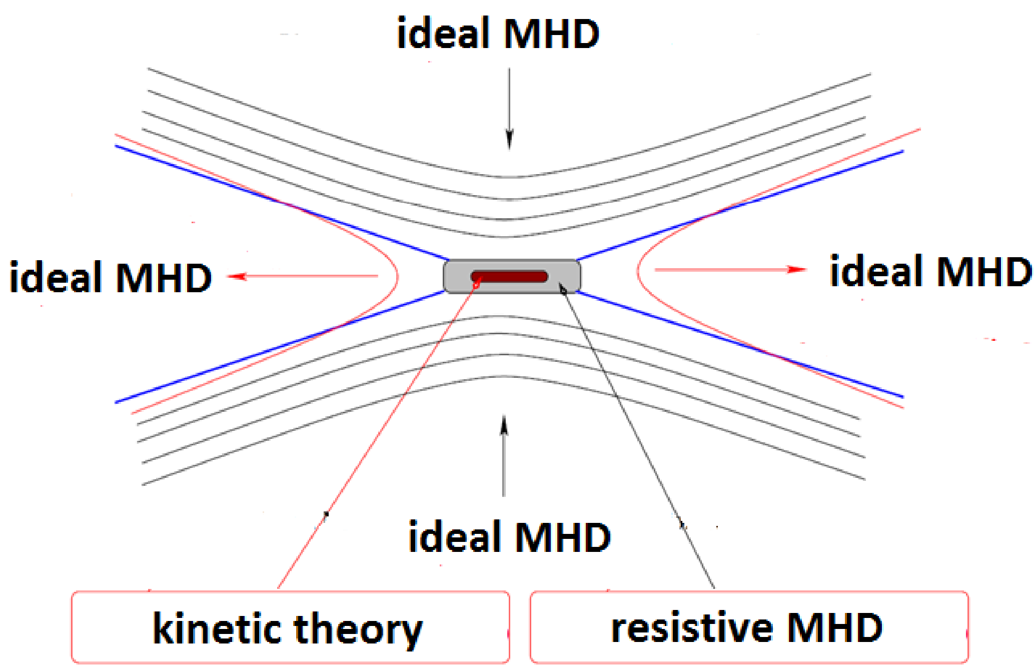
Accelerated, charged particles (electrons, positrons, protons) emit observable X-rays

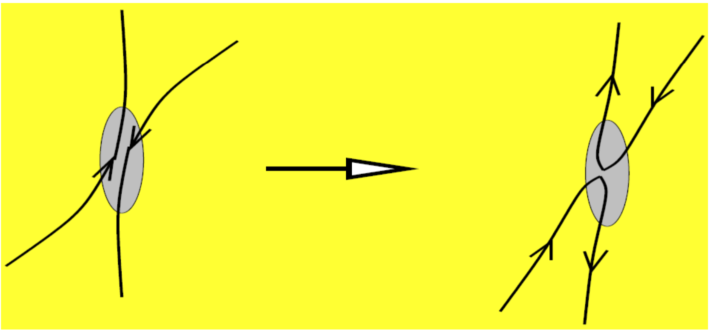
Reconnection is a possible generic mechanism behind flares and **particle acceleration**

[Images from NASA/JPL-Caltech, http://sdo.gsfc.nasa.gov/]

Magnetic reconnection

- Current dissipation through resistivity → Magnetic field reconnection
- Multi-scale character: fluid theory (MHD) → kinetic theory (particles)
- Excess magnetic energy → Particle acceleration in jets and flares





[Images obtained from Keppens, Coupling Multiple Scales, NBIA school 2013]

<u>How do we model phenomena at such different scales?</u>

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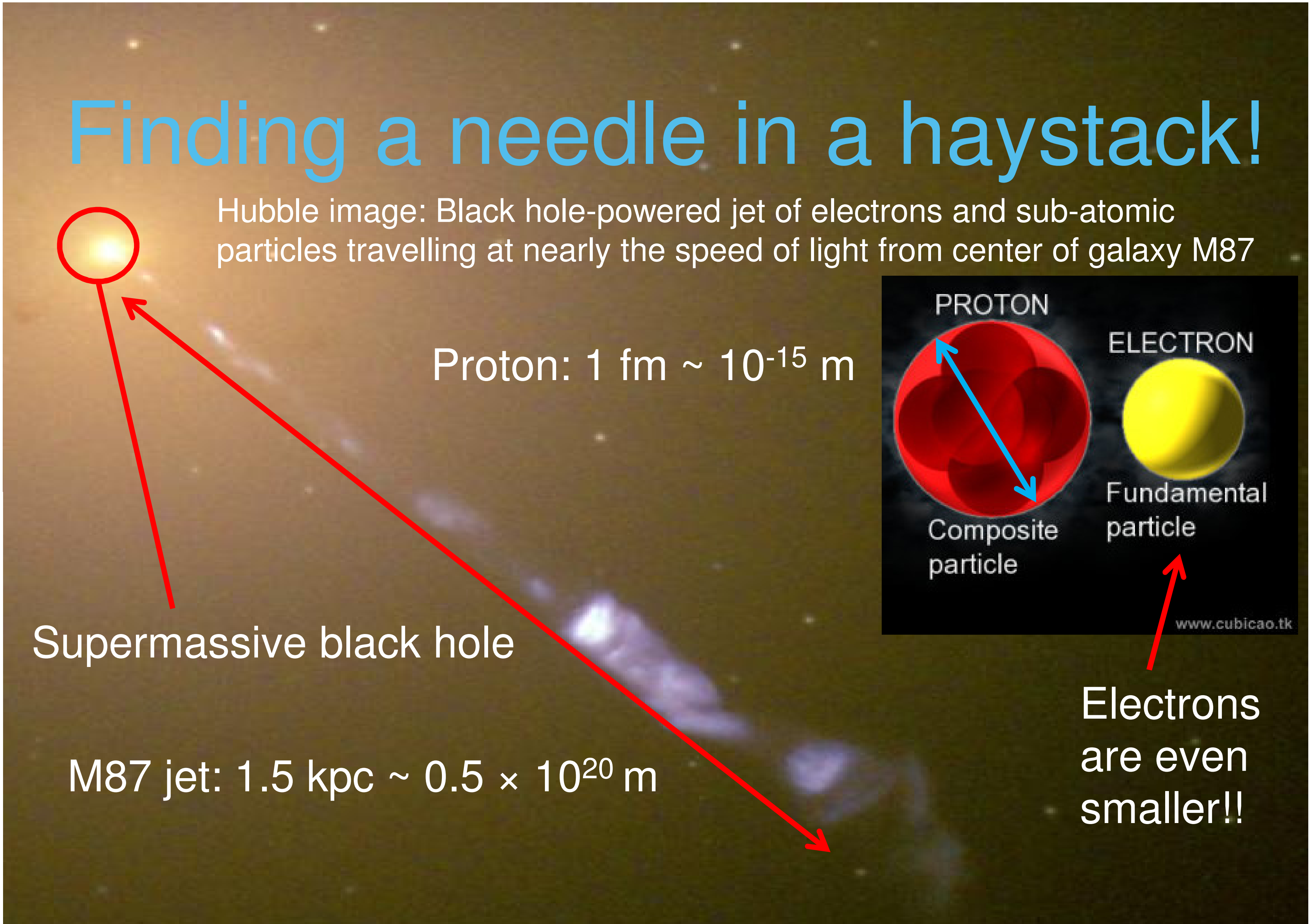

Finding a needle in a haystack!
Hubble image: Black hole-powered jet of electrons and sub-atomic particles travelling at nearly the speed of light from center of galaxy M87
Proton: 1 fm ~ 10^{-15} m
PROTON
ELECTRON
Composite particle
Fundamental particle
www.cubicao.tk
Supermassive black hole
M87 jet: 1.5 kpc ~ 0.5×10^{20} m
Electrons are even smaller!!

The continuum picture

$$\partial_{[\mu}F_{\nu\alpha]} = 0$$

$$\nabla_\nu F^{\mu\nu} = J^\mu / c$$

Coupling

Magnetohydro-dynamics (MHD)

$$\nabla_\mu\left(\rho_0 u^\mu\right) = 0$$

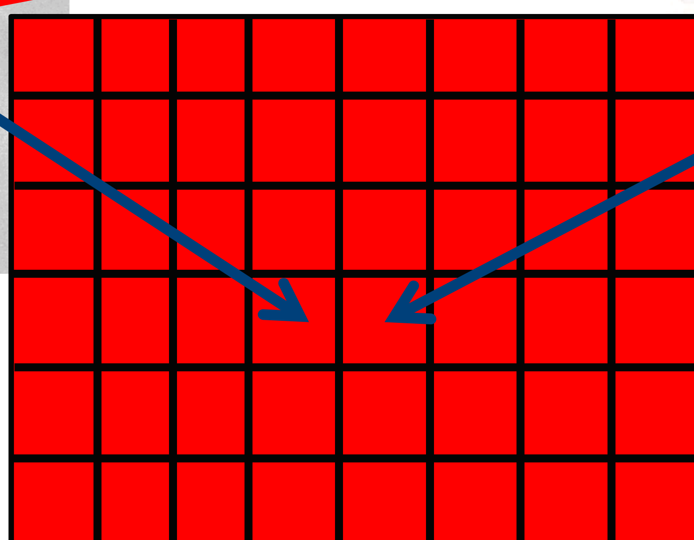
$$\nabla_\mu\left(T_f^{\mu\nu} + T_{EM}^{\mu\nu}\right) = 0$$

E(t), **B**(t)

ρ, **J**

ρ(t), **u**(t)

E, B



Maxwell's equations

Navier-Stokes equations

MPI-AMRVAC: parallel, grid-adaptive code

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Fluids in the universe

Magnetohydrodynamics describes a wide variety of events in the universe

From ultrarelativistic black hole jets …

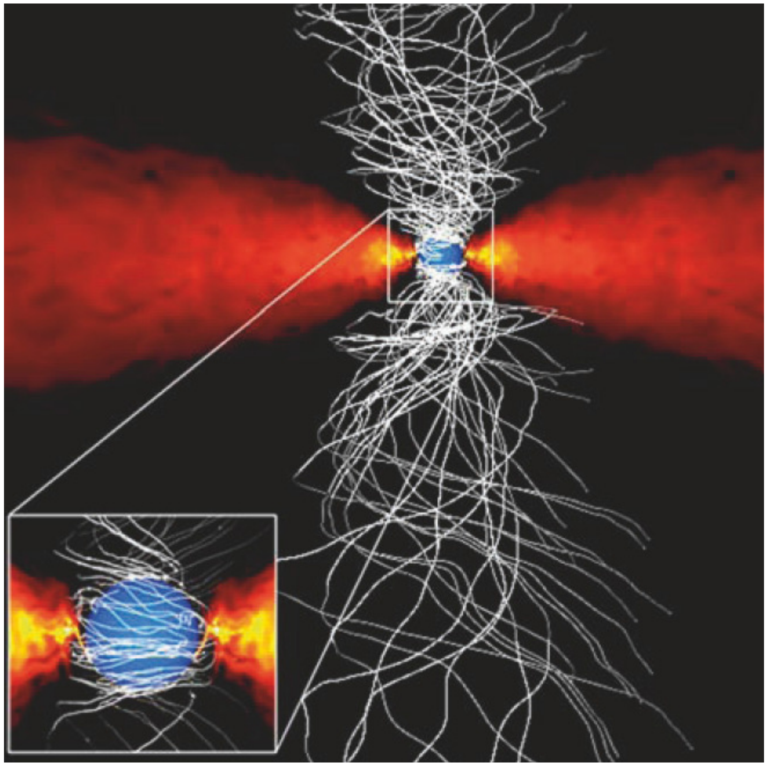

Image taken from Punsly, Black hole gravito-hydromagnetics (2008)

to solar flares

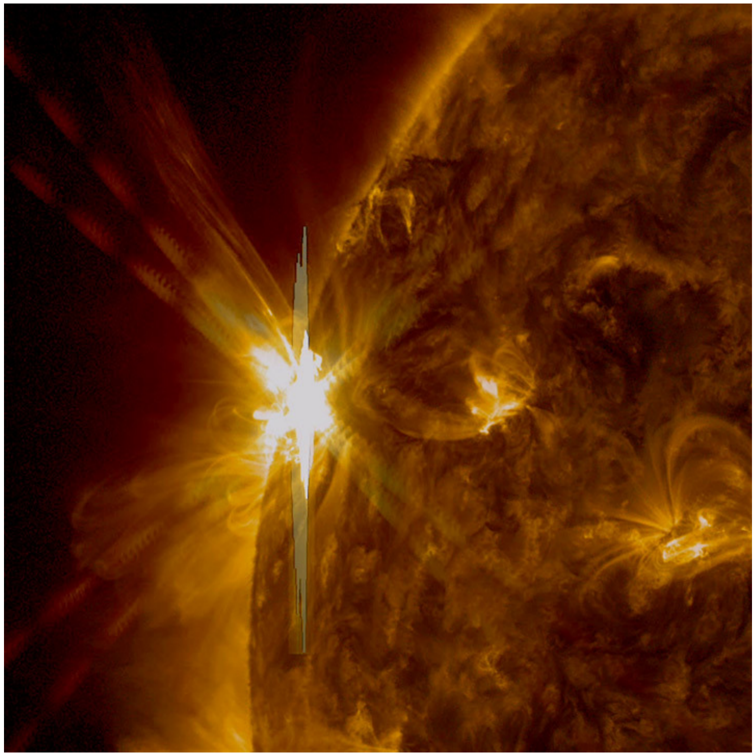

Image taken from http://sdo.gsfc.nasa.gov/

But …

additional physics needed for reconnection and particle-field interaction!

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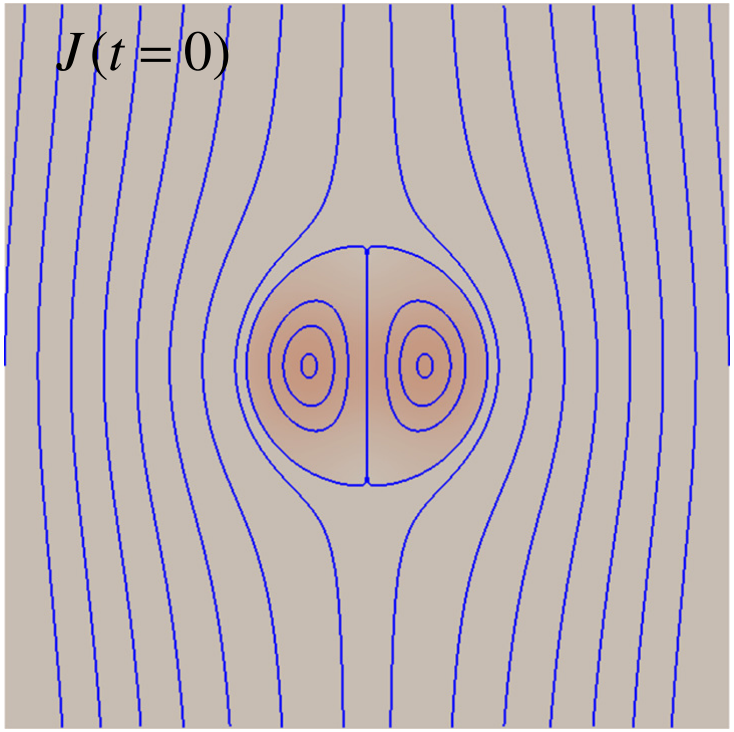
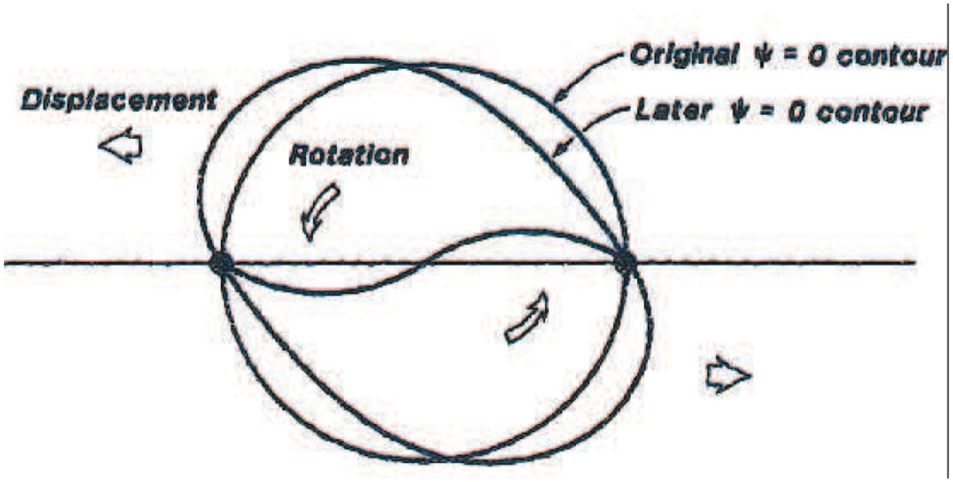
Initiating reconnection in MHD

- 2D force-free ideal MHD equilibrium: two repelling currents
- → Tilt instability and resistivity → Reconnection → Particle acceleration





Sketch taken from Lankalapalli et al, JCP 225 (2007)

- 3D effects → Kink (in)stability

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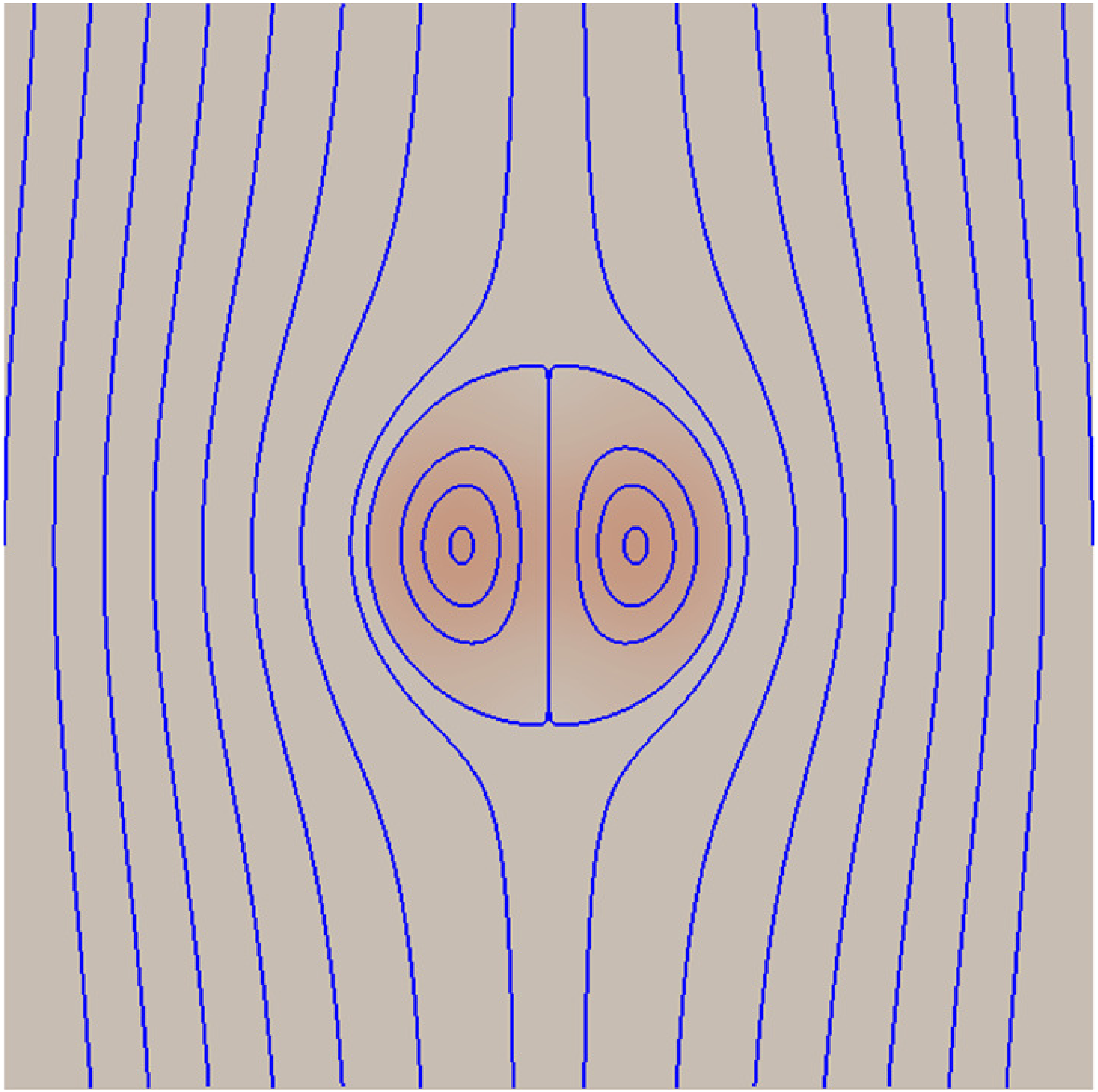
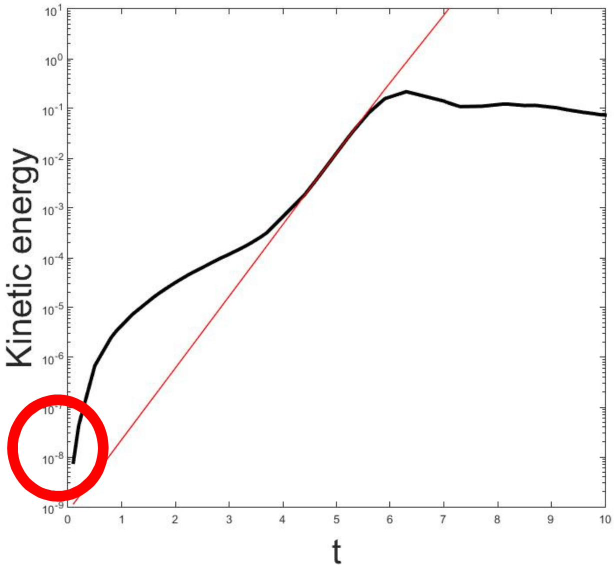
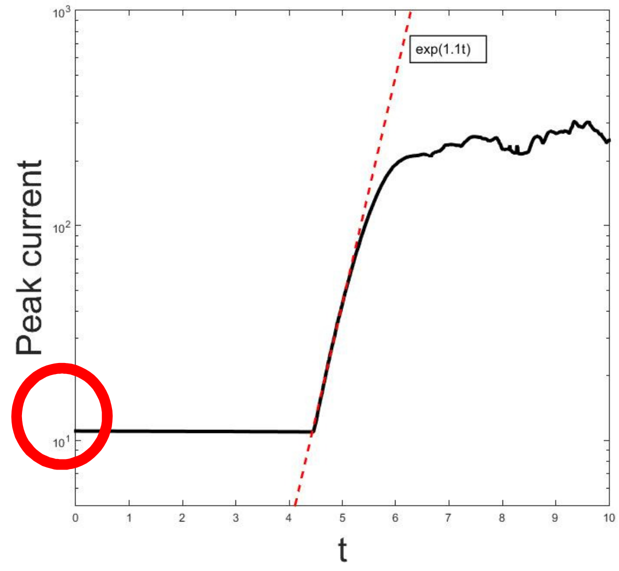


J(t=0)





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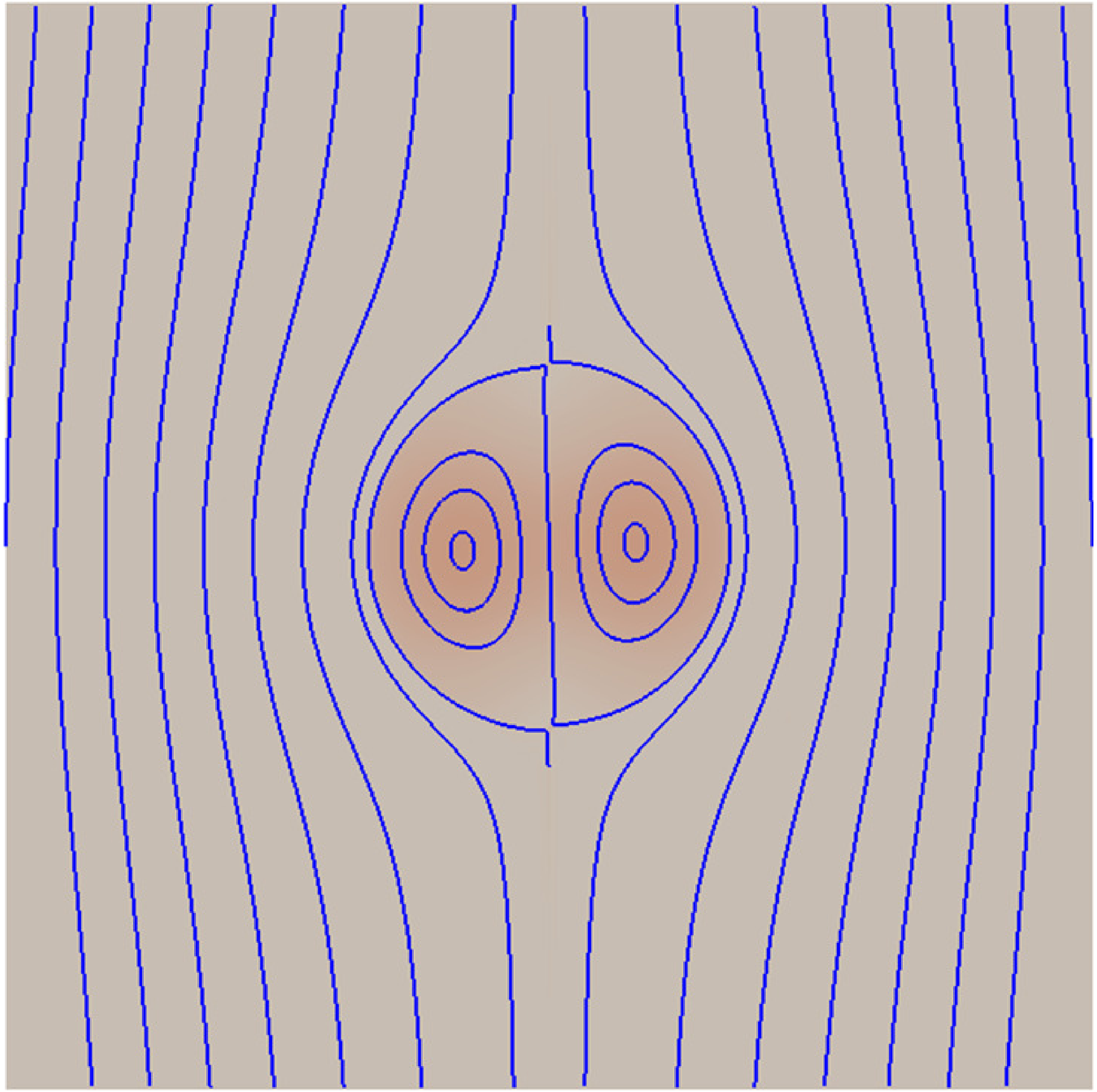

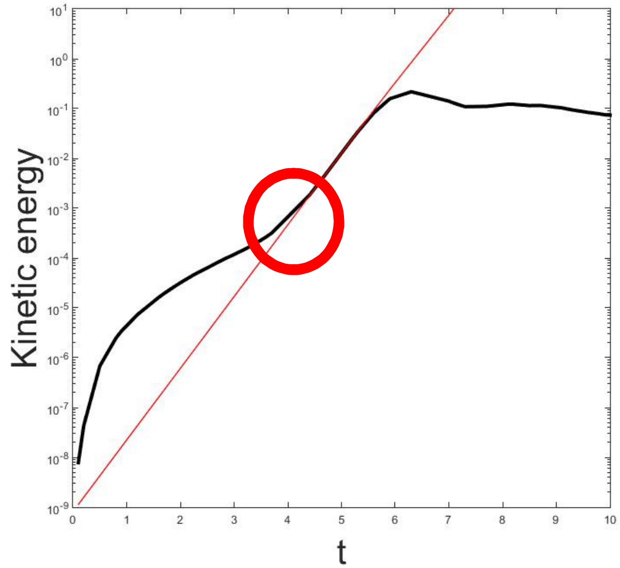

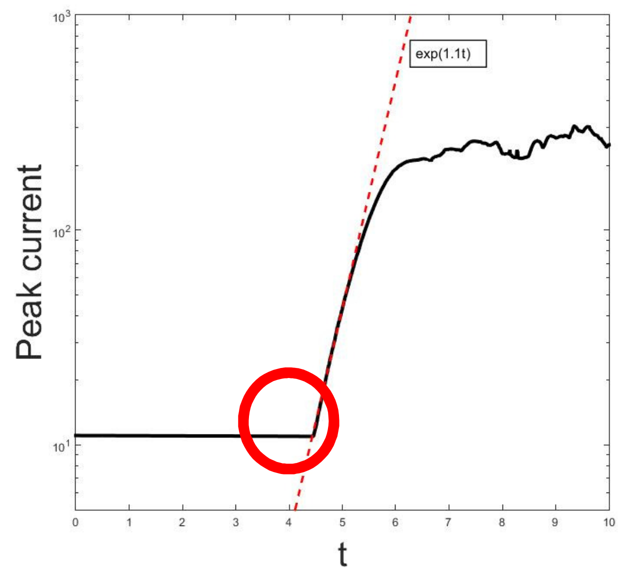

J(t=4)

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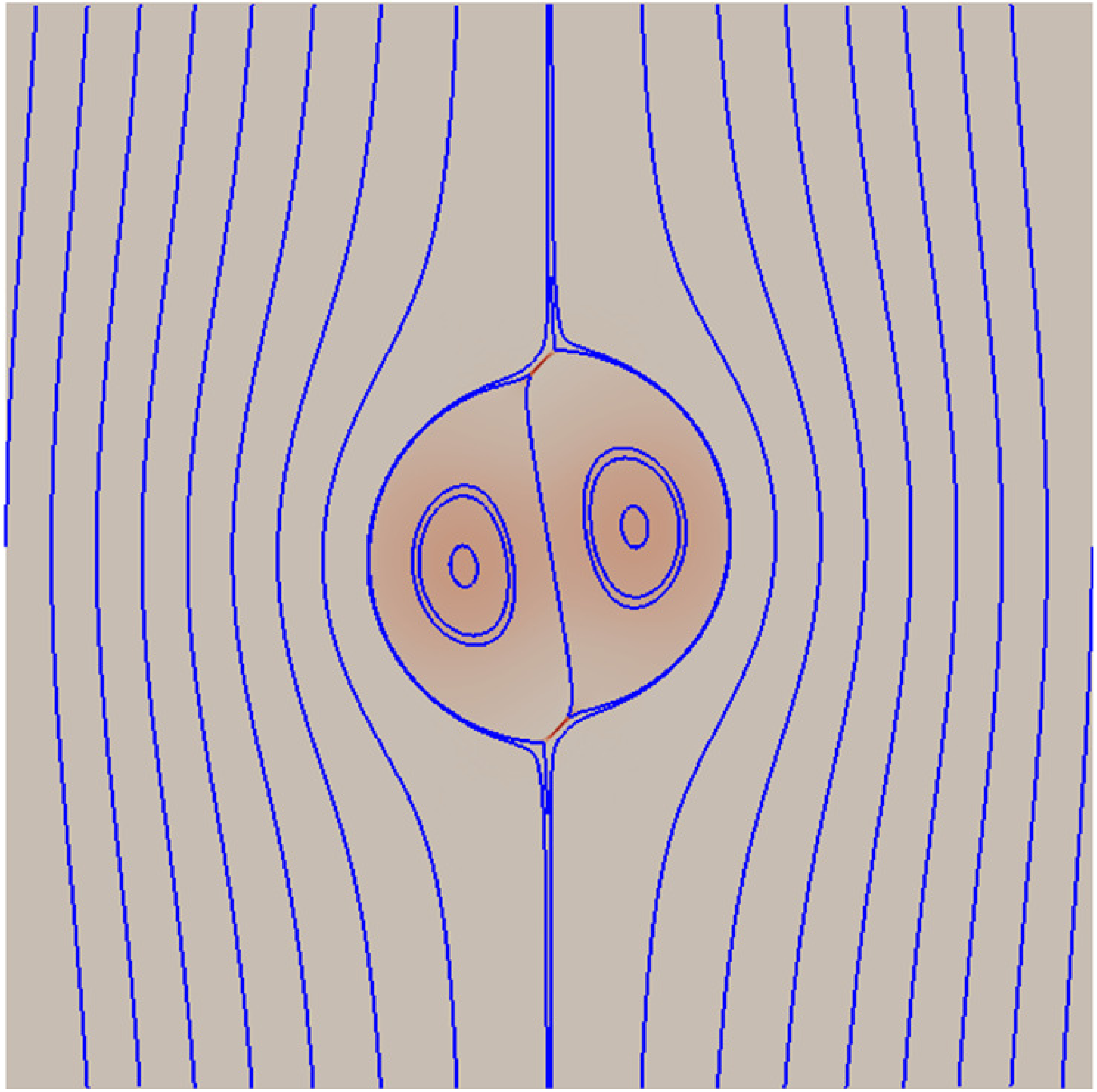
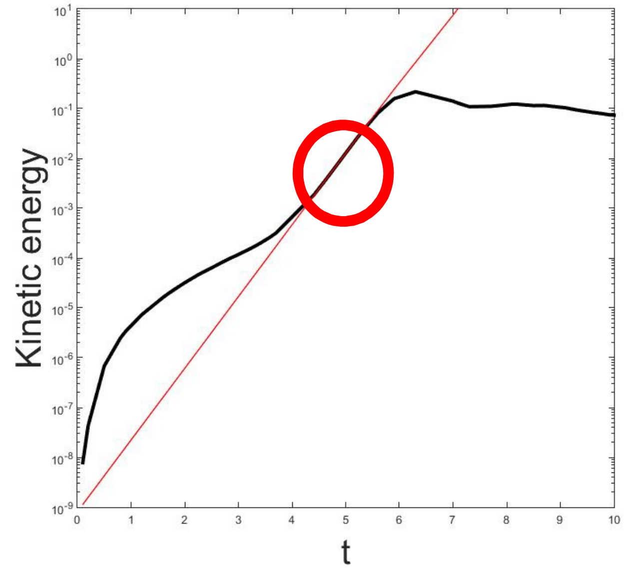
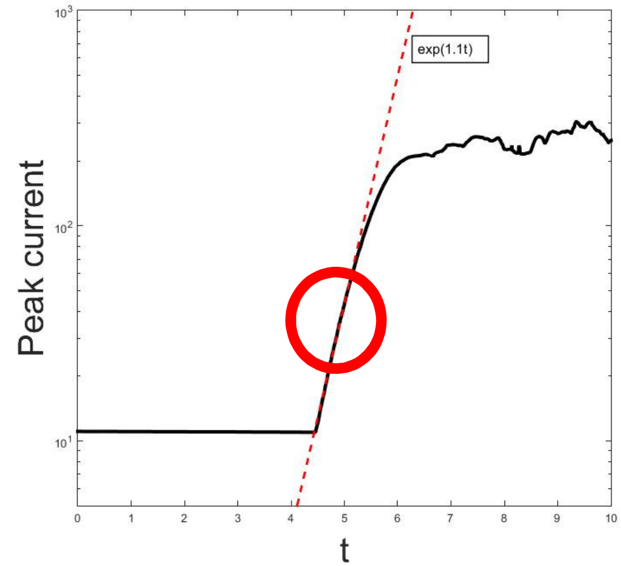
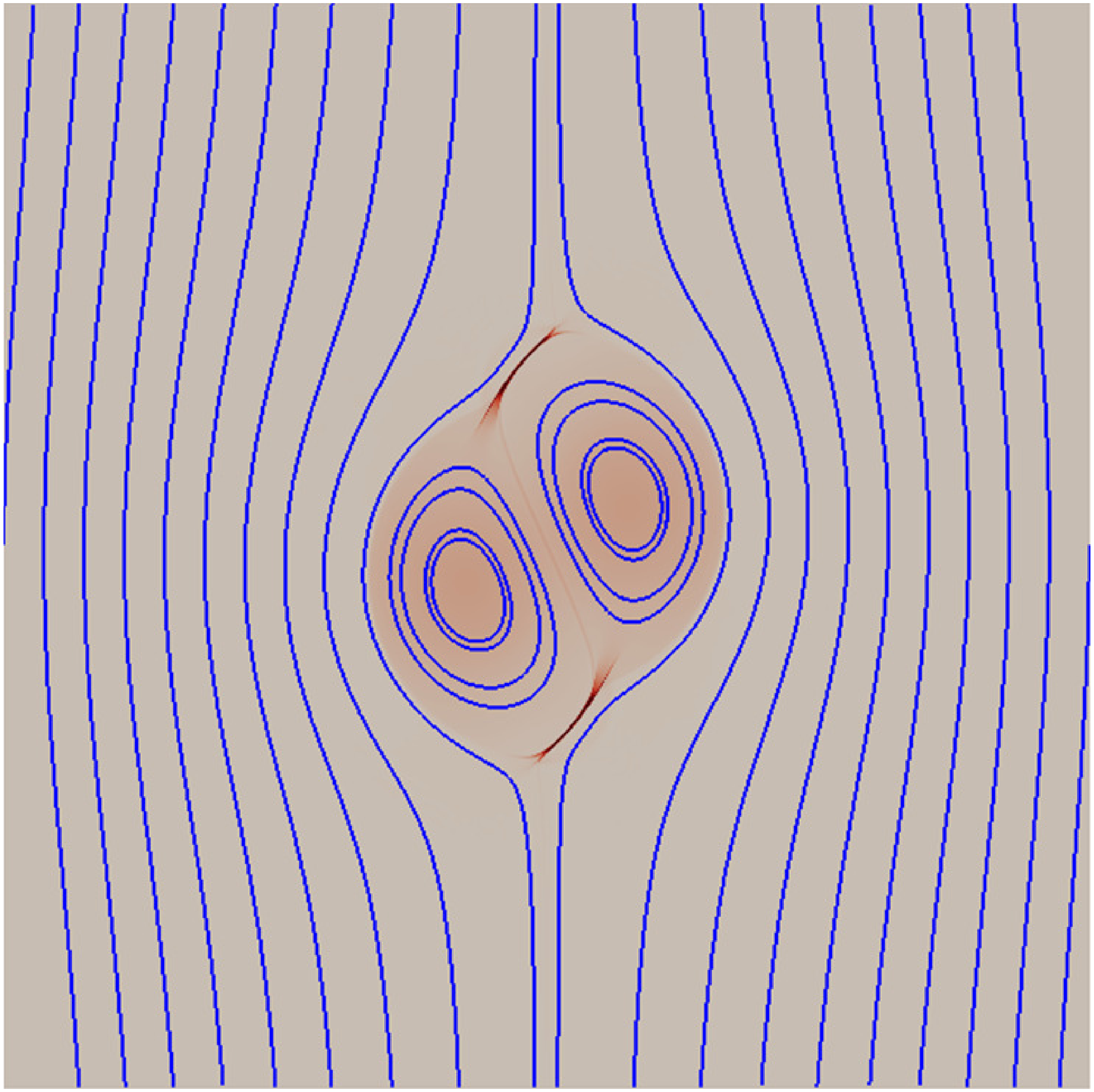


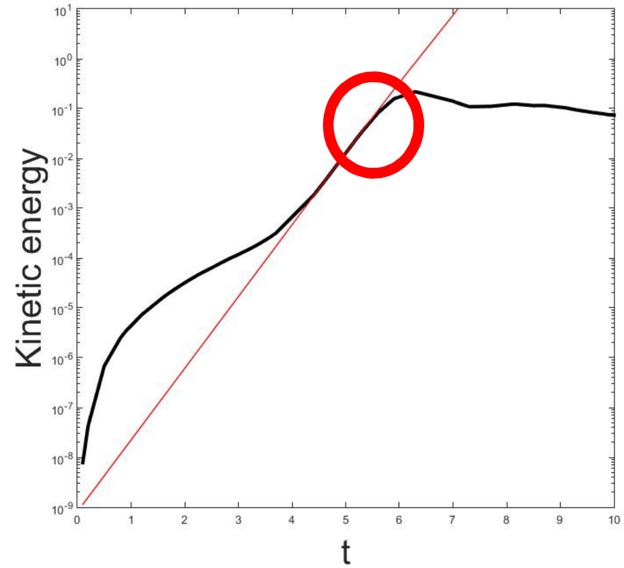


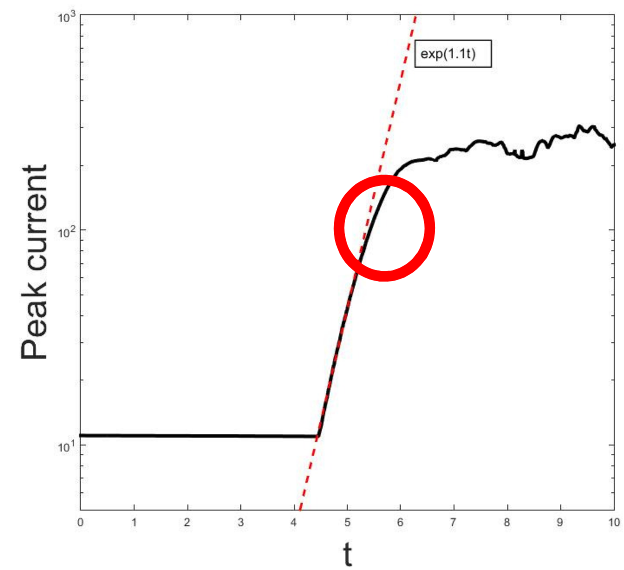


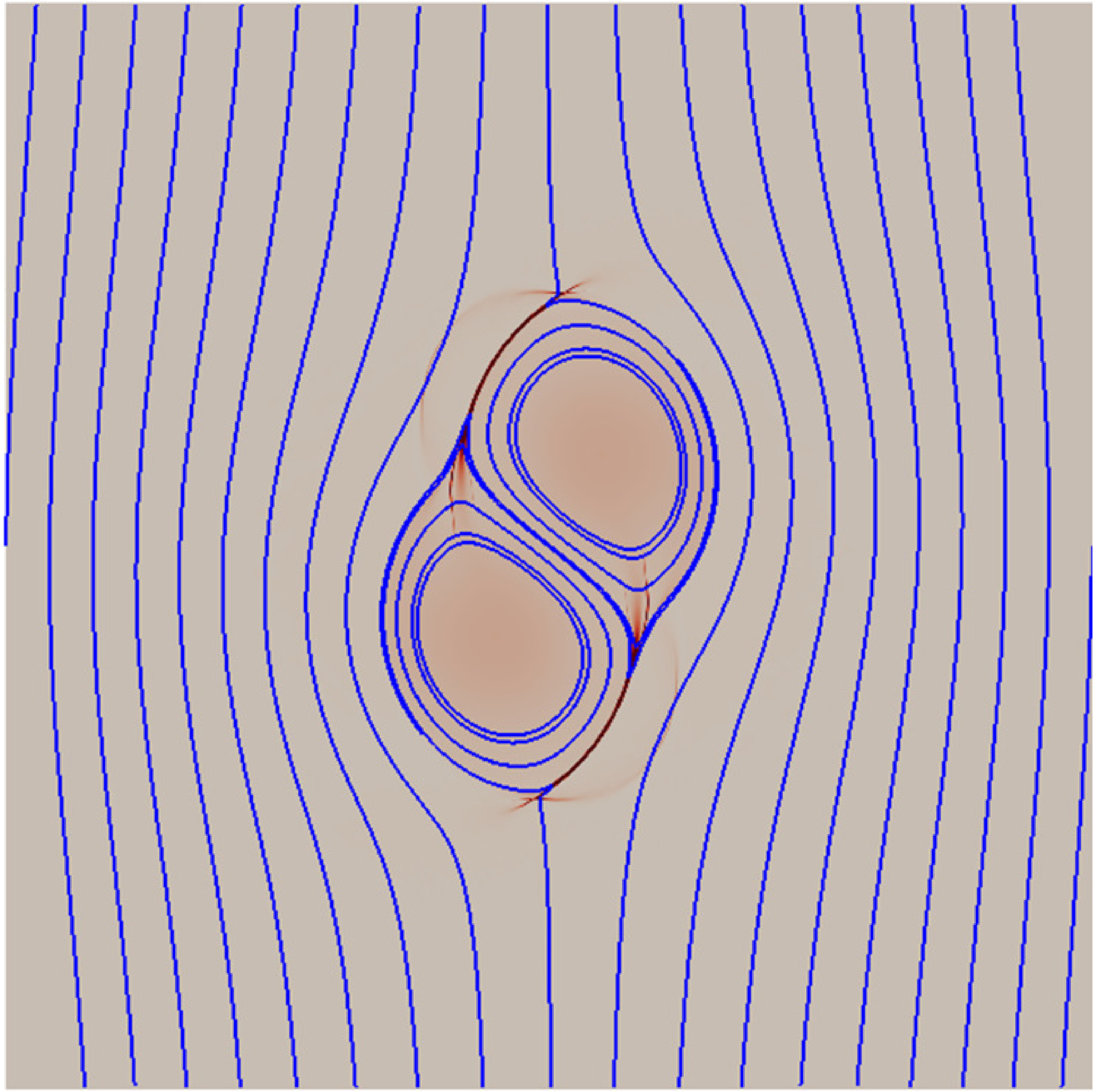
J(t=5)

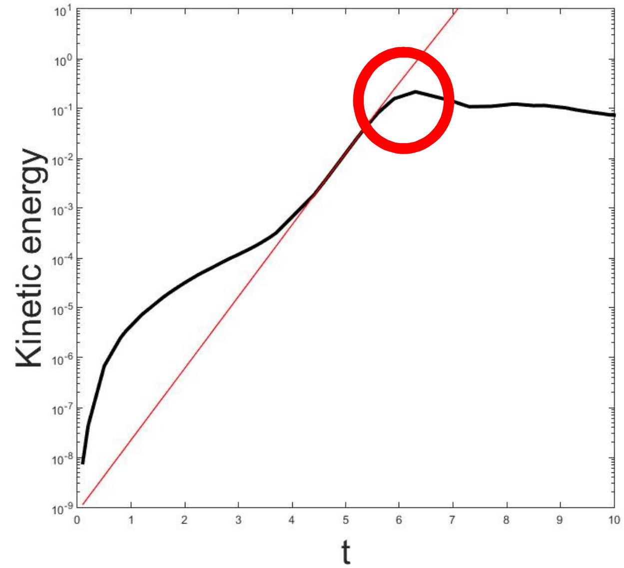


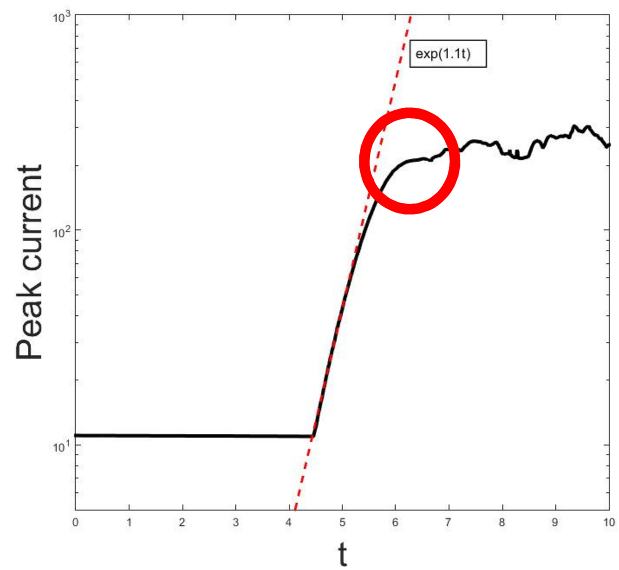
J(t=5.5)











J(t=6)

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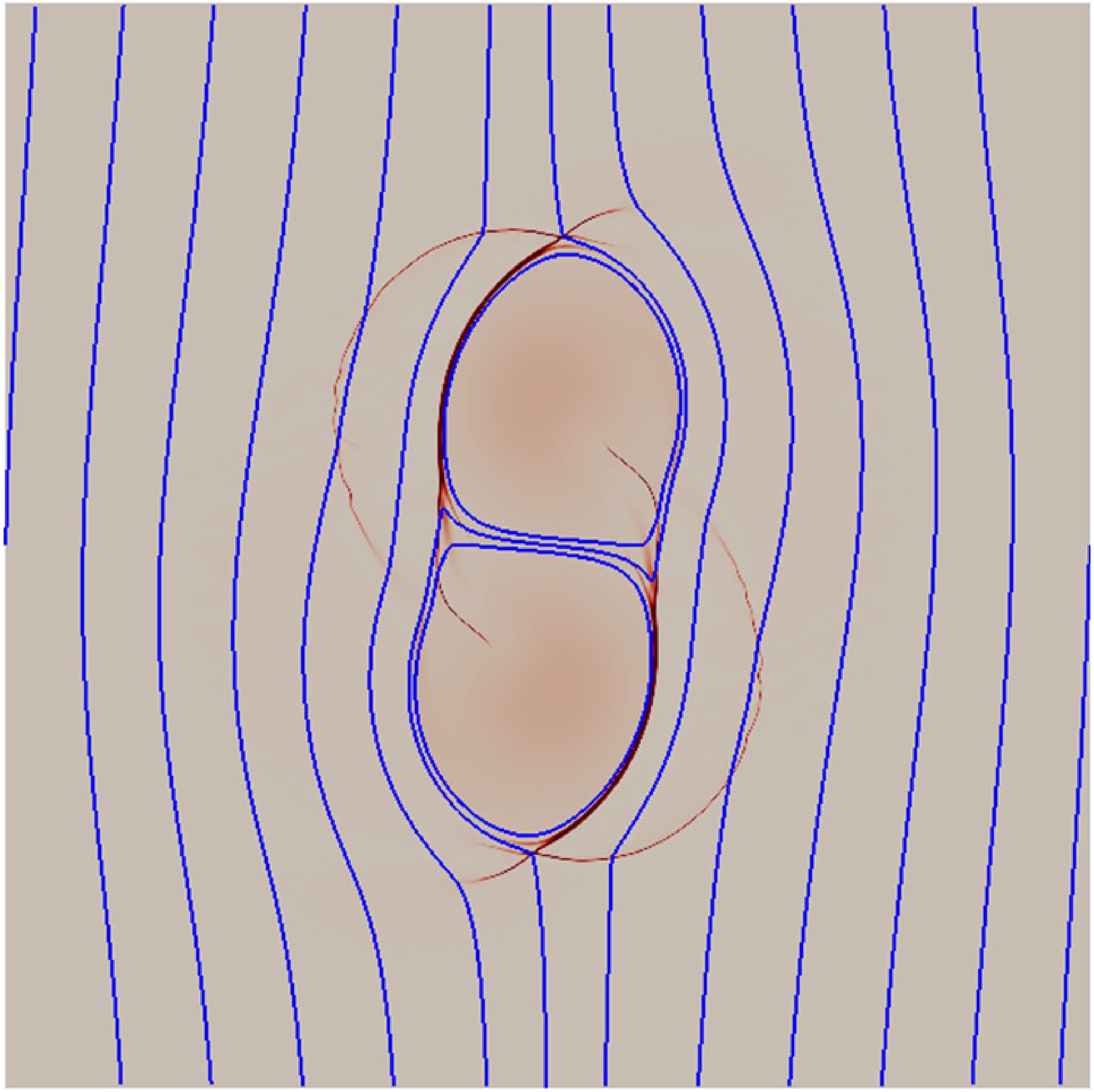
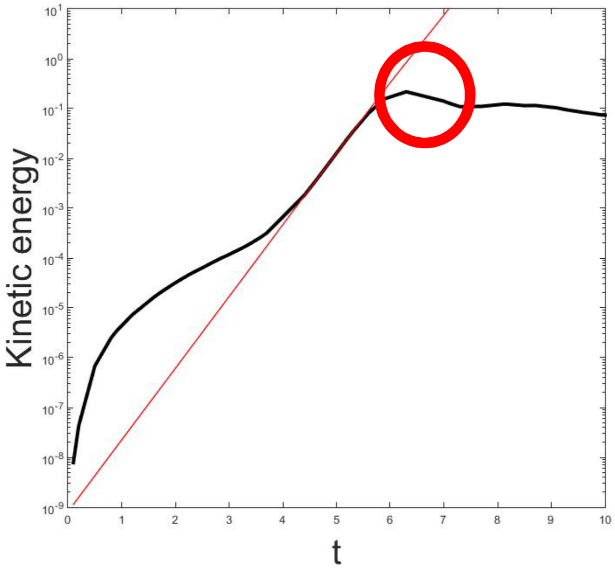
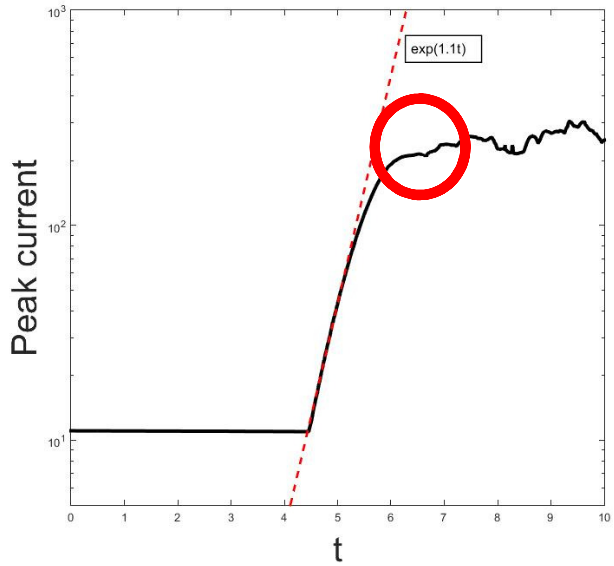
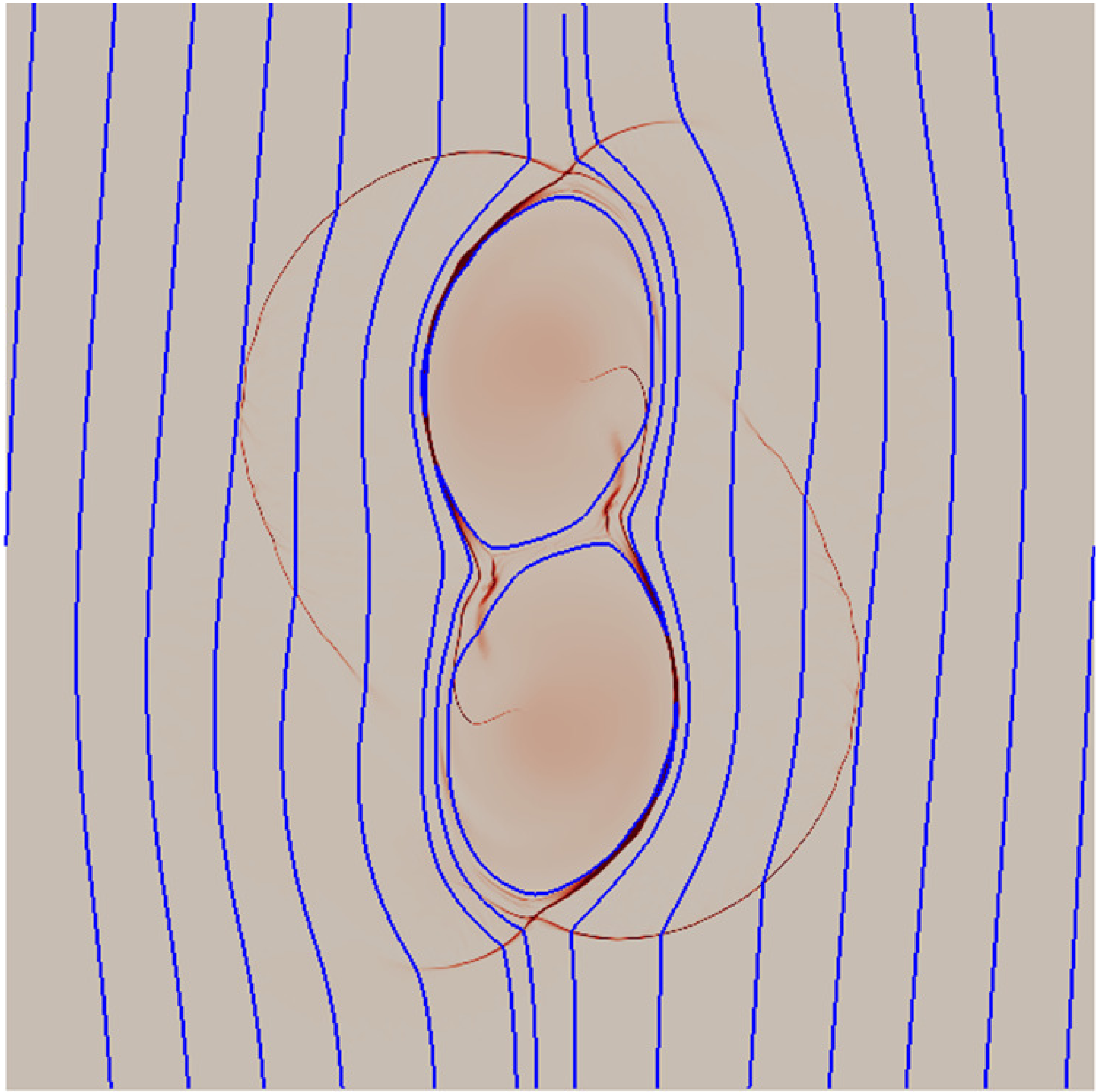
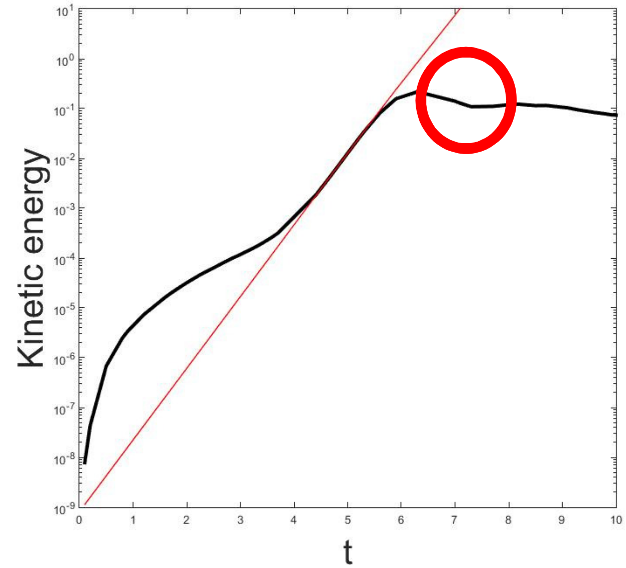


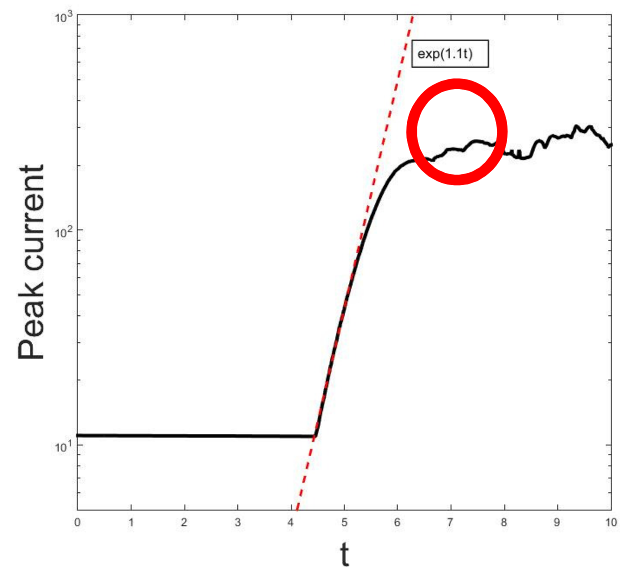




J(t=6.5)







J(t=7)

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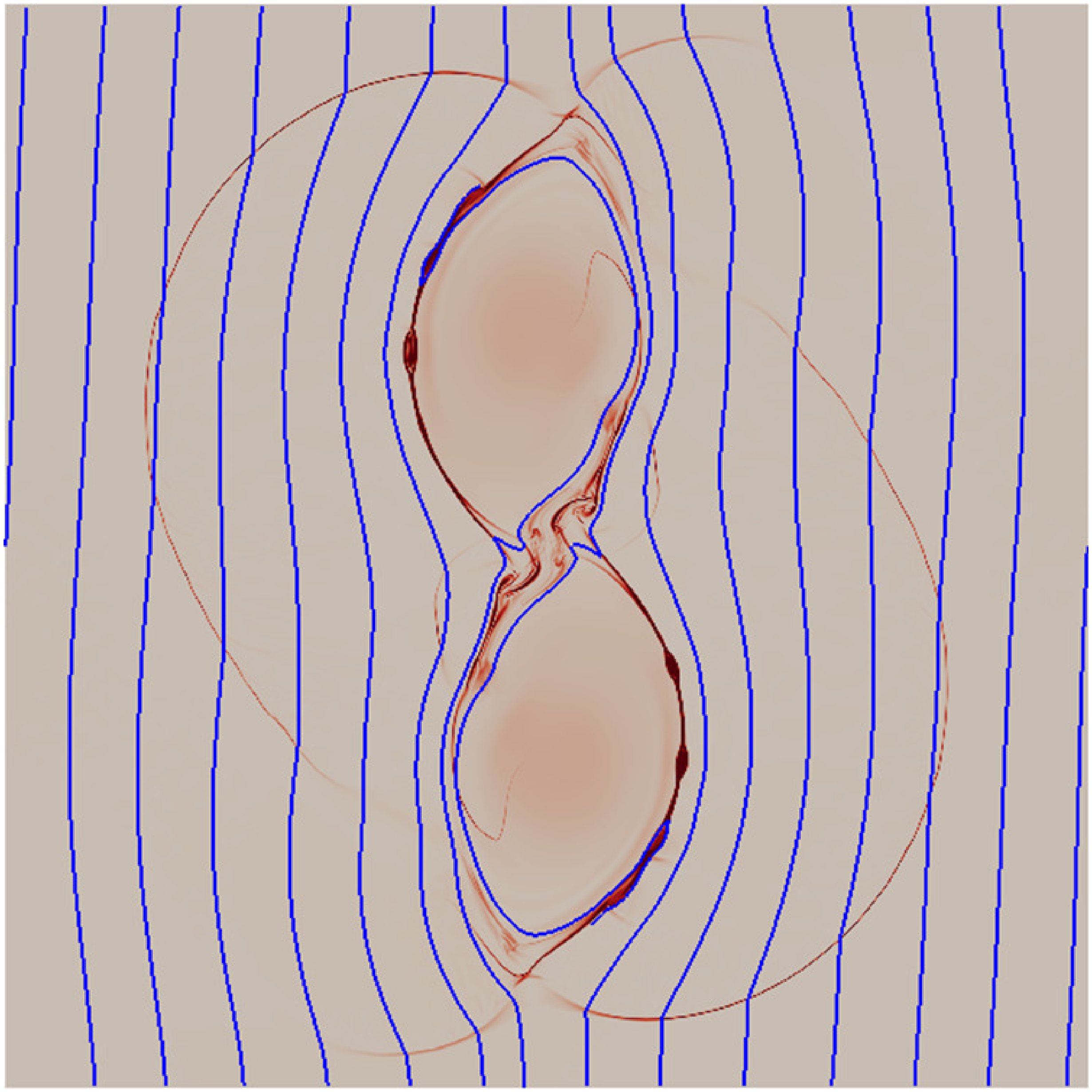

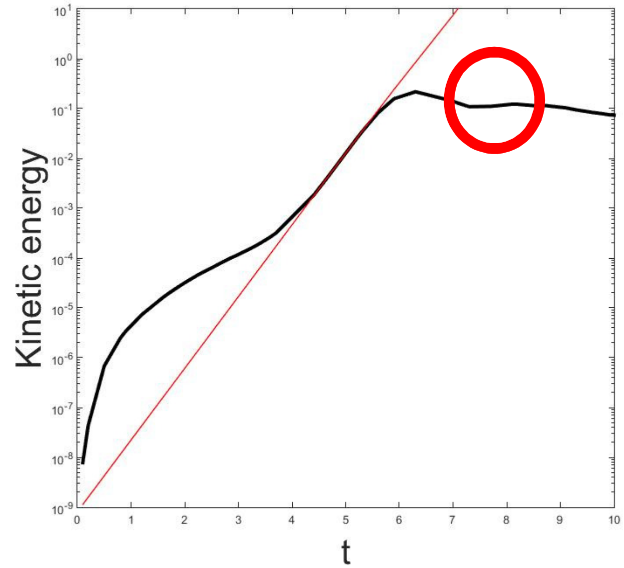

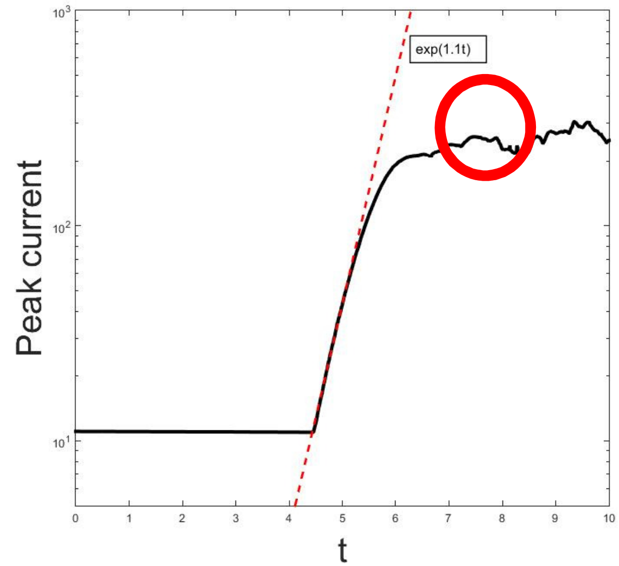

J(t=7.5)

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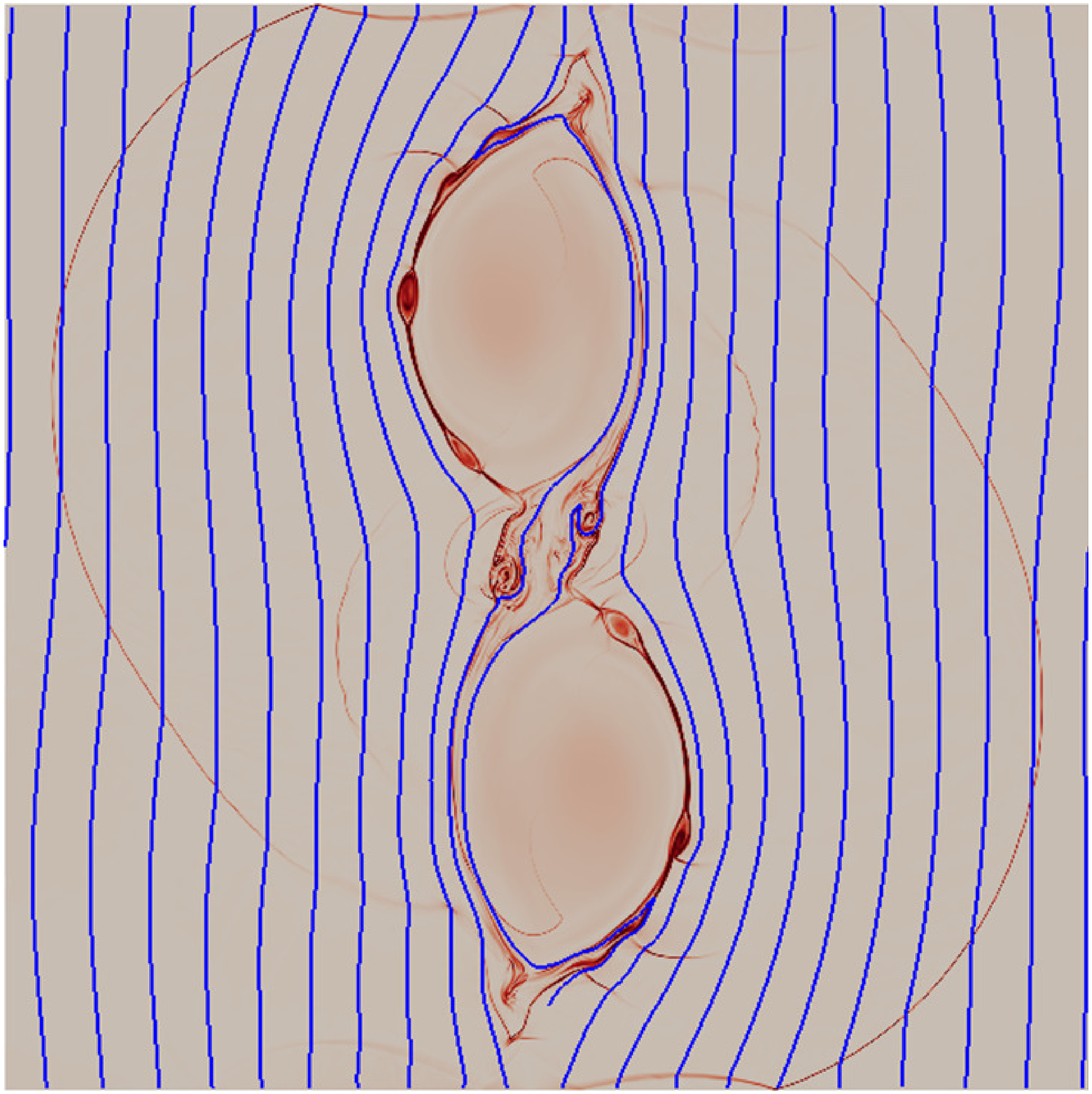

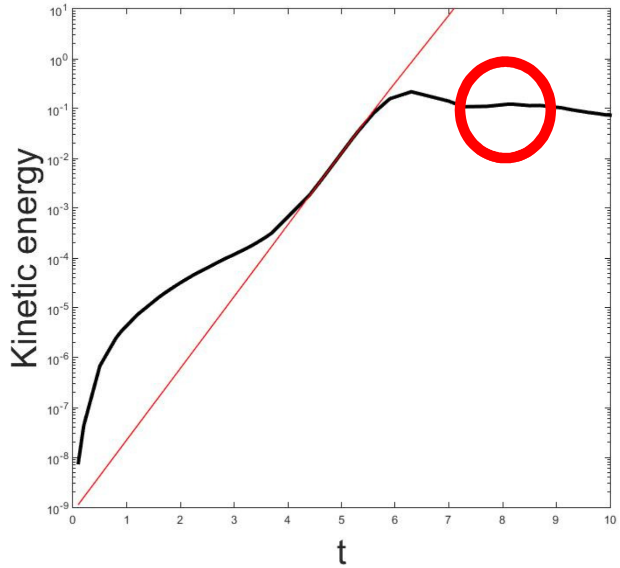

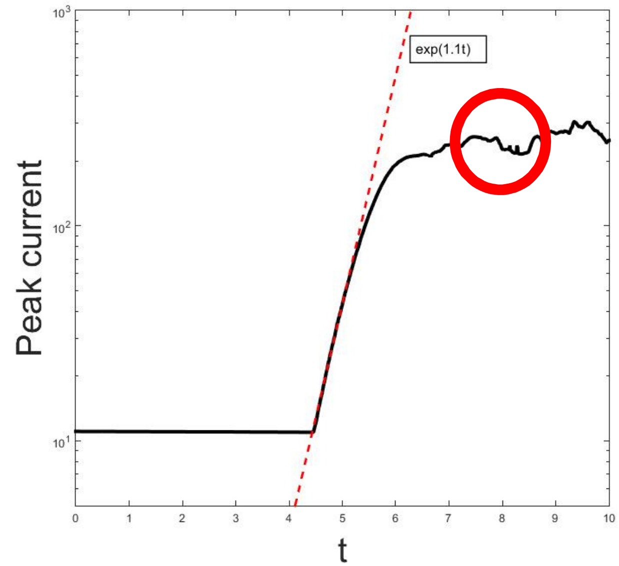

J(t=8)

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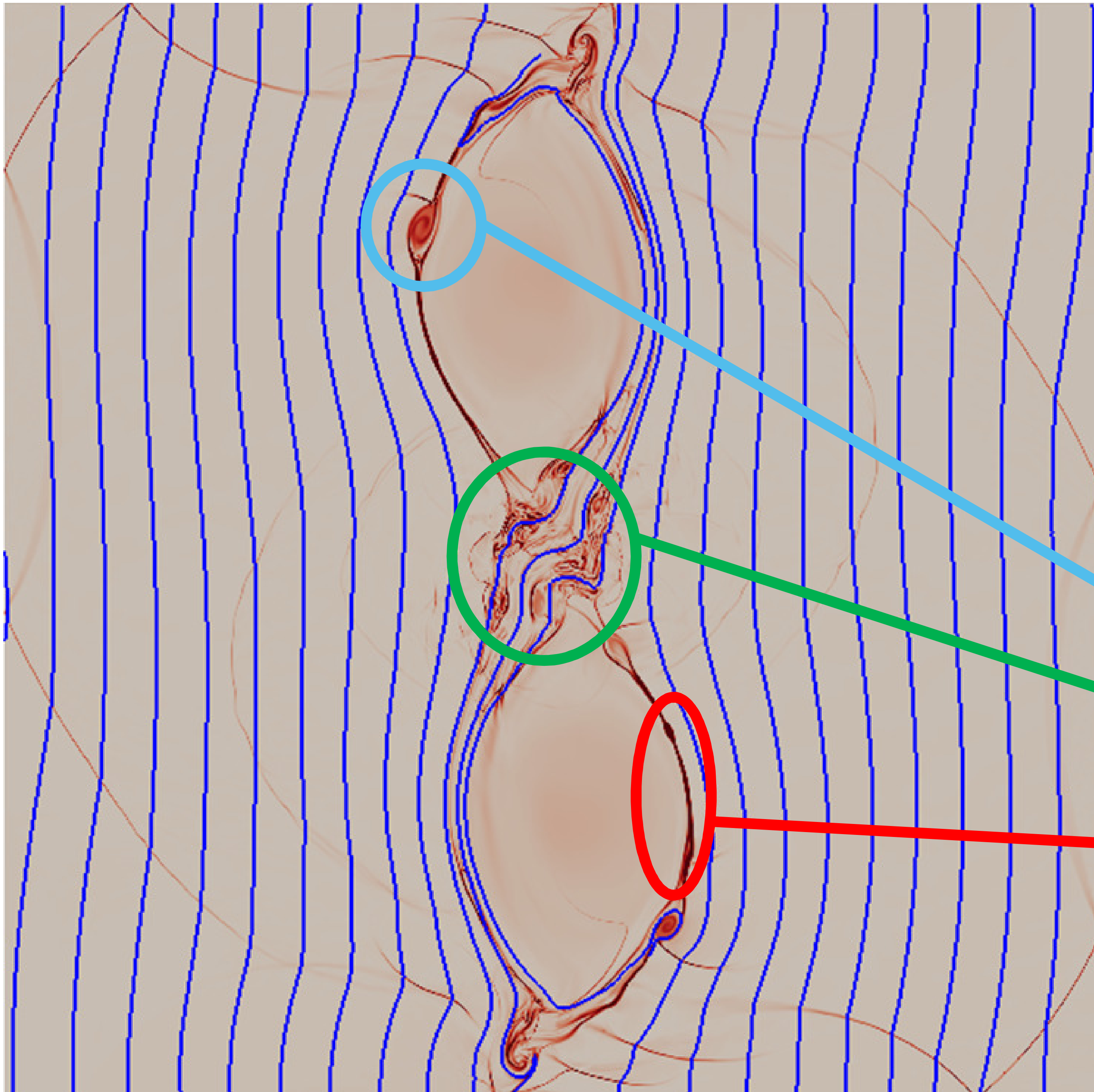

J(t=8.5)

- β = 0.04
- Similar behaviour for larger β, but delayed
- Secondary islands
- Reconnection
- Fast forming current sheets

→ Particle acceleration

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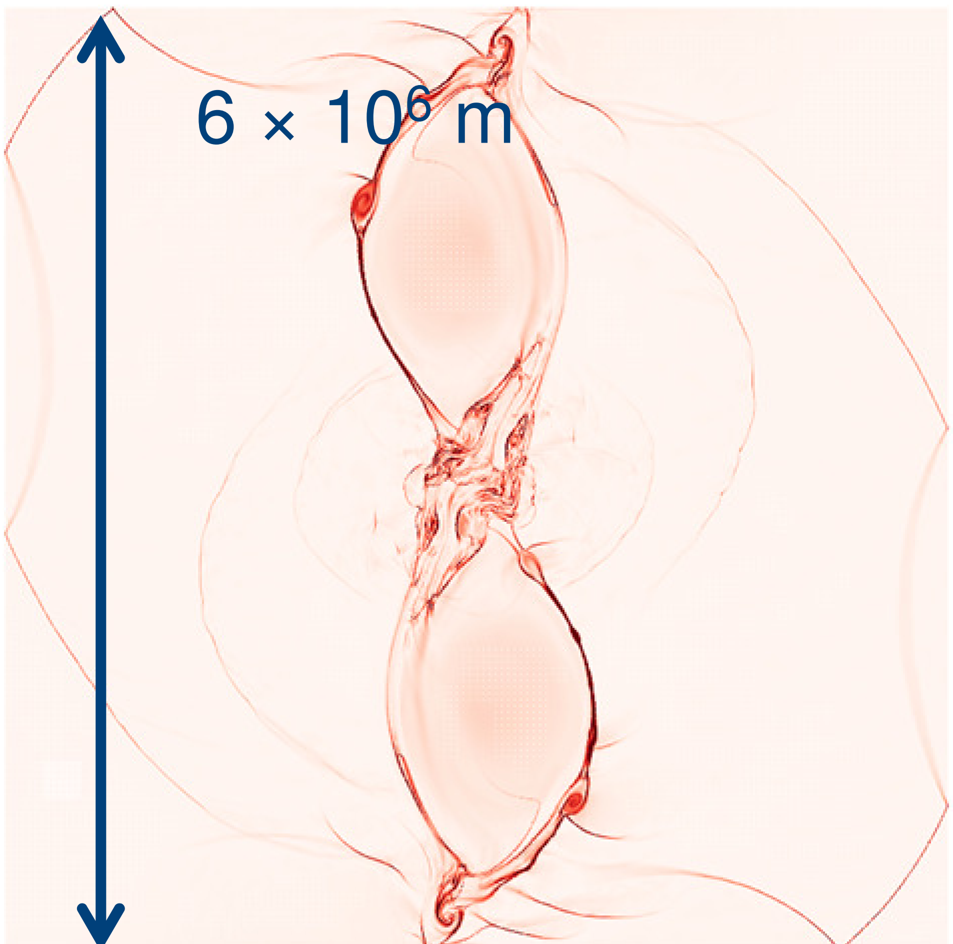
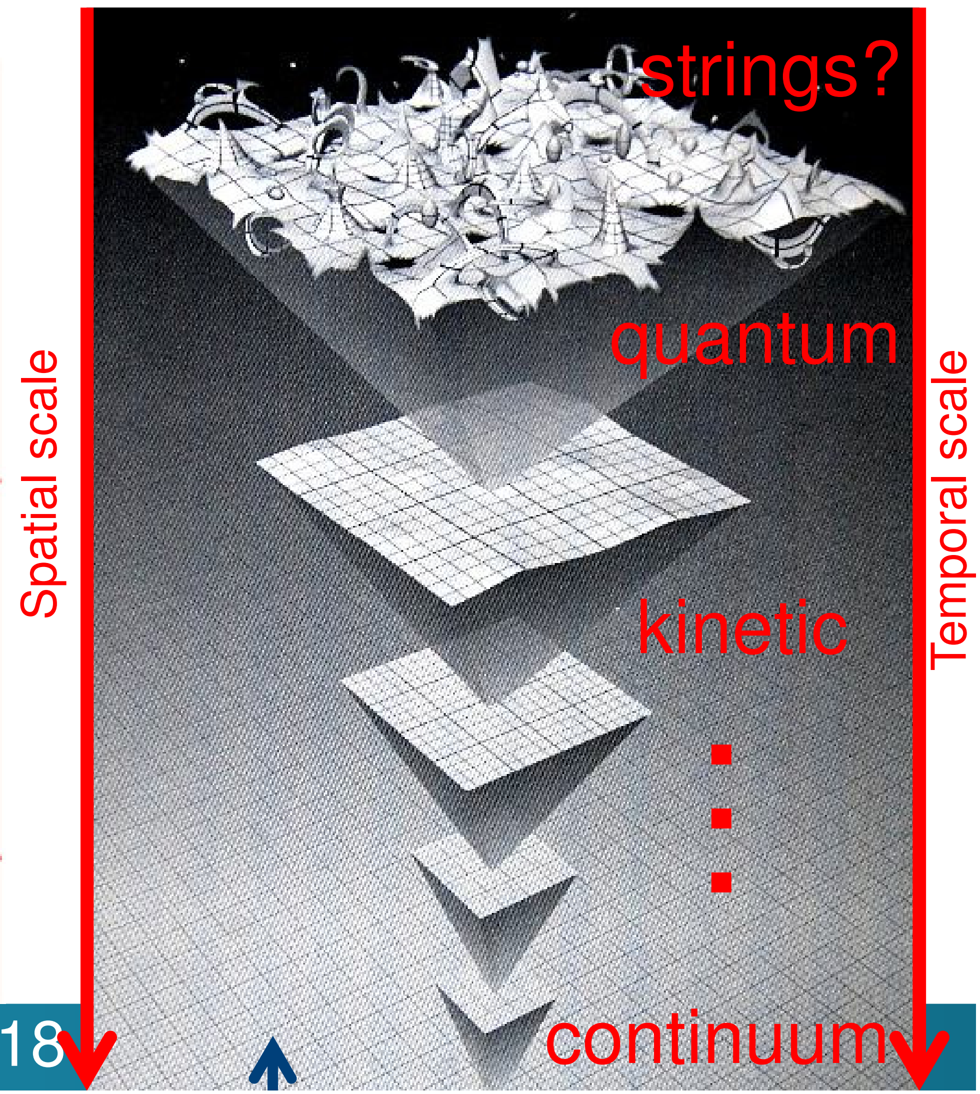
How far can we zoom in?

Current J on 2400^2 grid cells

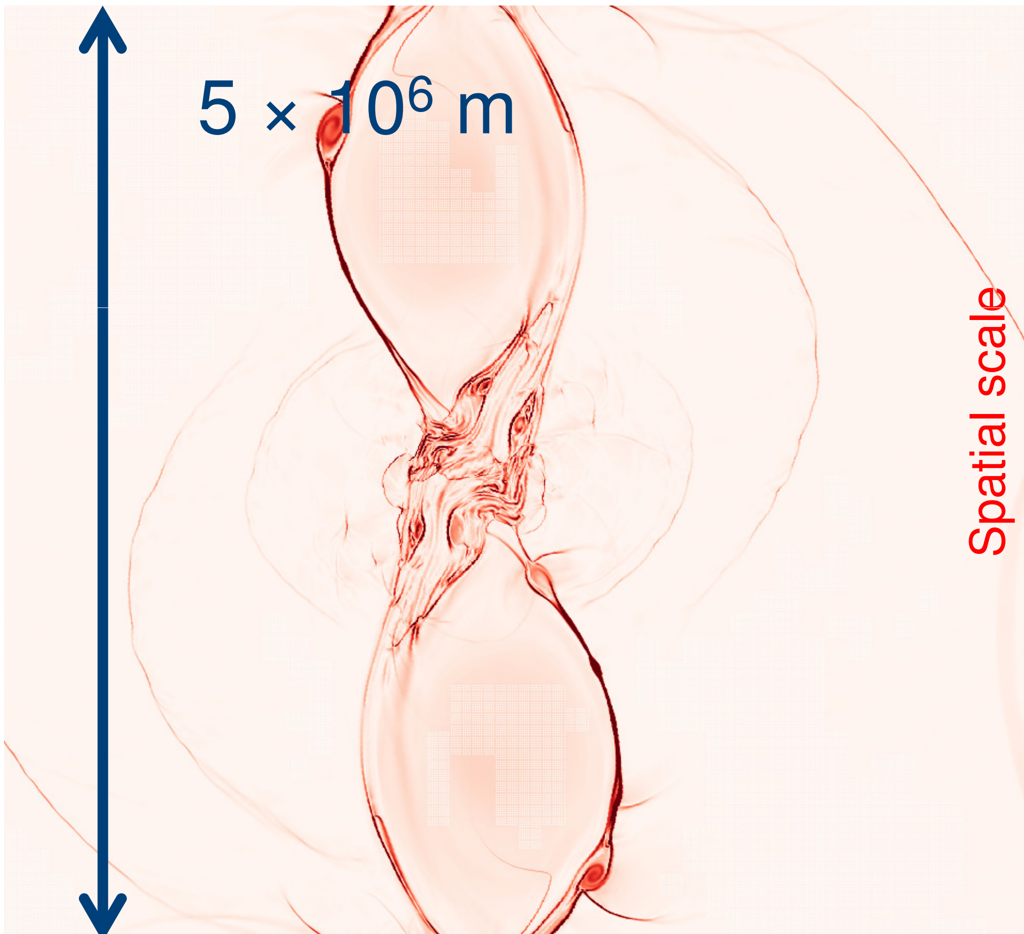


From “The Elegant Universe” by B. Greene



How far can we zoom in?

Current J on 2400^2 grid cells



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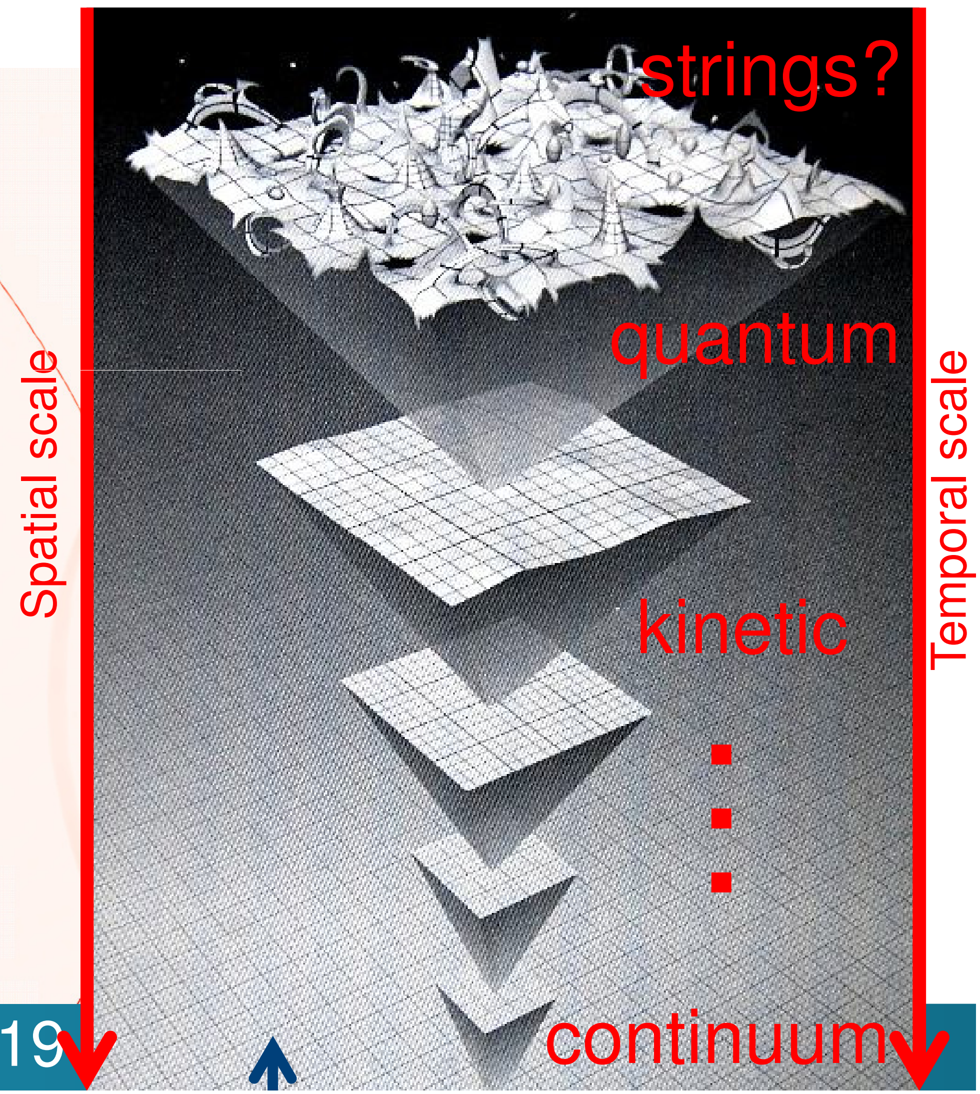
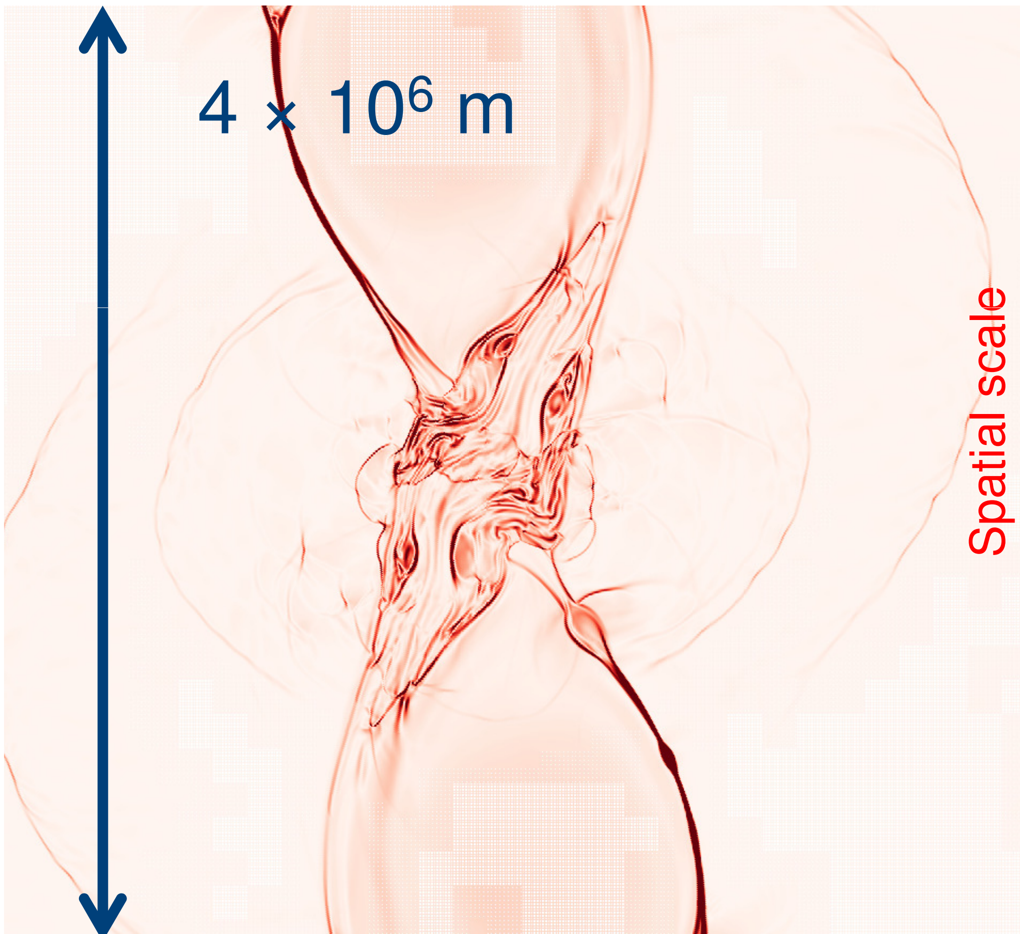


How far can we zoom in?

Current J on 2400² grid cells



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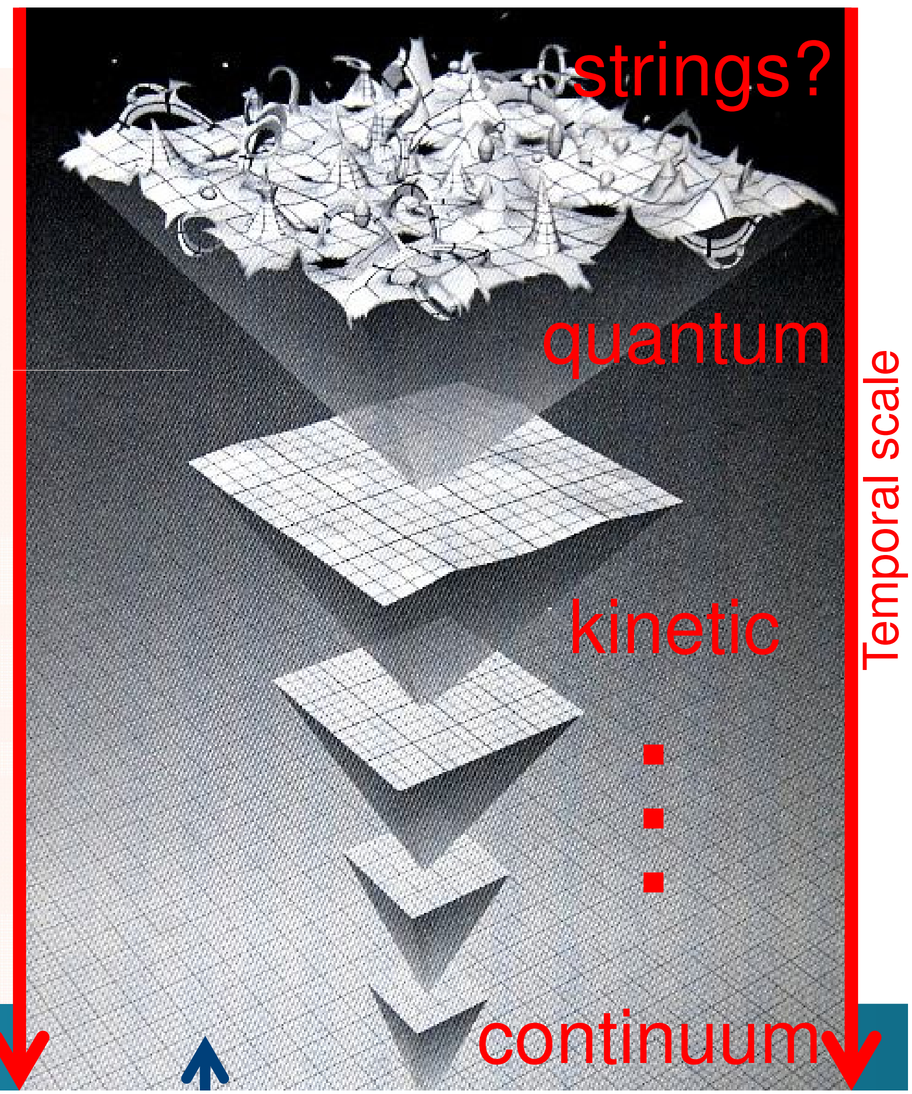

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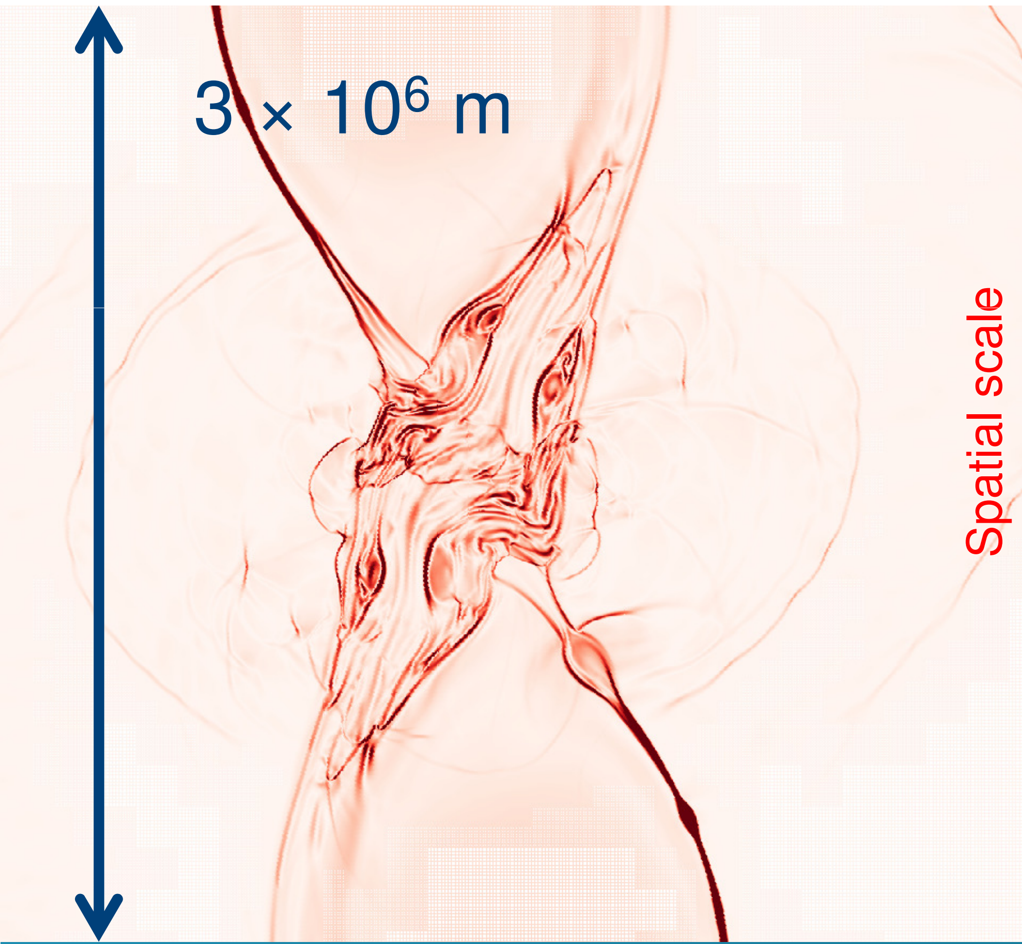
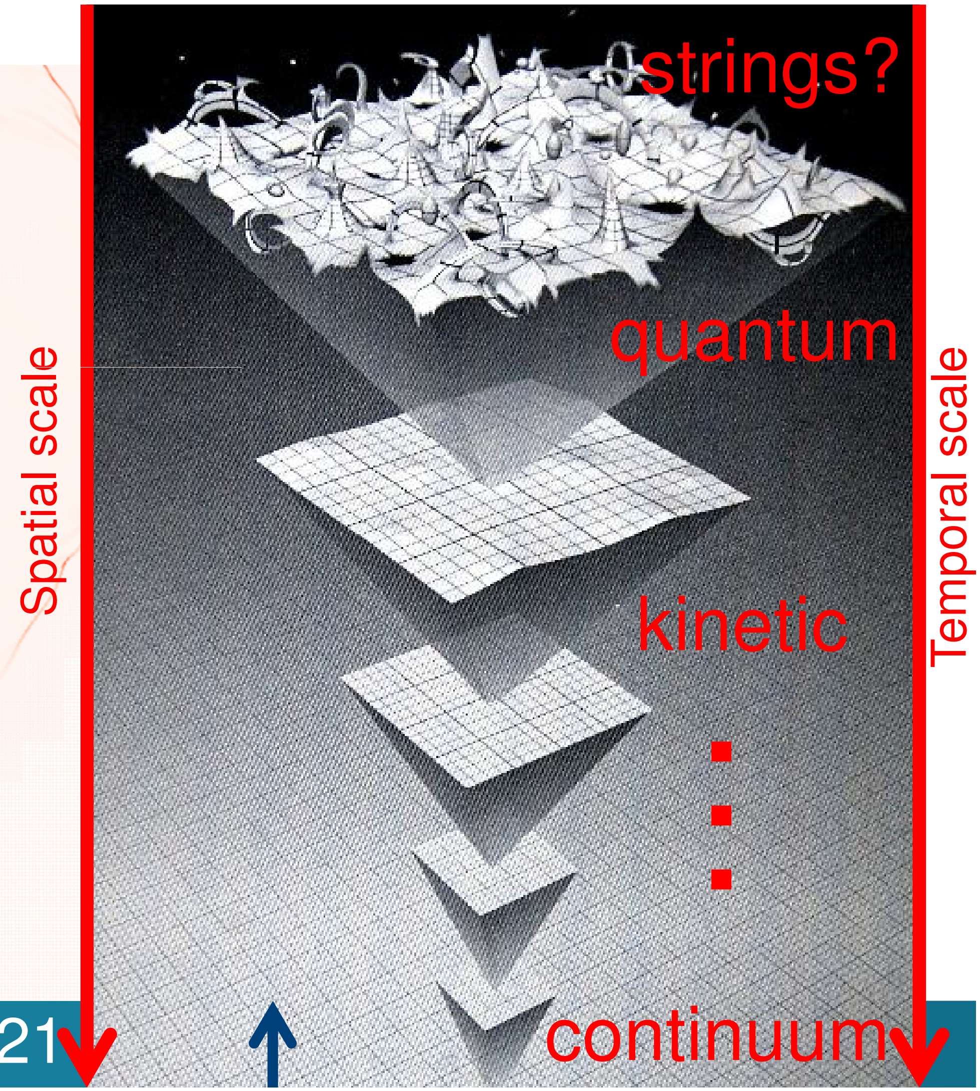
Current J on 2400² grid cells





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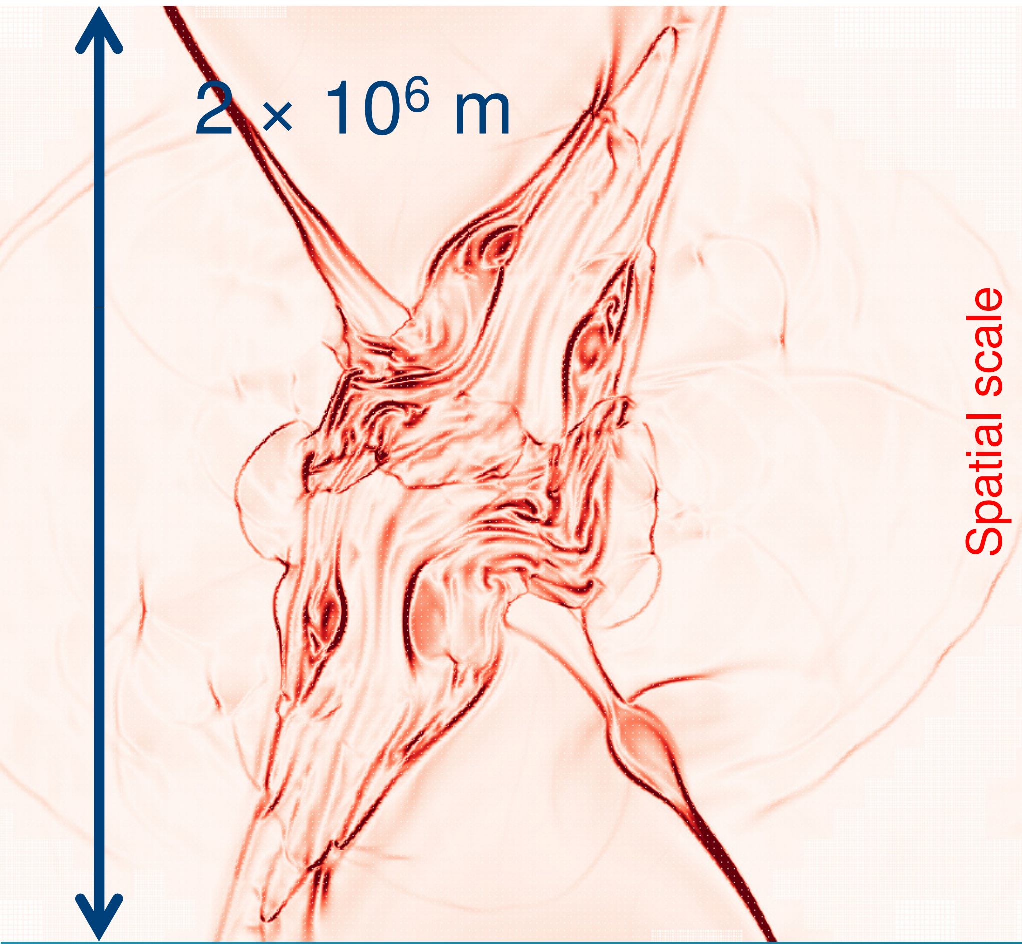

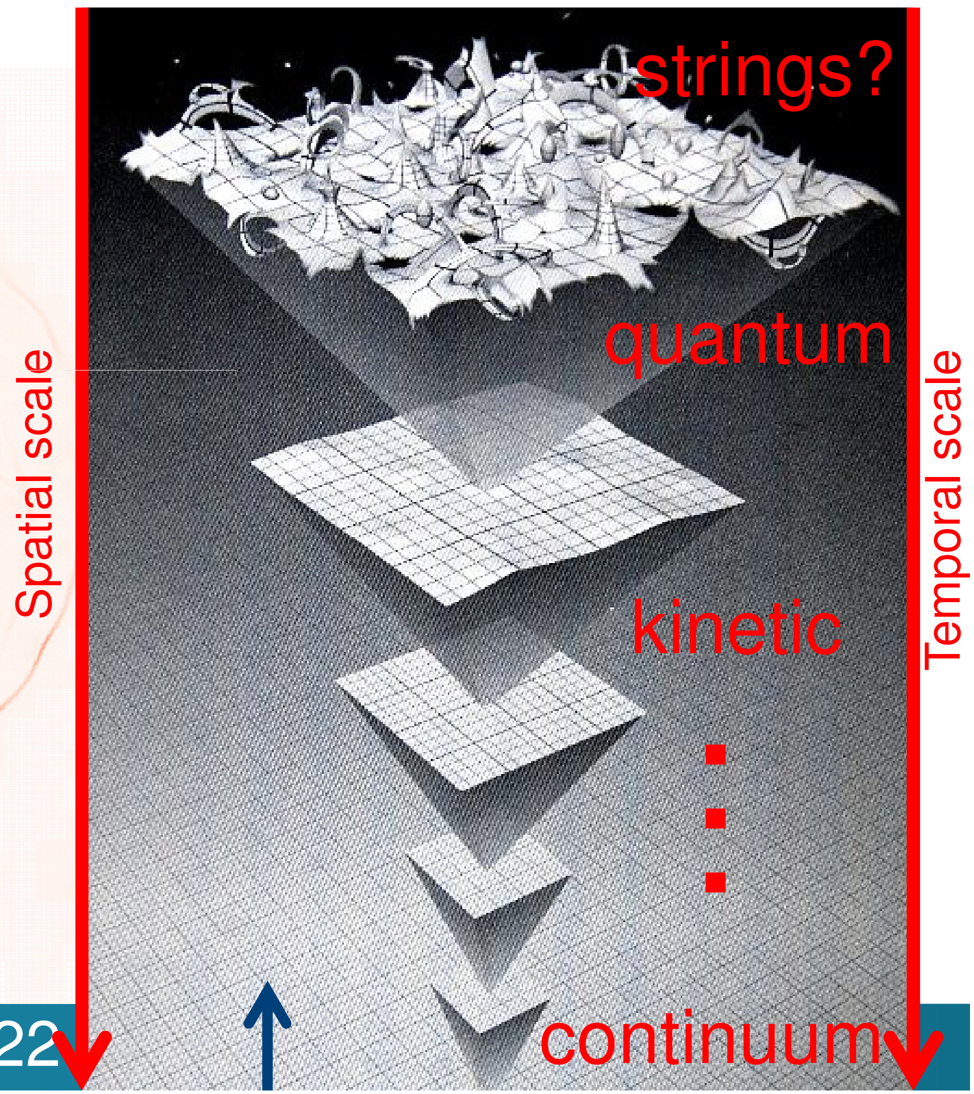

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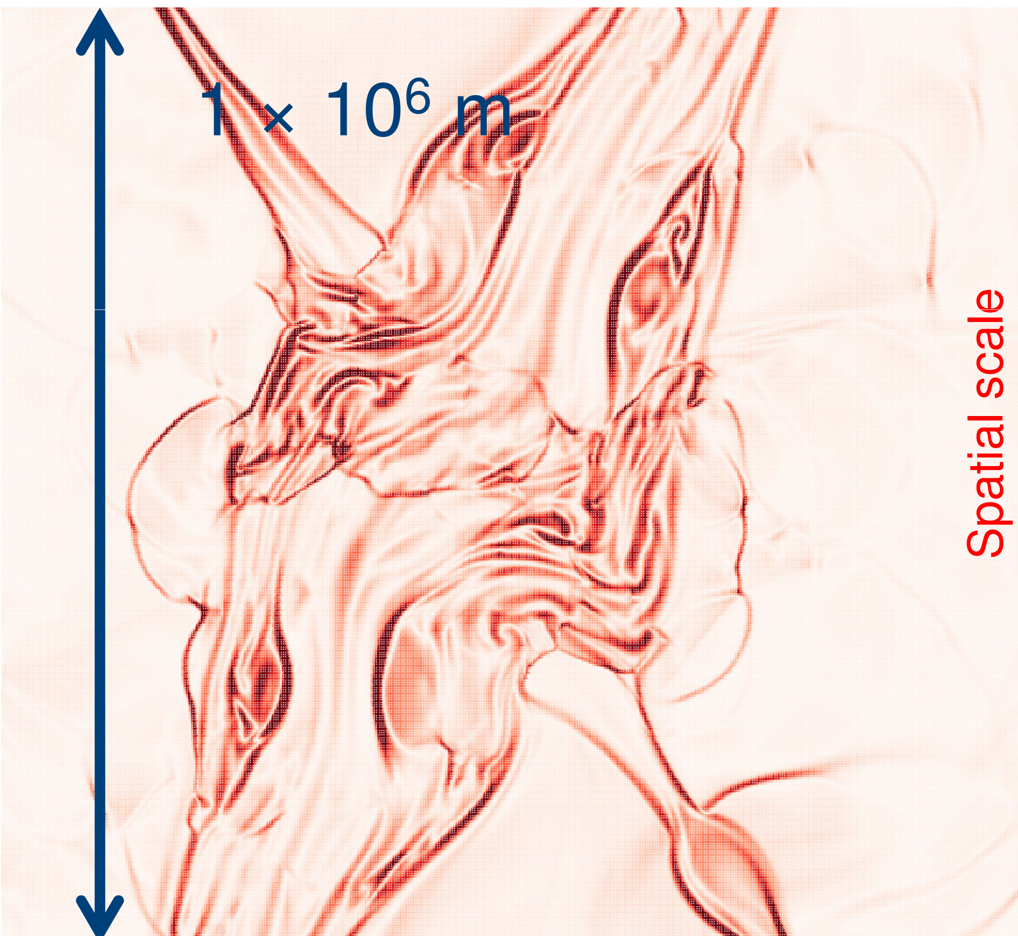
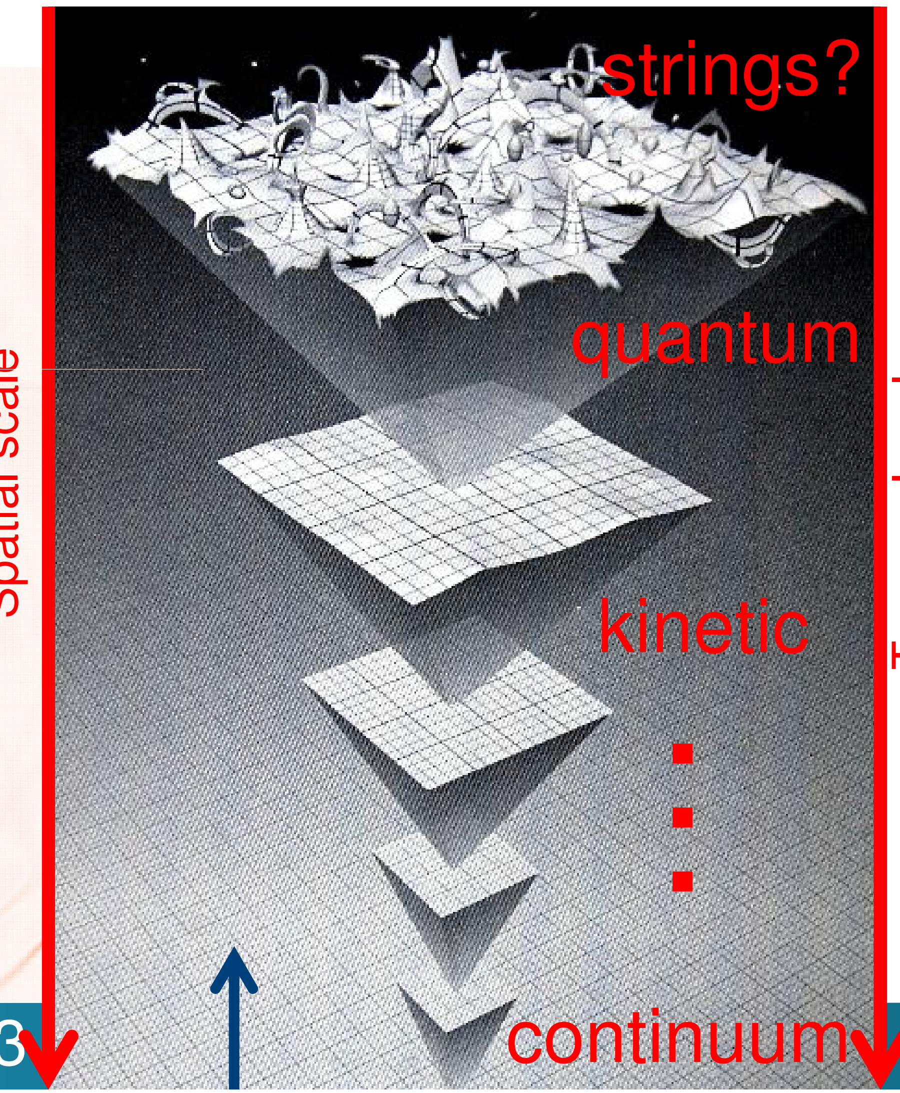
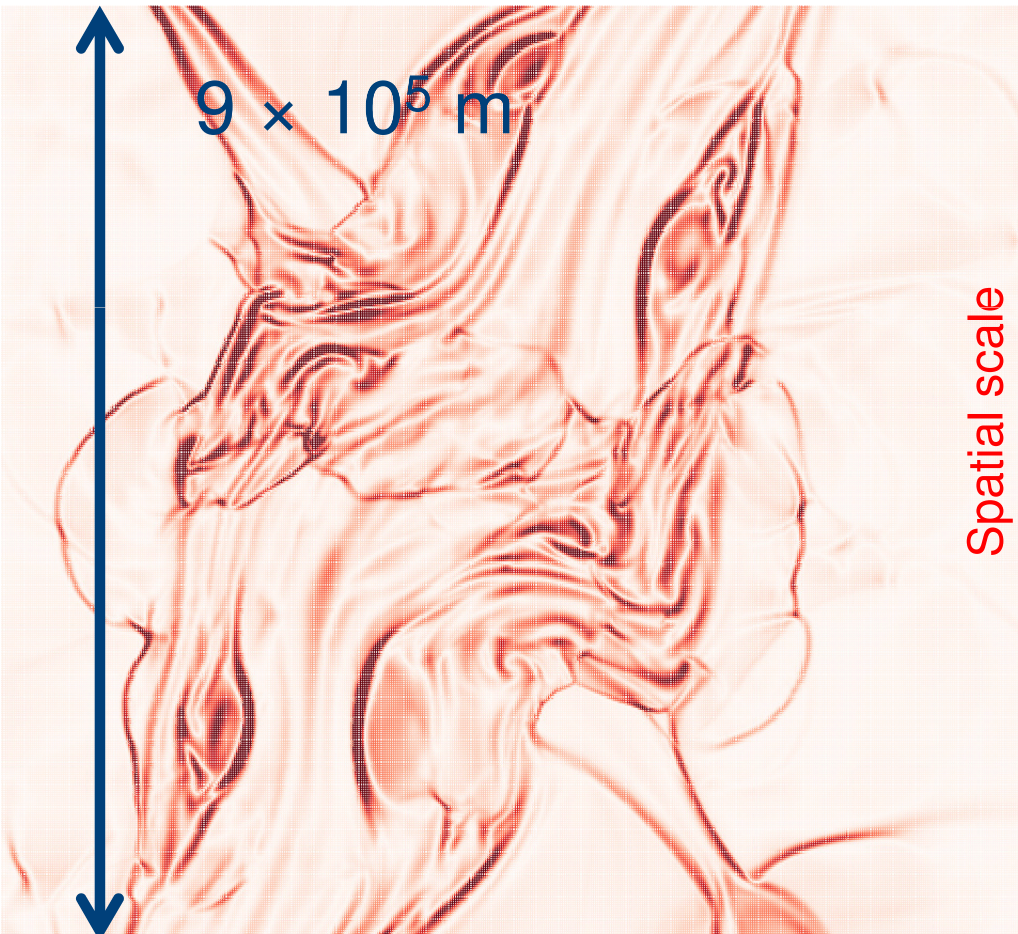




How far can we zoom in?

Current J on 2400^2 grid cells



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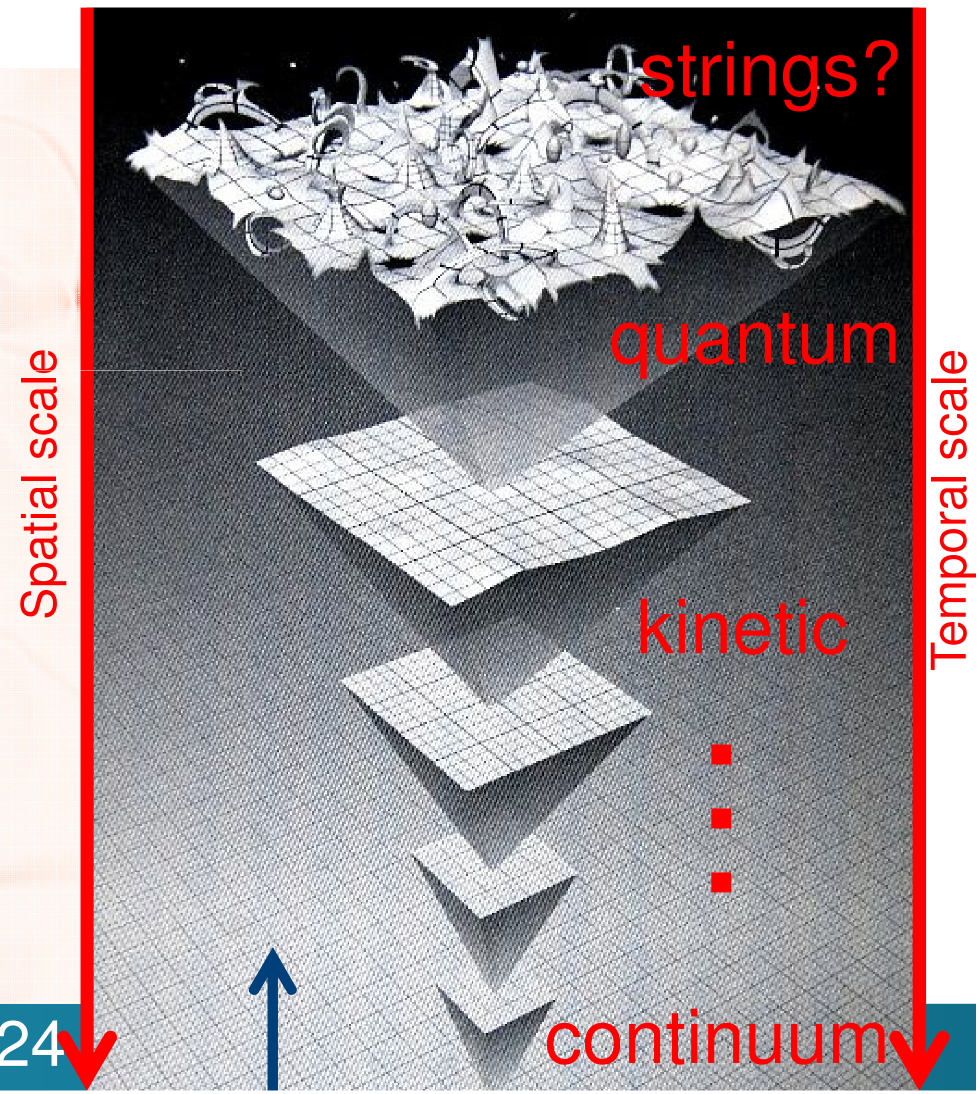

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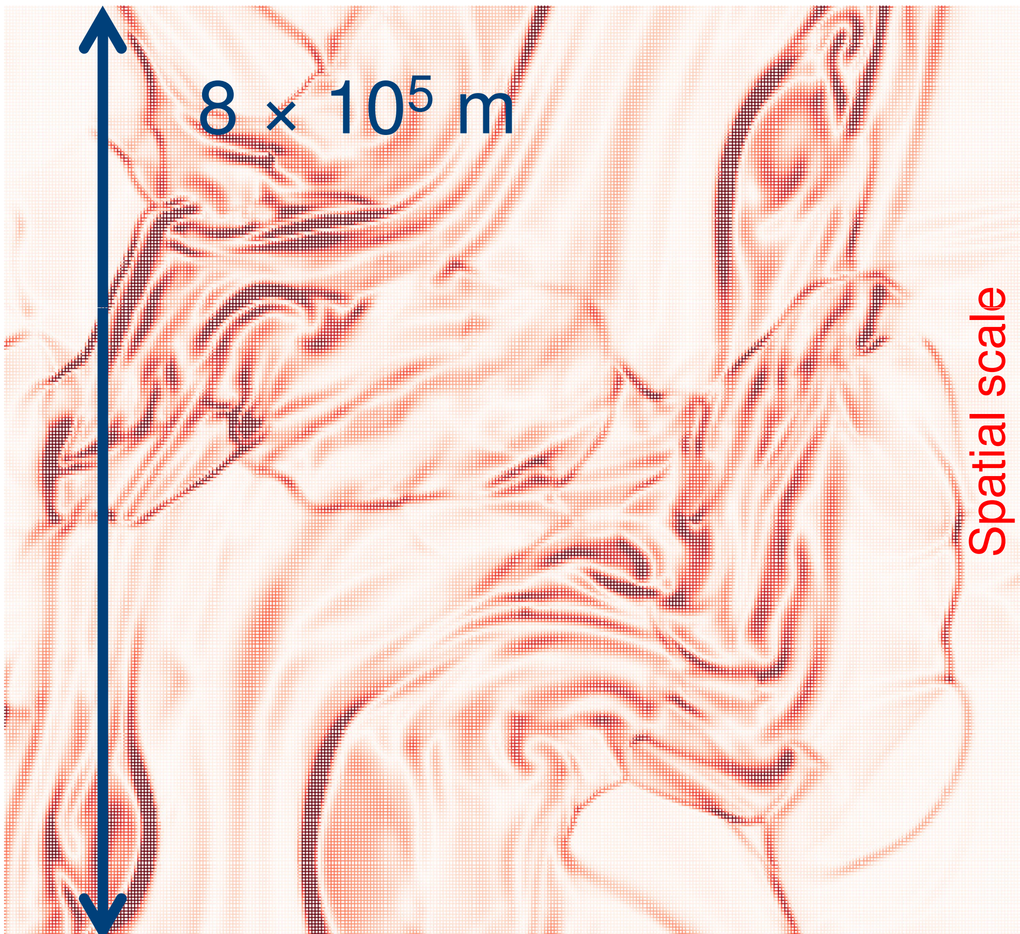
Current J on 2400^2 grid cells



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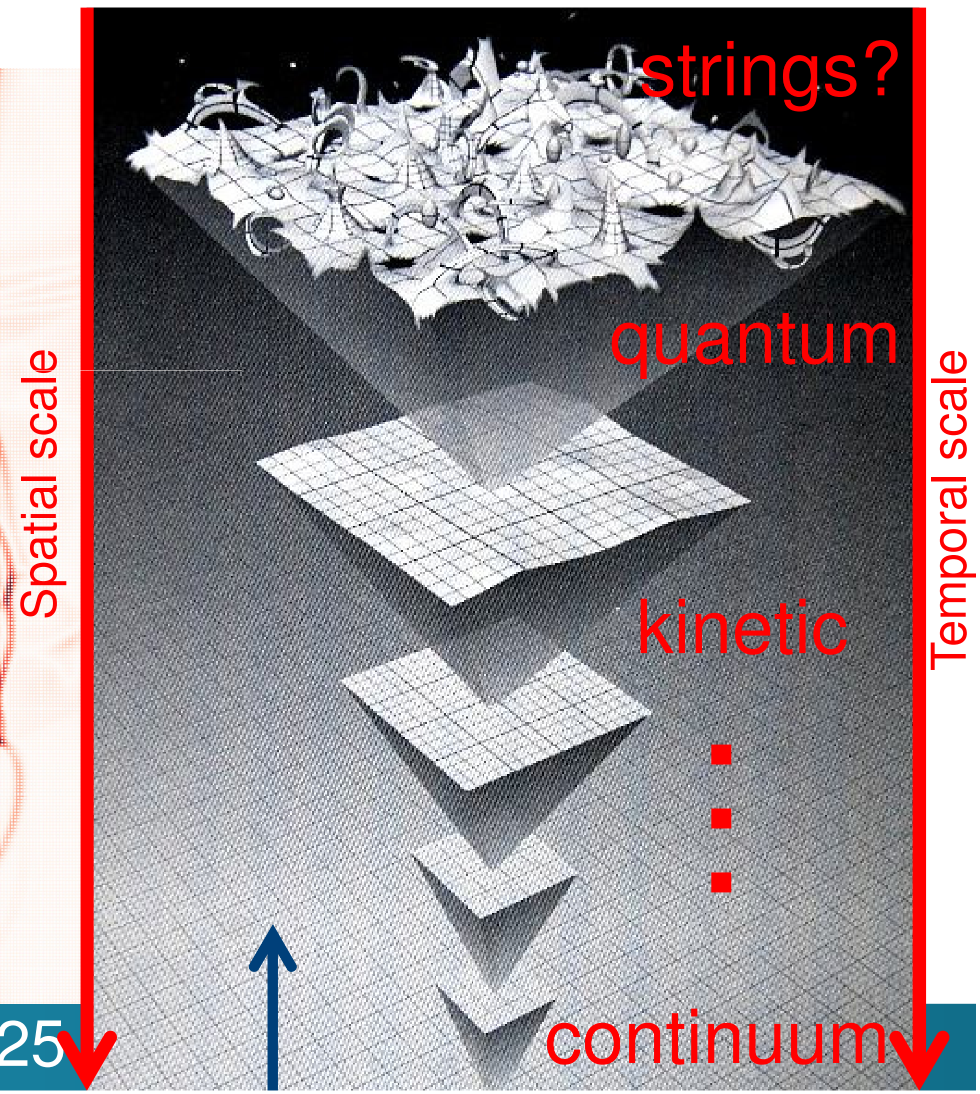
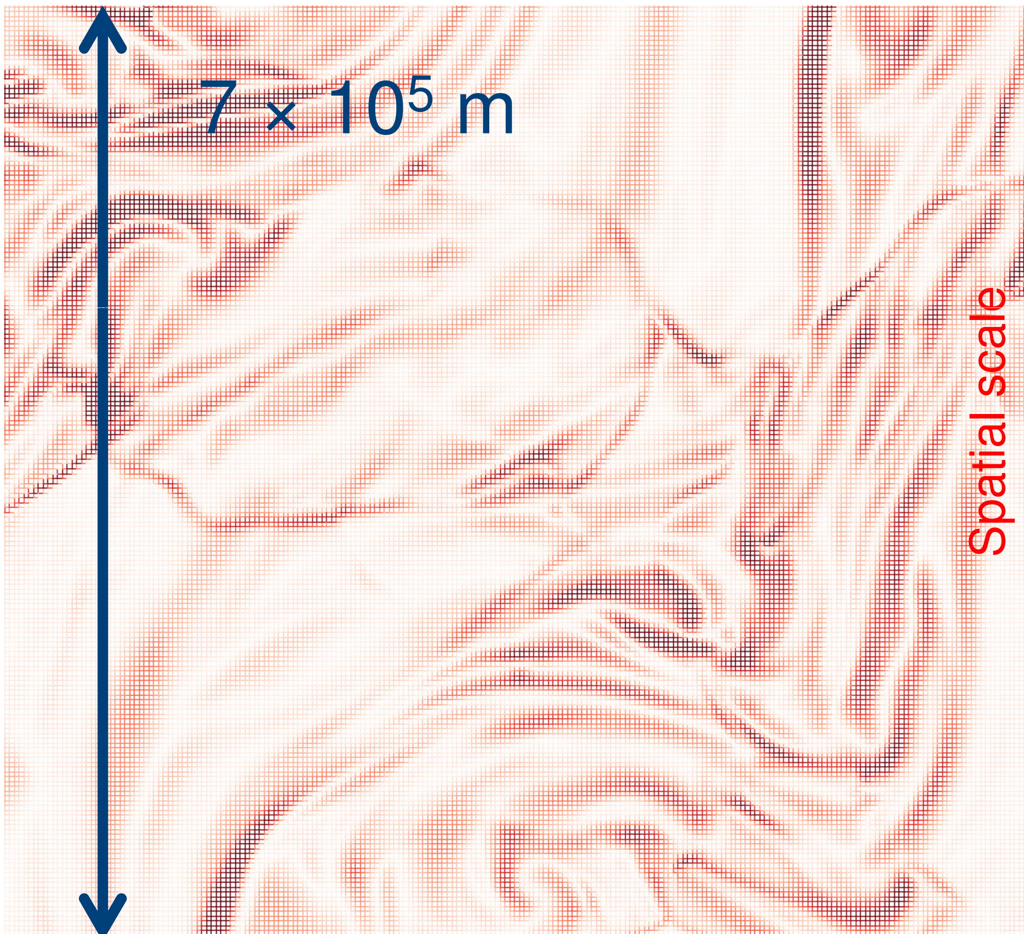


How far can we zoom in?

Current **J** on 2400^2 grid cells



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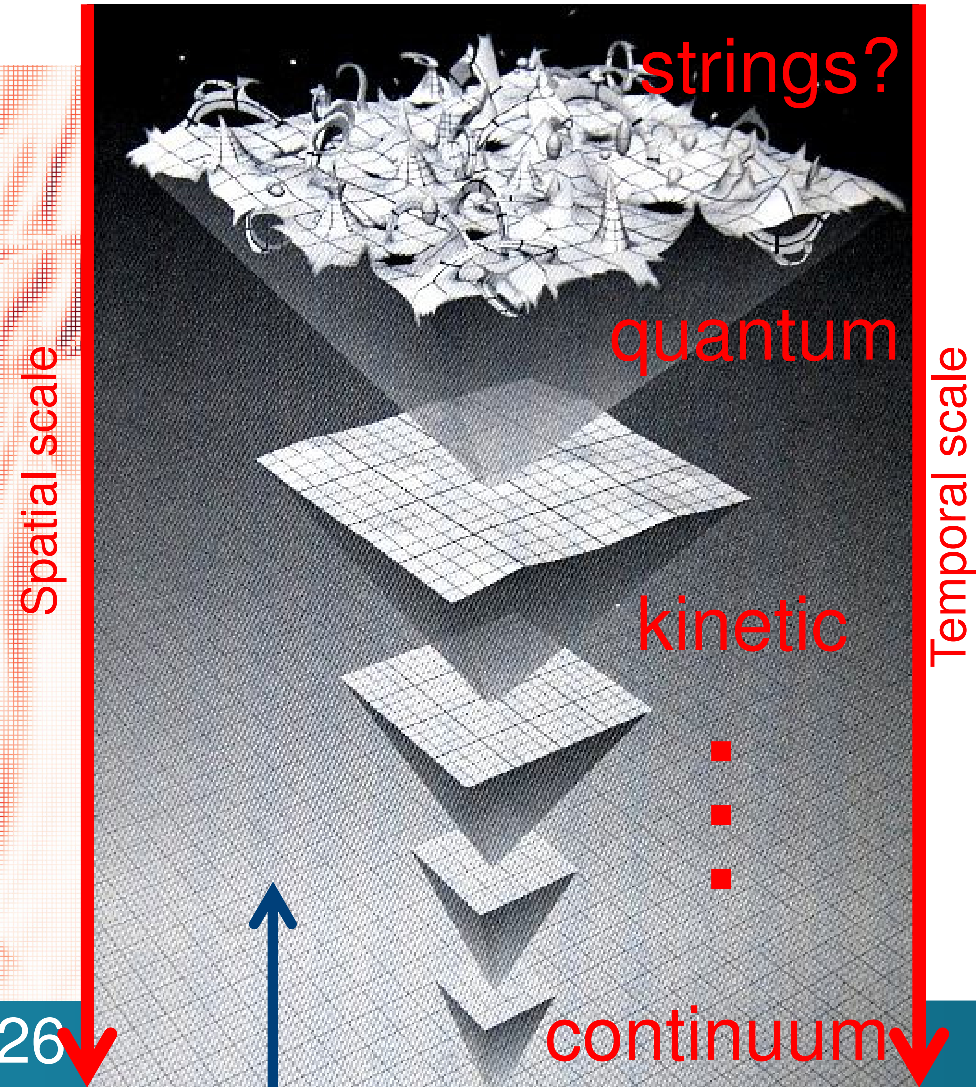
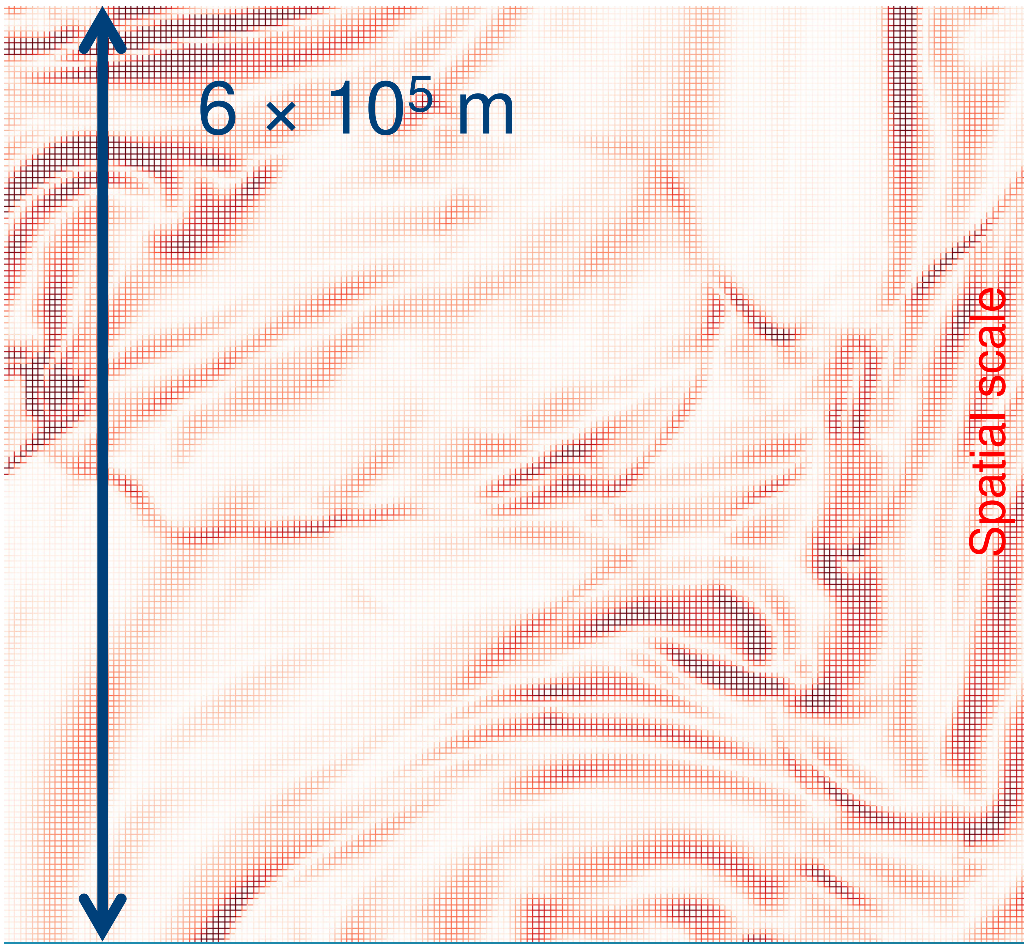


How far can we zoom in?

Current J on 2400² grid cells



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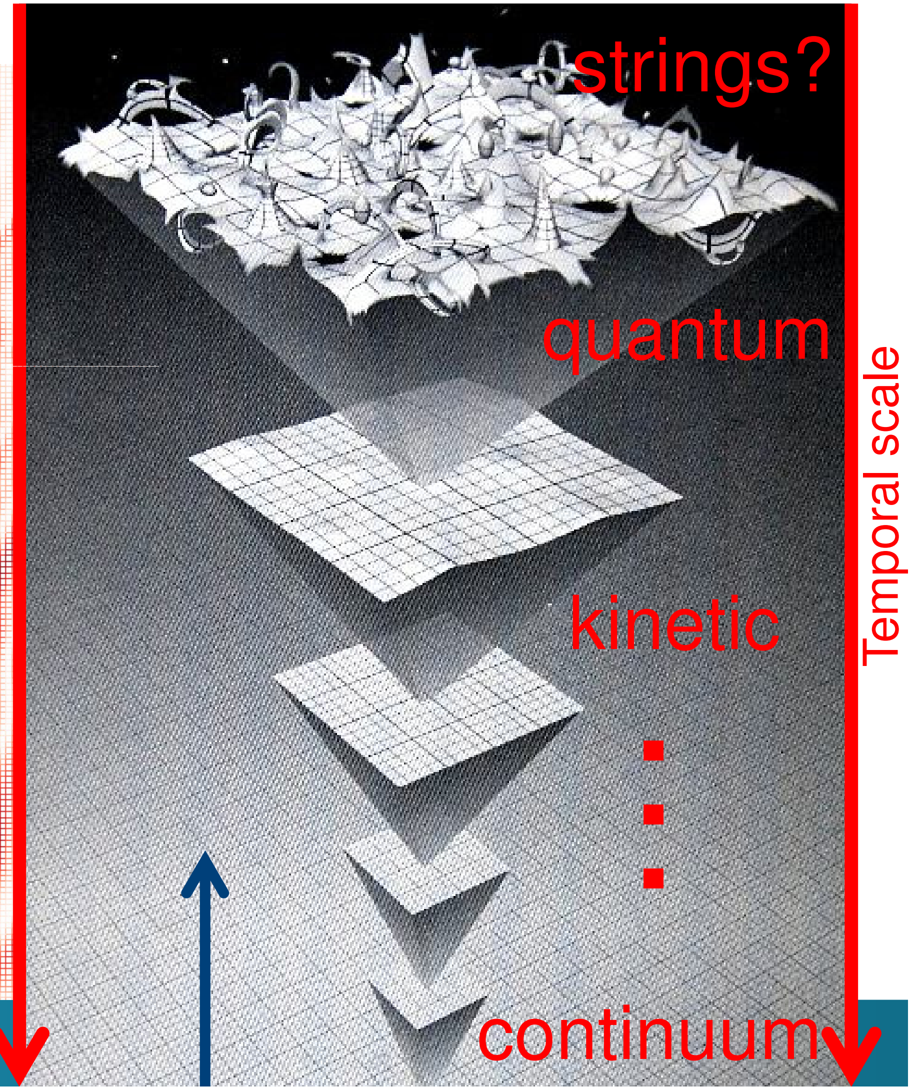
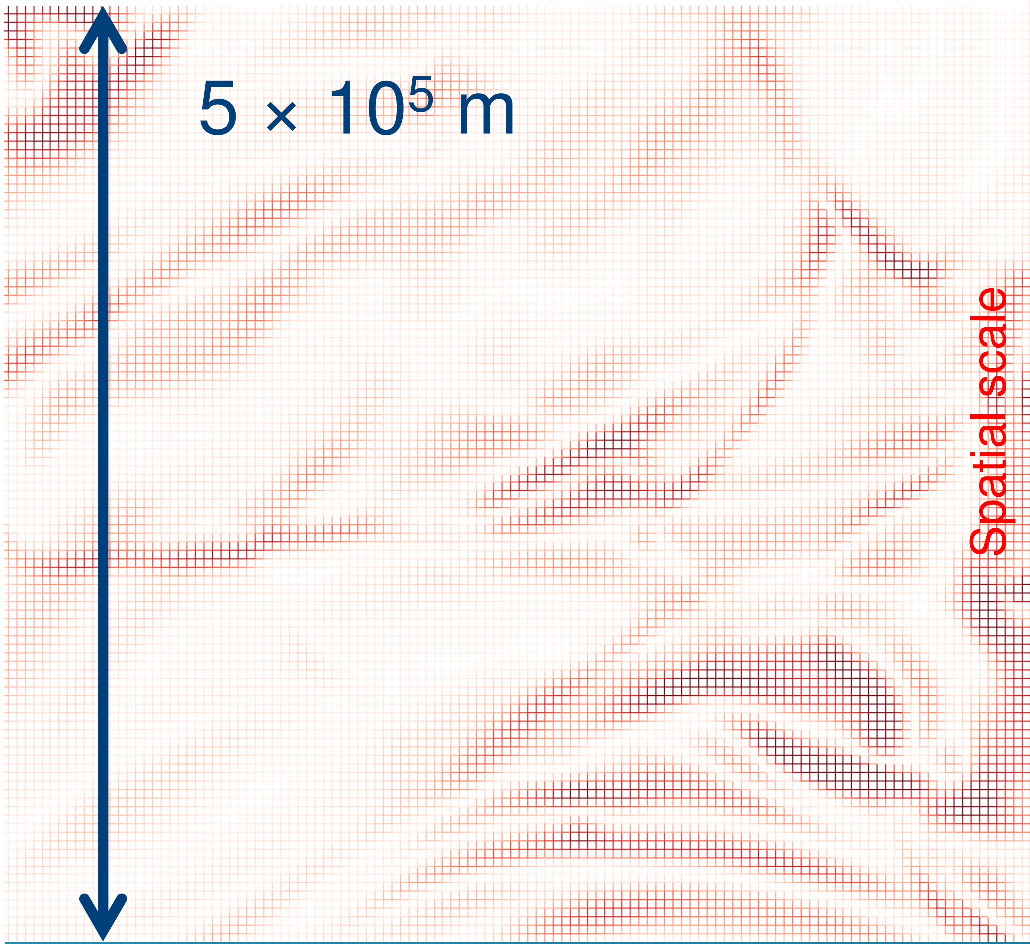


How far can we zoom in?

Current J on 2400² grid cells



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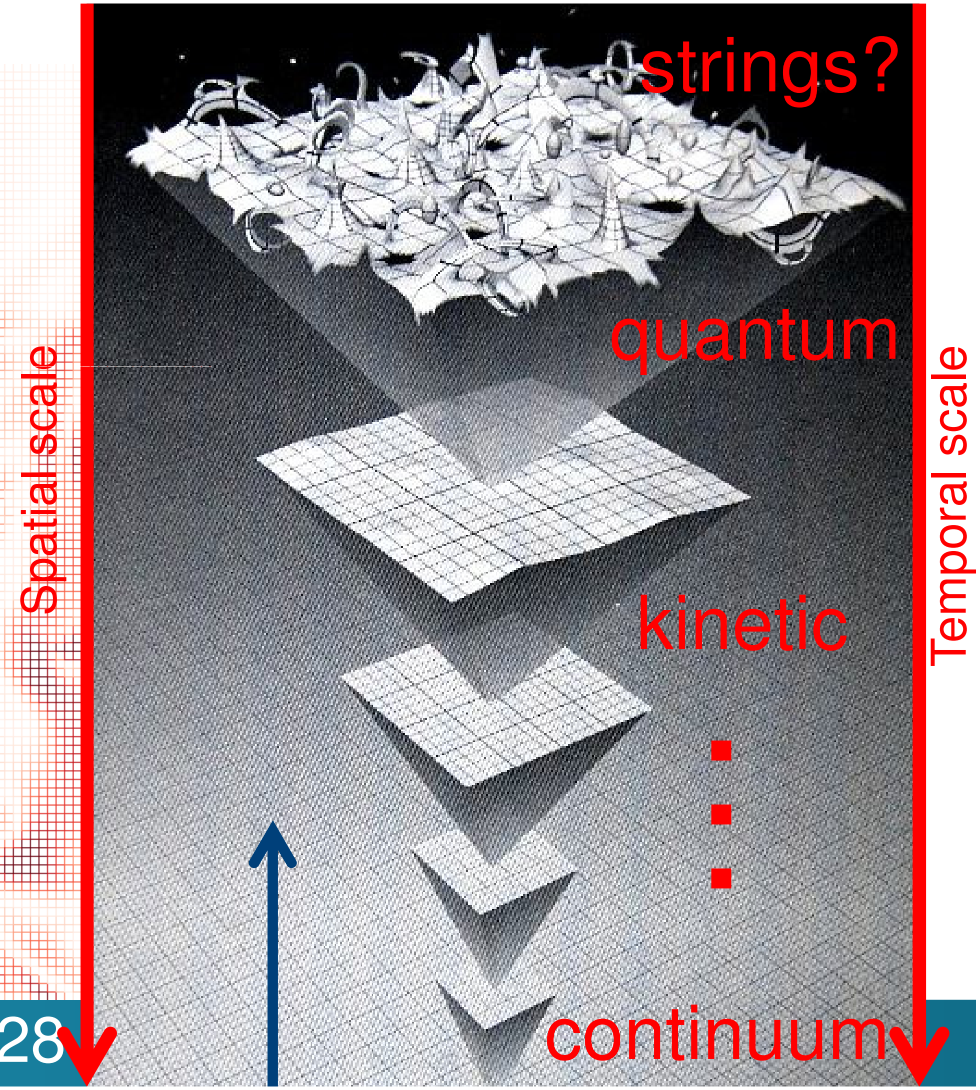
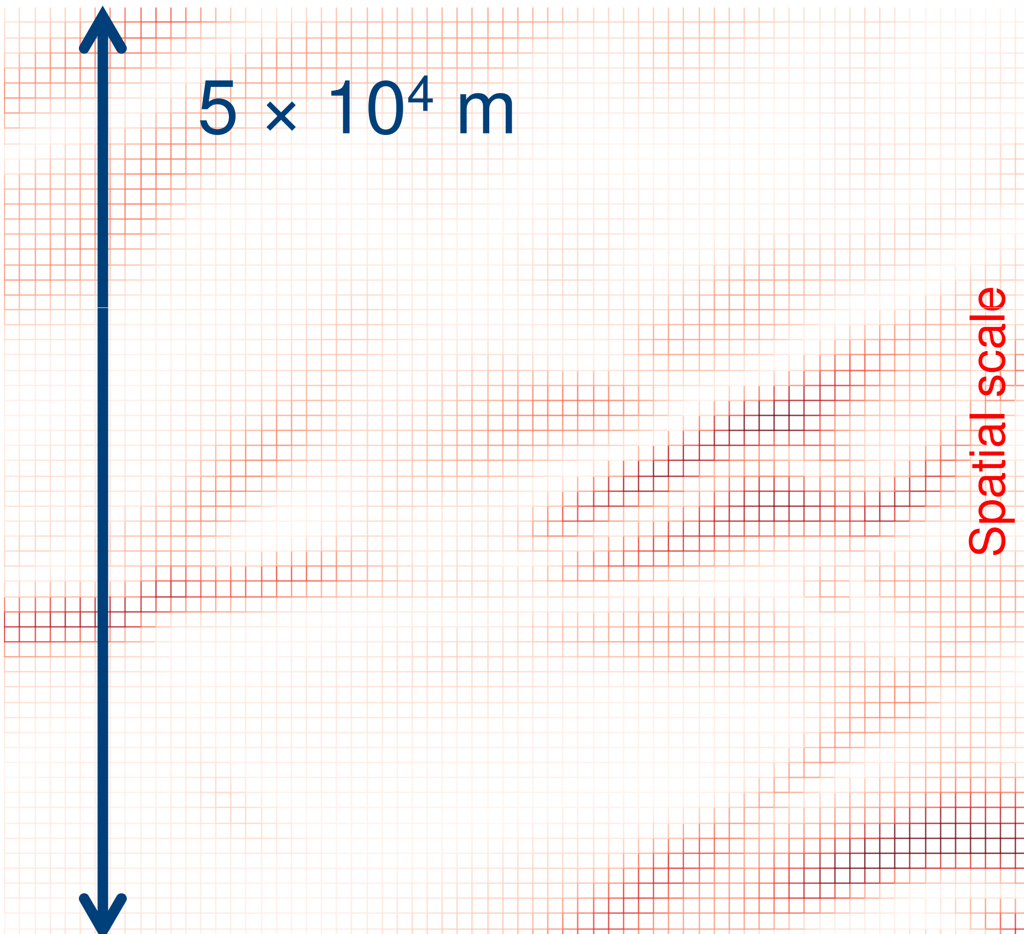


How far can we zoom in?

Current **J** on 2400^2 grid cells



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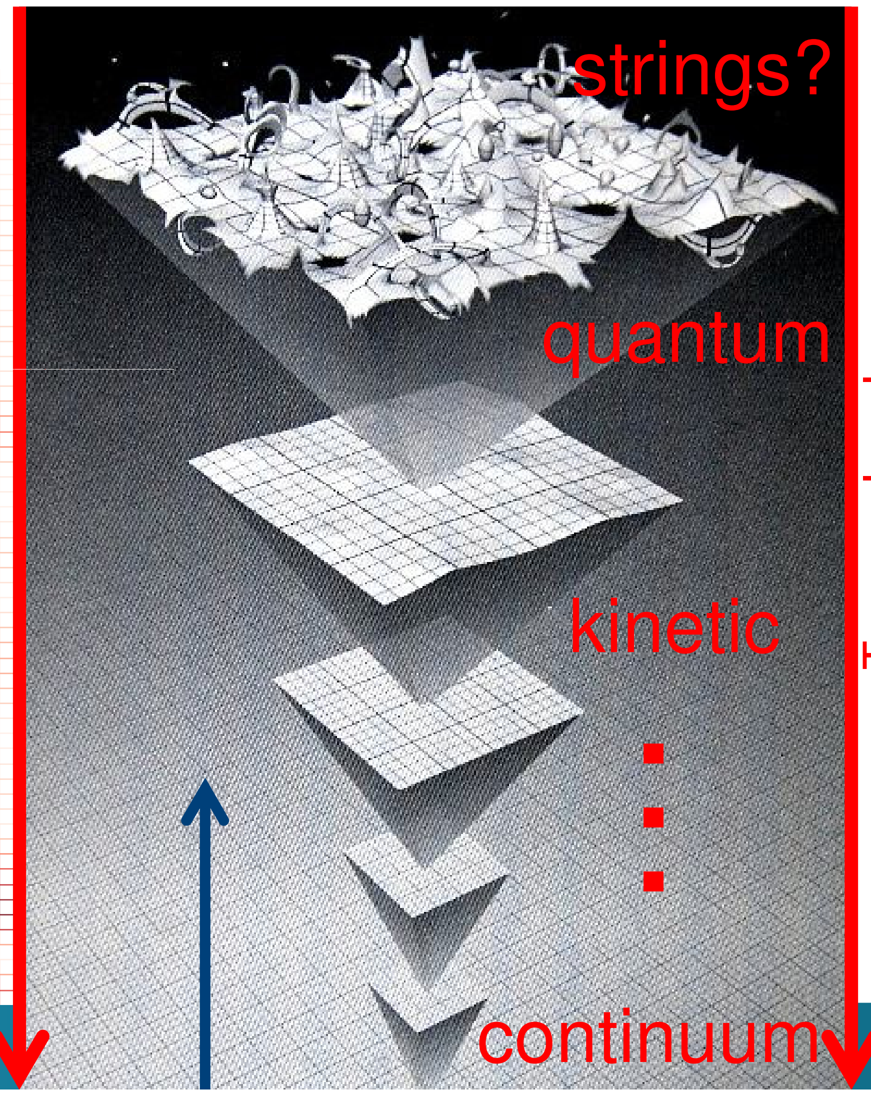

How far can we zoom in?

Current J on 2400^2 grid cells

1×10^4 m

Mesh refinement allows to see MHD fine structure

The limits of the continuum (MHD) picture are reached.

No new information is gained from zooming in further

We need particles!

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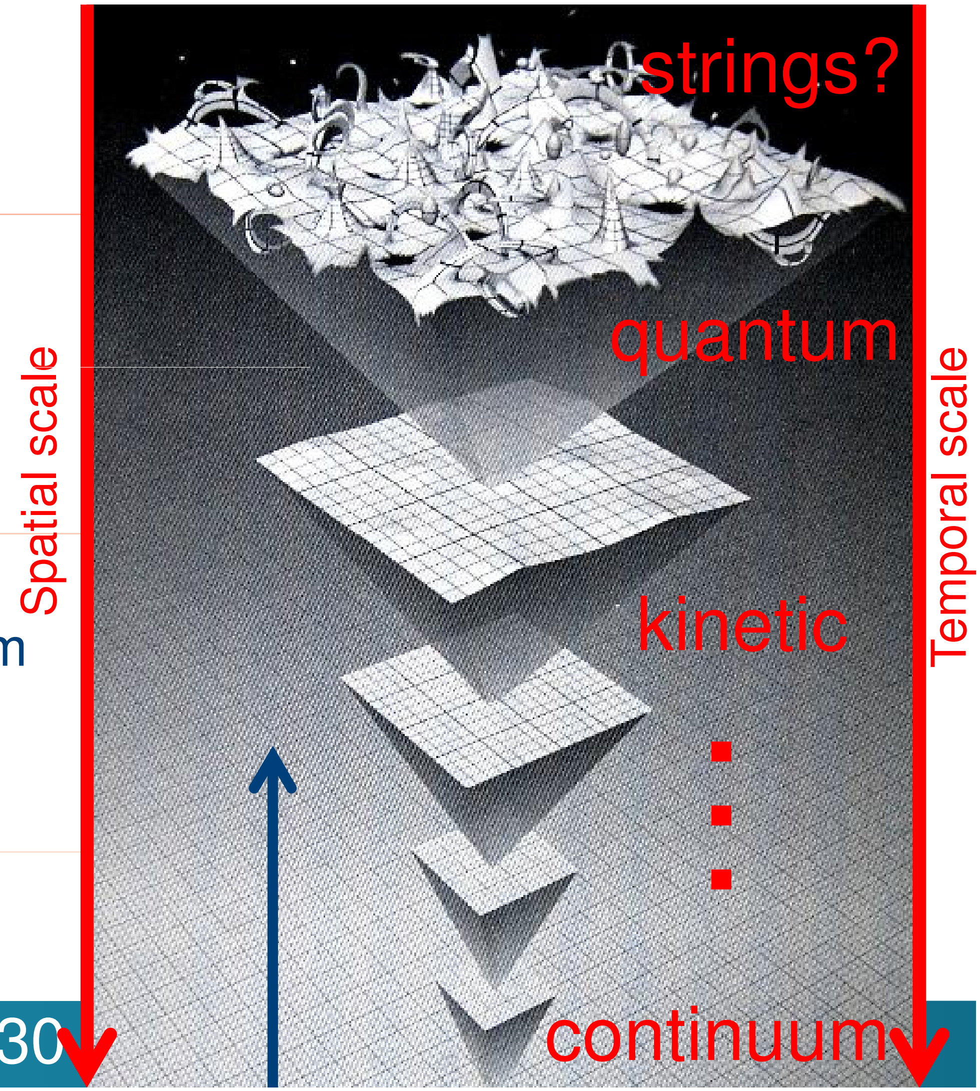
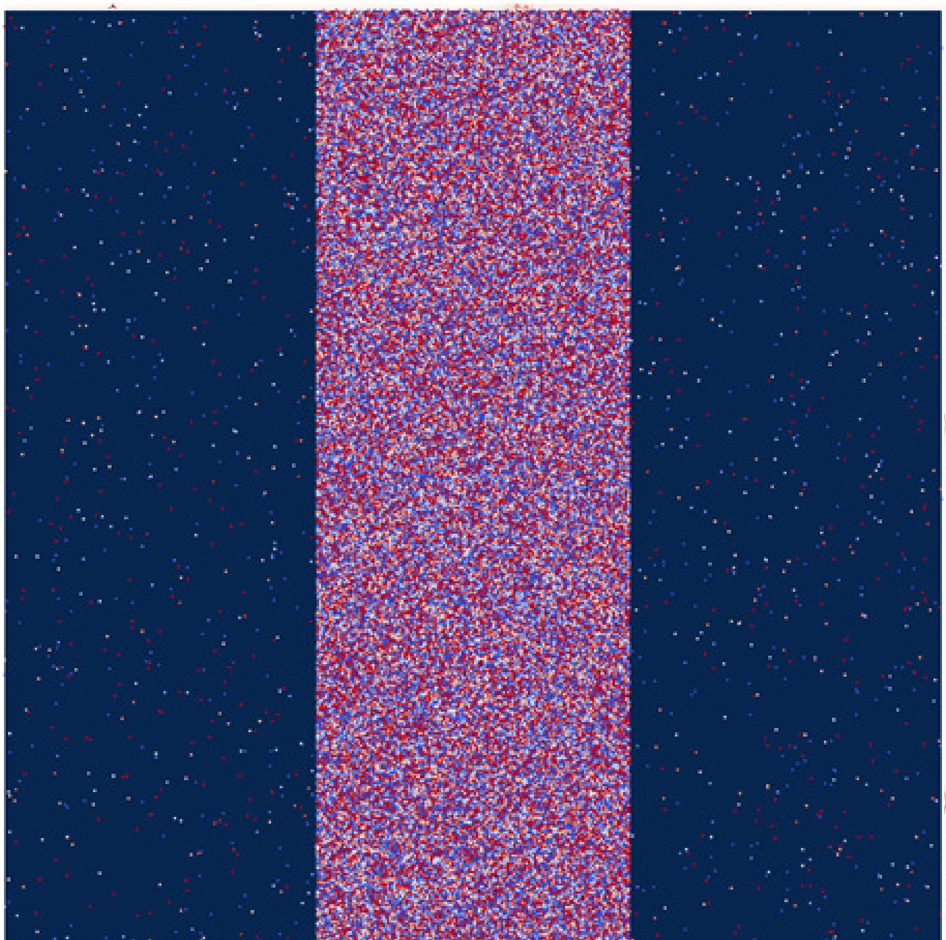
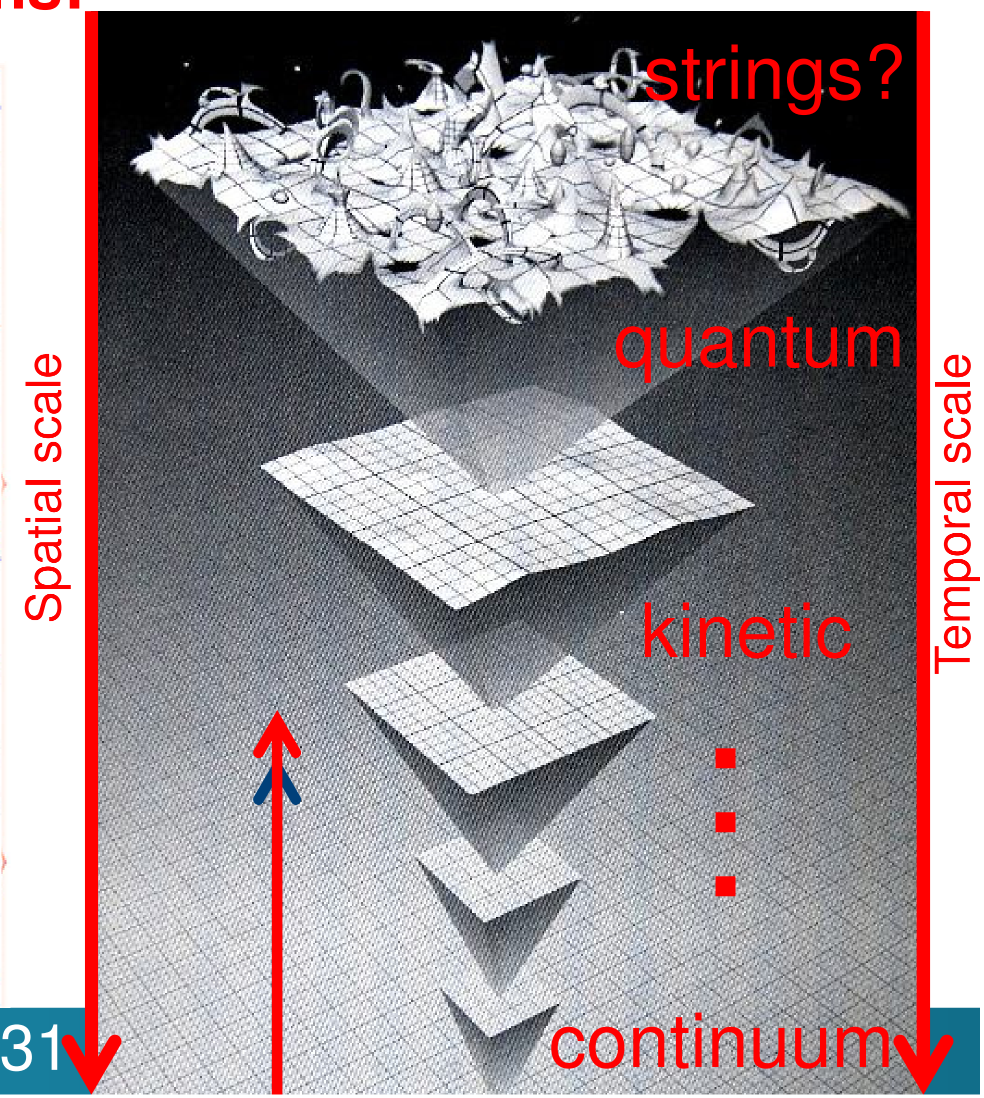


So… What can we do?!

Test particles in MHD evolutions!



From "The Elegant Universe" by B. Greene



Which particles are we interested in?

Two populations of particles are considered

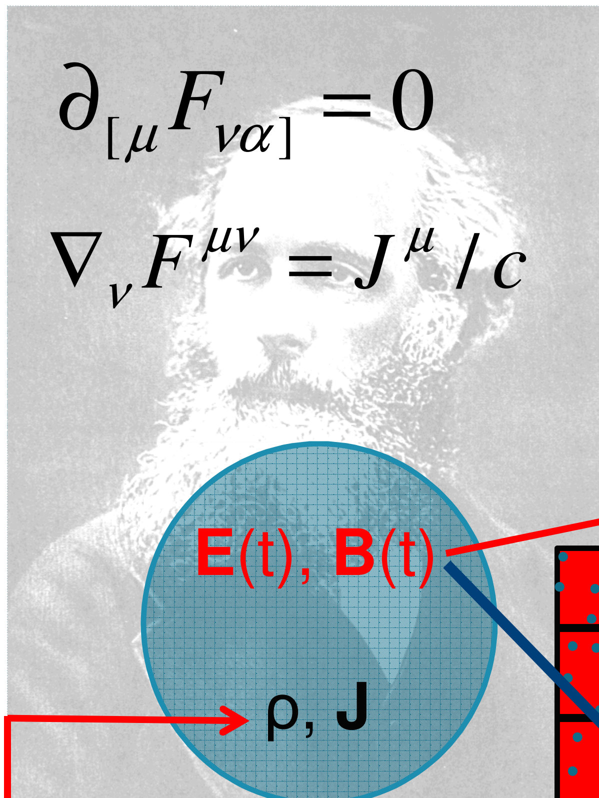
- Thermal plasma described by a Maxwellian distribution.
  → MHD is a satisfactory description.
  → The largest scales of the system are studied.

- Non-thermal plasma, with highly accelerated particles and a power law distribution.
  → Relativistic particle equations of motion are required.
  → The microscopic scales of the system are studied.

… and now with particles

$$\partial_{[\mu} F_{\nu\alpha]} = 0$$

$$\nabla_\nu F^{\mu\nu} = J^\mu / c$$

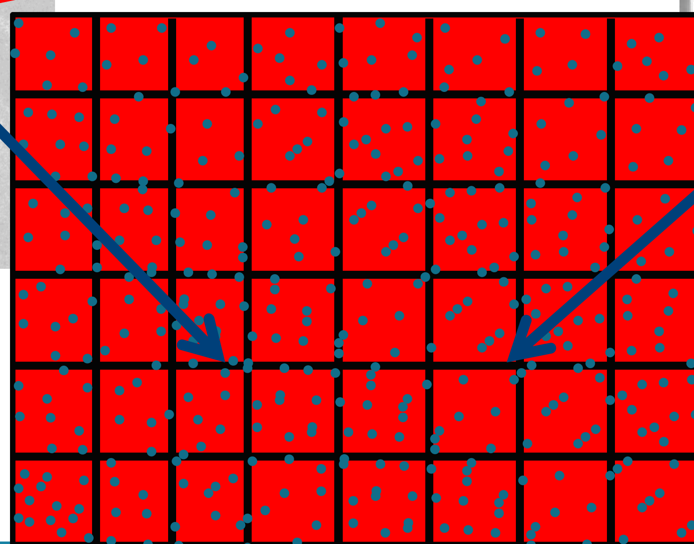


Maxwell's equations

Still obtained from fluid equations

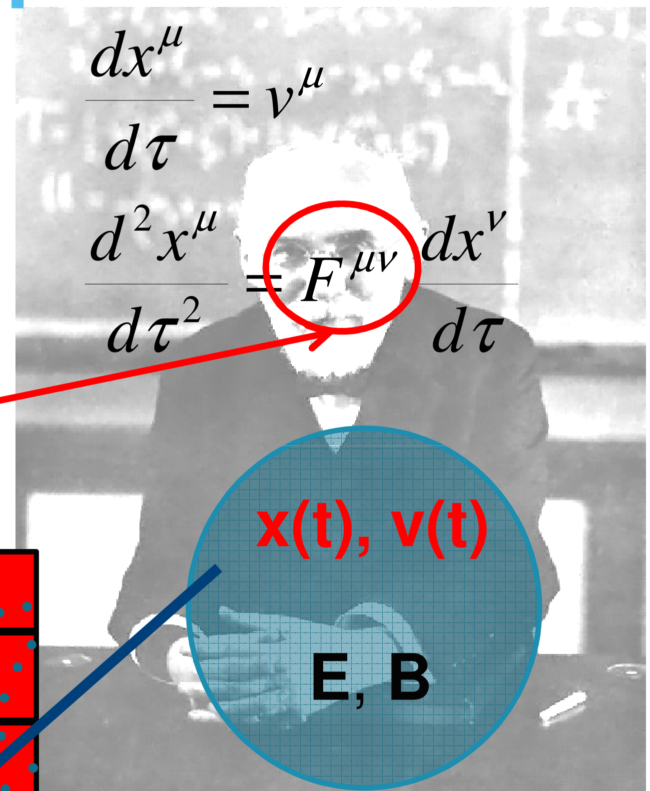
Particles follow fields

Relativistic test particles in MHD snapshots



$$\frac{dx^\mu}{d\tau} = v^\mu$$

$$\frac{d^2x^\mu}{d\tau^2} = F^{\mu\nu}\frac{dx^\nu}{d\tau}$$



Relativistic equations of motion

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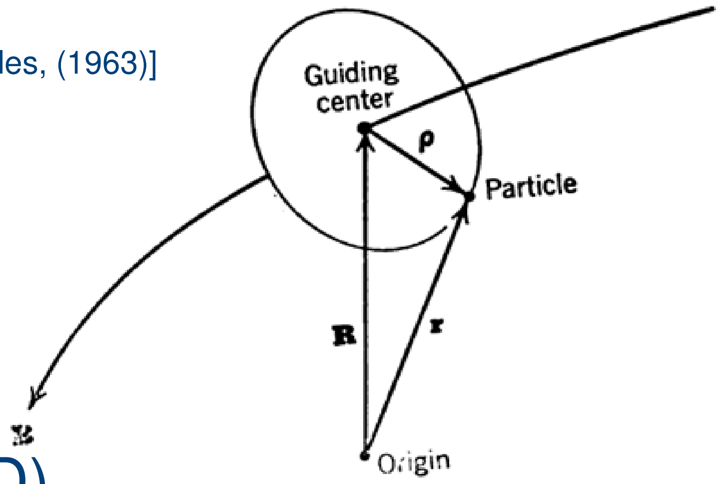
[Northrop, The Adiabatic Motion of Charged Particles, (1963)]



Assumptions

- Low $\beta = 2p/B^2$ (e.g. solar flares)
- Low $\sigma \rightarrow v_A << c$ (non-relativistic MHD)
- $\partial t_{particle} << \partial t_{MHD}$ → relativistic particles in MHD evolutions
- MHD → magnetic field, velocity and density
- Test particles → collisions and effect on fields ignored
- Gyroradius $R_c = \frac{\gamma m v_\perp}{Bq} \sim 10^{-1} m - 10^{-3} m << \text{grid cell size} \sim 10^3 m$

  → Replace particles position by its guiding centre

Particle	$B\ [T]$	$T\ [K]$	$n\ [m^{-3}]$	$v_{thermal}\ [m/s]$	$\beta\ [-]$	$R_c\ [m]$	$\gamma\ [-]$
Electron	$0.03\ T$	$10^6\ K$	$10^{16}\ m^{-3}$	$5.5 \times 10^7\ ms^{-1}$	0.0004	$10^{-3}\ m$	1.0002
Proton	$0.03\ T$	$10^6\ K$	$10^{16}\ m^{-3}$	$1.3 \times 10^6\ ms^{-1}$	0.0004	$4.4 \times 10^{-2}\ m$	1.0000

[Goedbloed & Poedts, Principles of Magnetohydrodynamics, (2004)]

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Guiding centre approximation

- Charged particle trajectories (spatial part of Lorentz equation):

$$\dot{\mathbf{r}} = \frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{E} - \frac{q}{m}\mathbf{B}\wedge\mathbf{v} = \frac{q}{m}\mathbf{E} - \boldsymbol{\Omega}\wedge\mathbf{v} \quad \text{with} \quad \boldsymbol{\Omega} = \Omega(\mathbf{B}/B) = \frac{qB}{m}(\mathbf{B}/B)$$

- Replace particle position by guiding centre position $\mathbf{r} = \mathbf{R} + \boldsymbol{\rho}$
- Expand equation of motion in $\frac{1}{\Omega}\frac{d}{dt}$ and time-average over gyration period

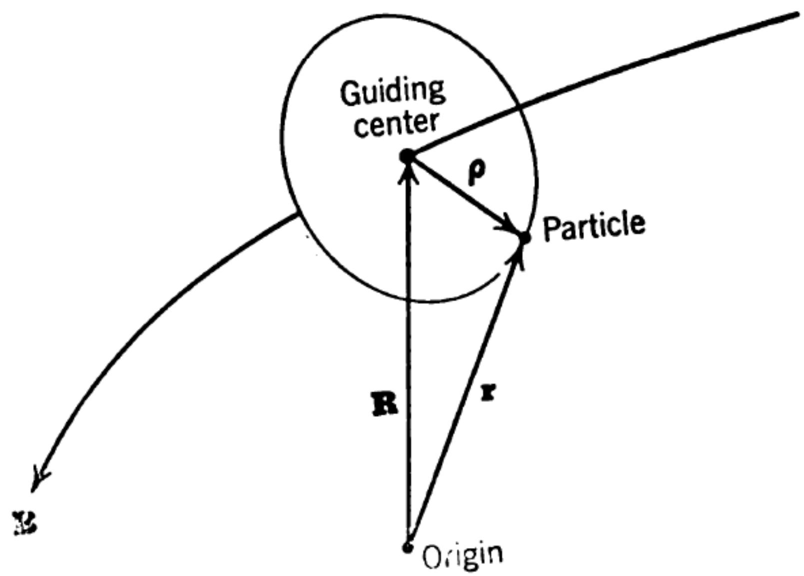

Figure 1.1. The charged particle and its guiding center.

Orbital motion ignored. Valid if:

- $\mathbf{B} \neq \mathbf{0}$ throughout domain
- Gyro-radii smaller than characteristic distance over which fields change:

$$v_\perp / \Omega L << 1$$

[Northrop, The Adiabatic Motion of Charged Particles, (1963)]

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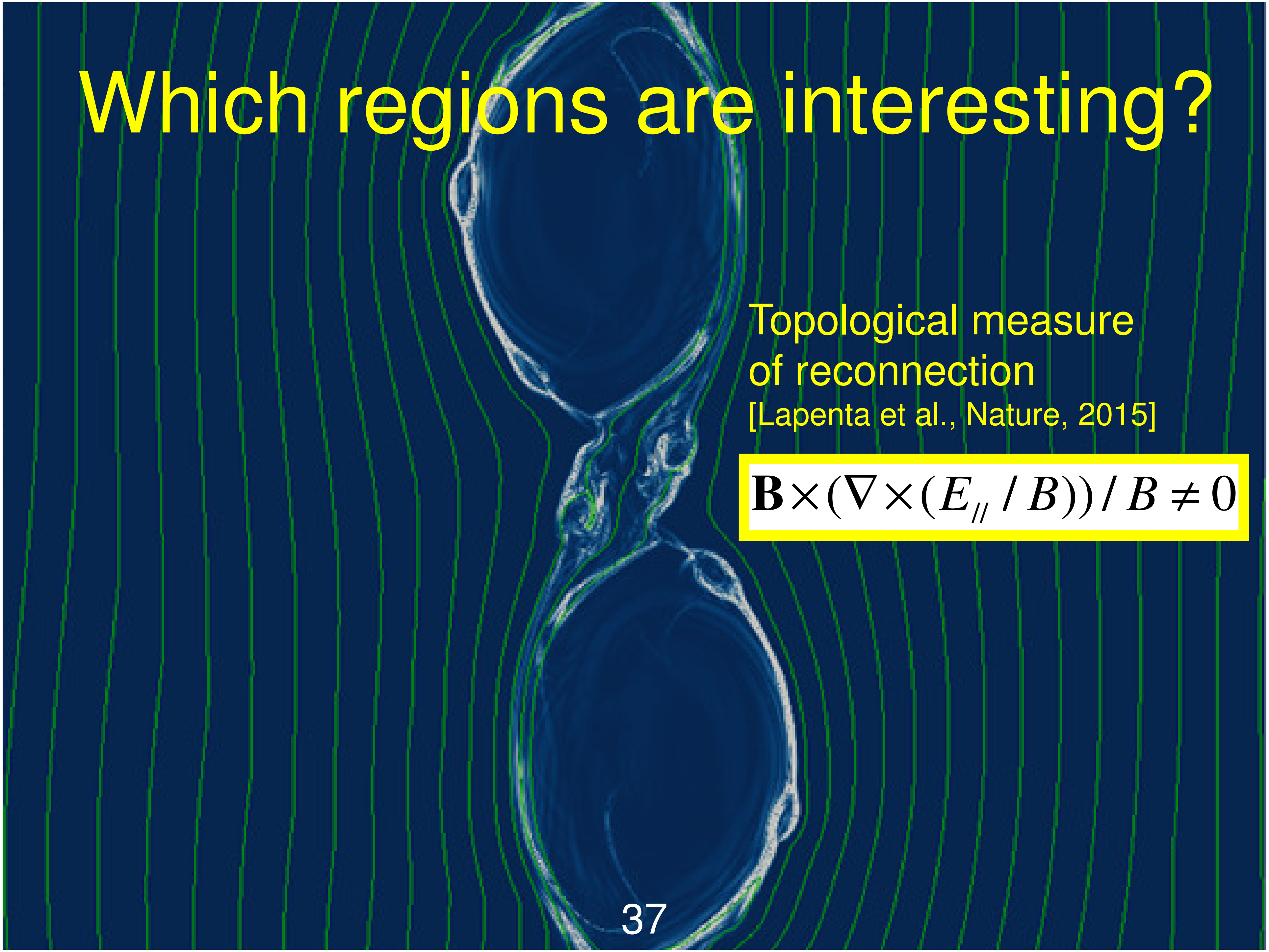
So for now..

- Fluid models: Good for global dynamics and energetics
  But.. fail to tell you anything about kinetic processes
- Kinetic models: The opposite...

→ Assume fluid models are largely correct and see how test particles behave in the global flow:

- Acceleration mechanisms
- Particle orbits and drifts
- Non-thermal distributions
- (Radiation)

Which regions are interesting?

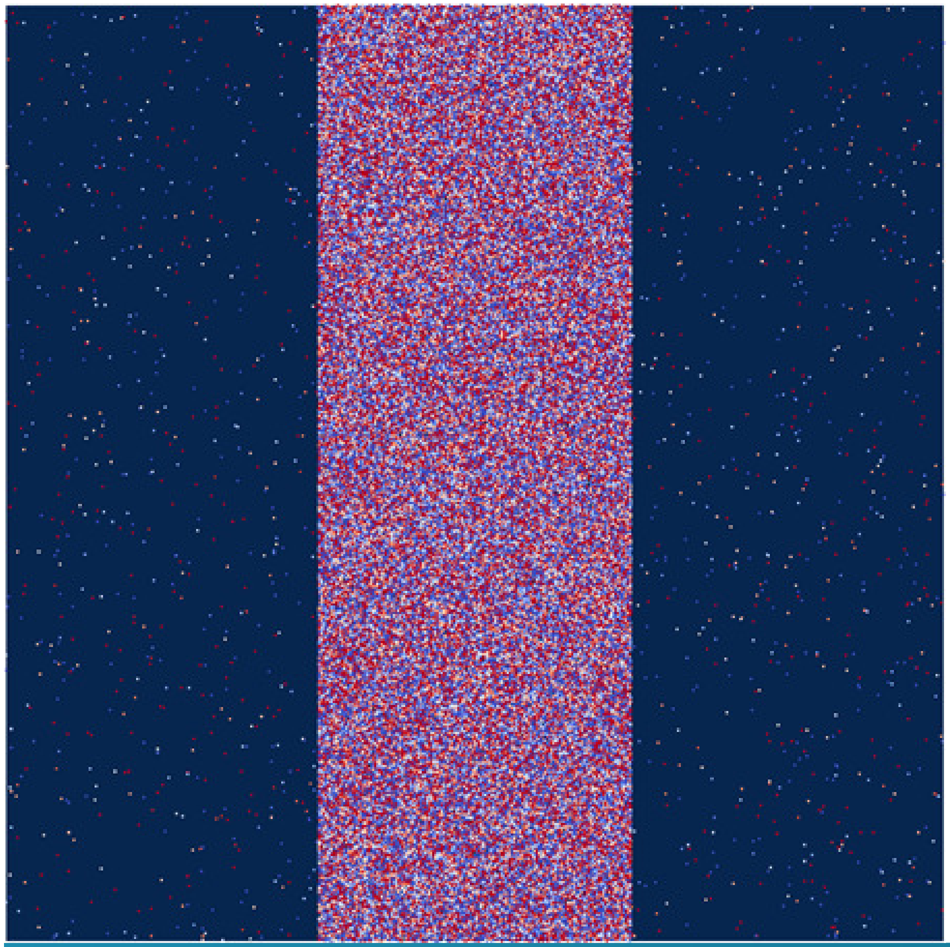


Topological measure
of reconnection
[Lapenta et al., Nature, 2015]

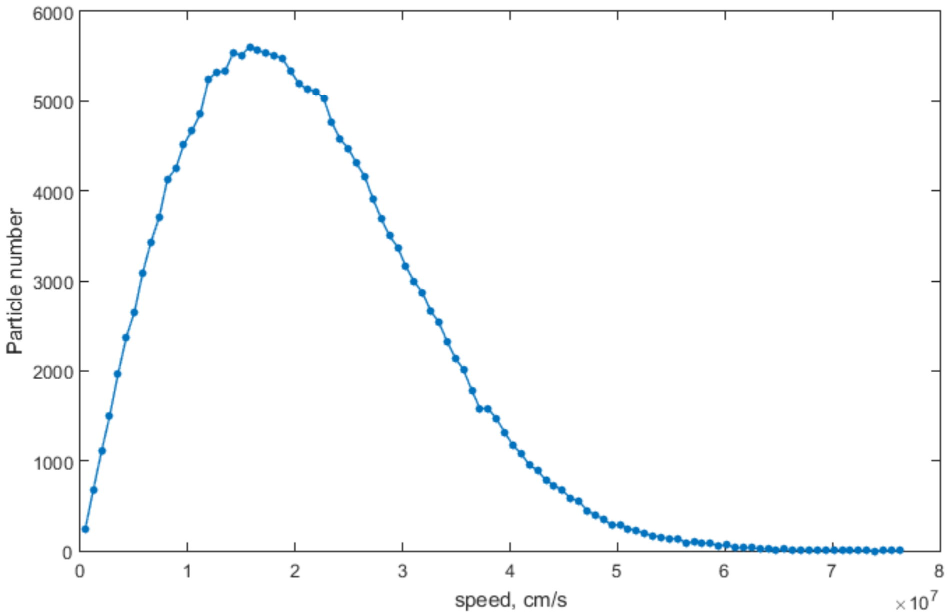
$$\mathbf{B}\times(\nabla\times(E_{//}/B))/B\neq 0$$

Test particles initialisation

Particles coloured by parallel velocity



- 200.000 Maxwellian electrons/protons
- Randomly and uniformly initialised
- 99% in area current channels



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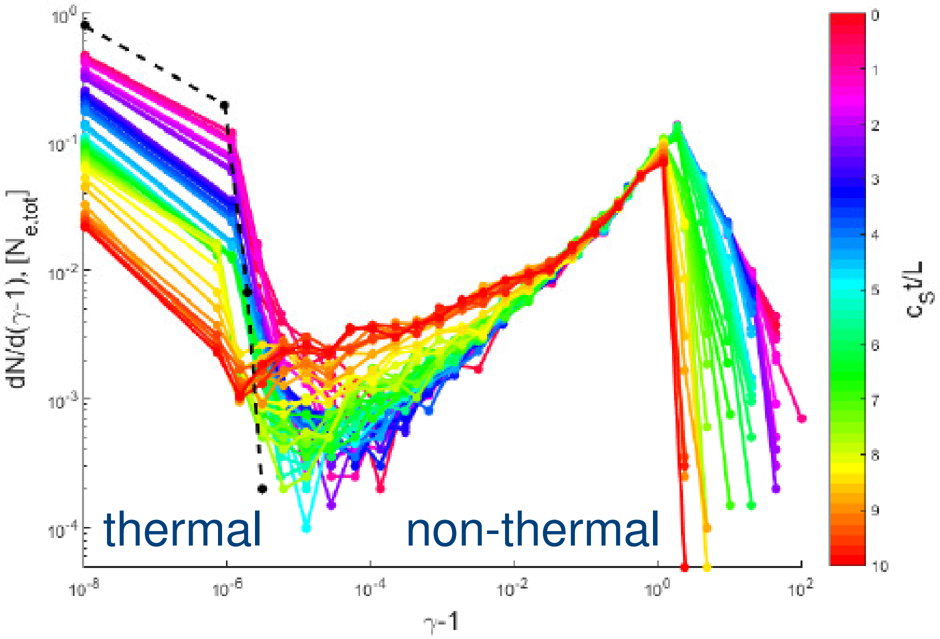
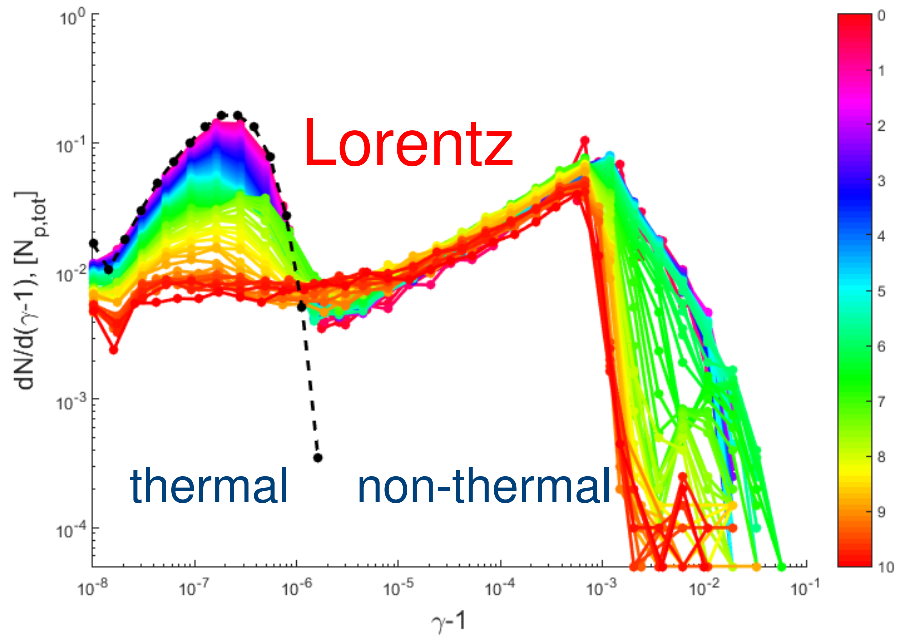
2.5D results: Energy distribution

20.000 Electrons



20.000 Protons



- Particle distribution develops high energy tail
- Thermal bath is applied in periodic direction
- Guiding centre approximation valid

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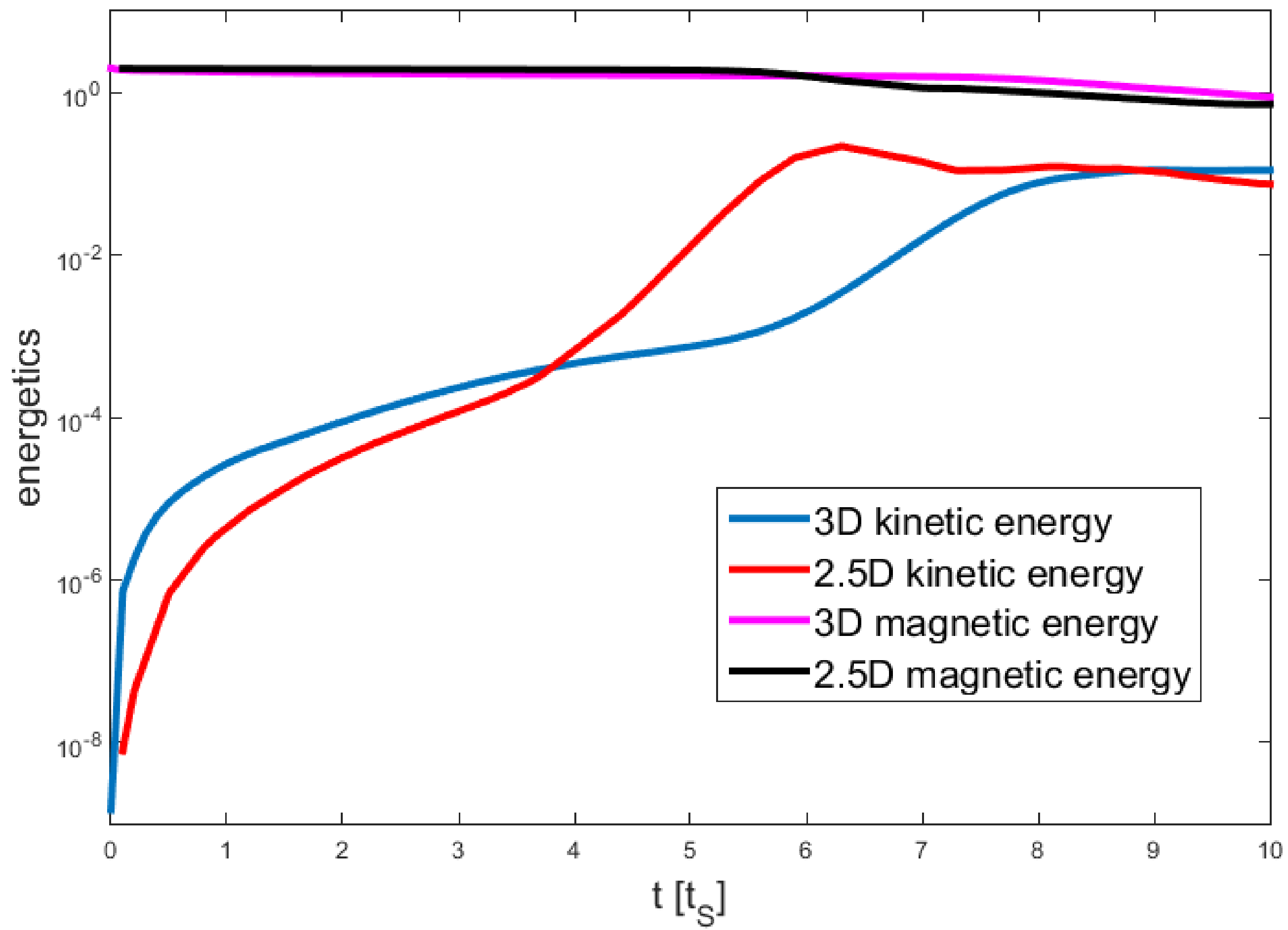
3D MHD effects

- Magnetic tension delays linear growth phase of instability



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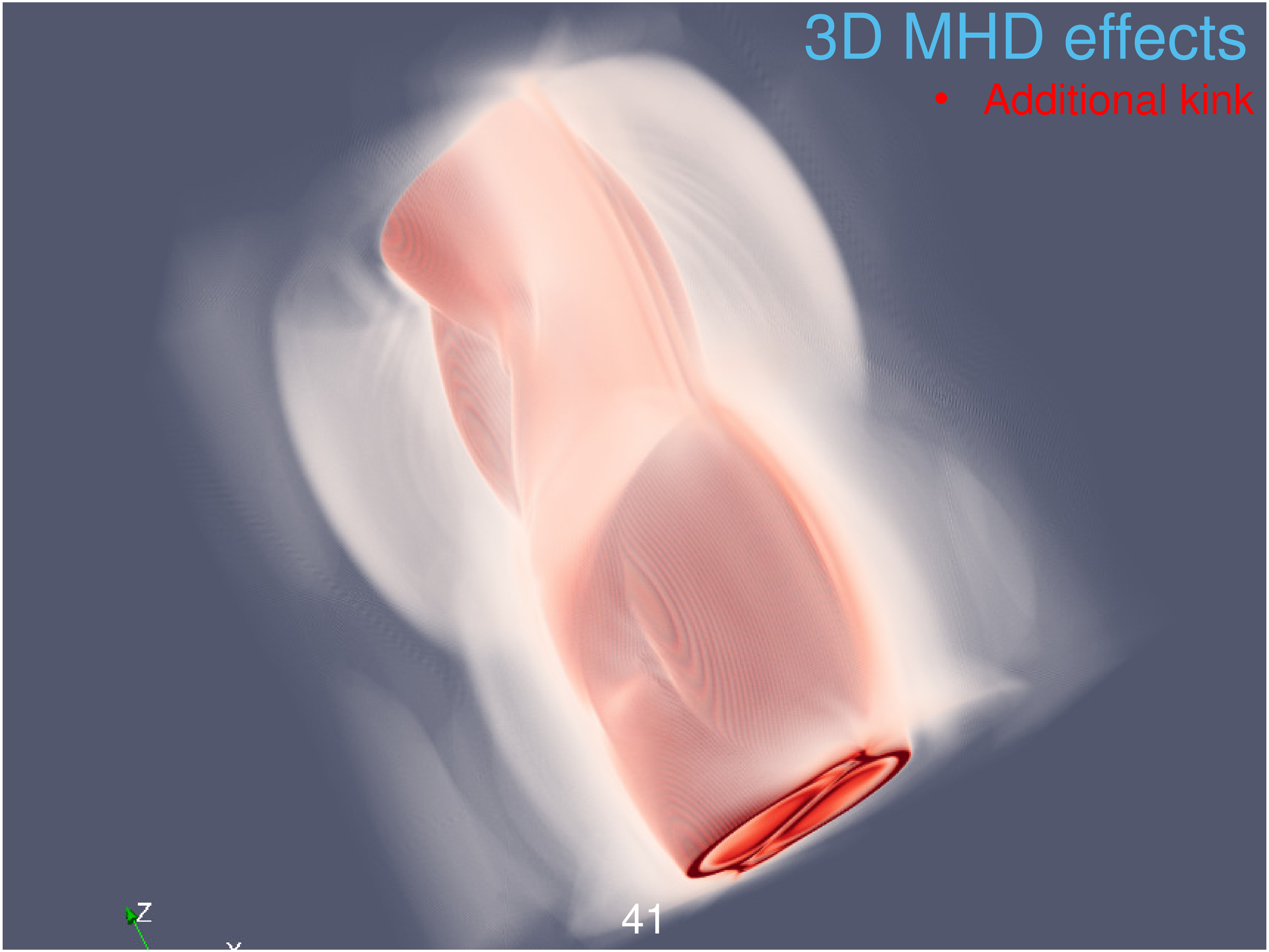
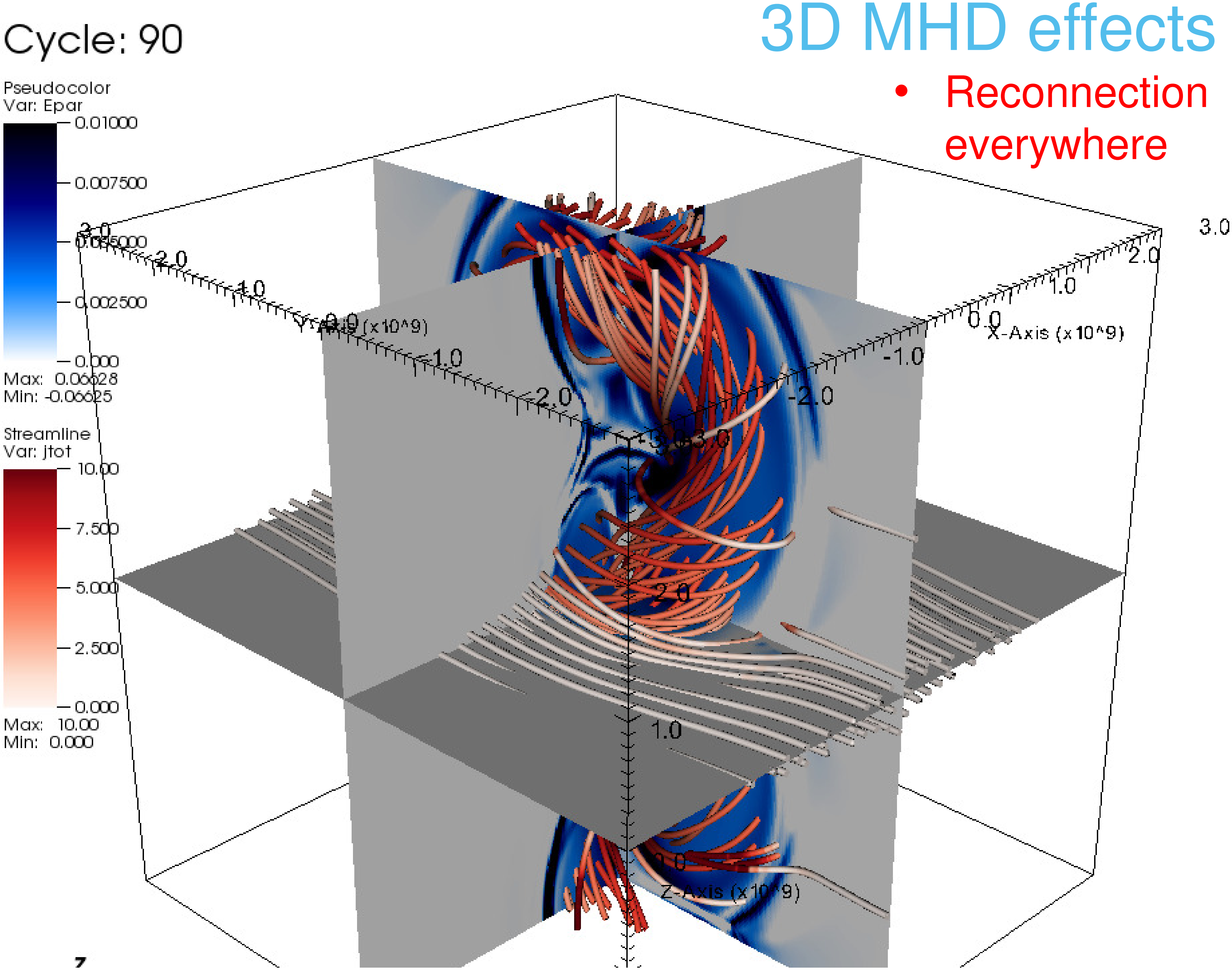
3D MHD effects

- Additional kink



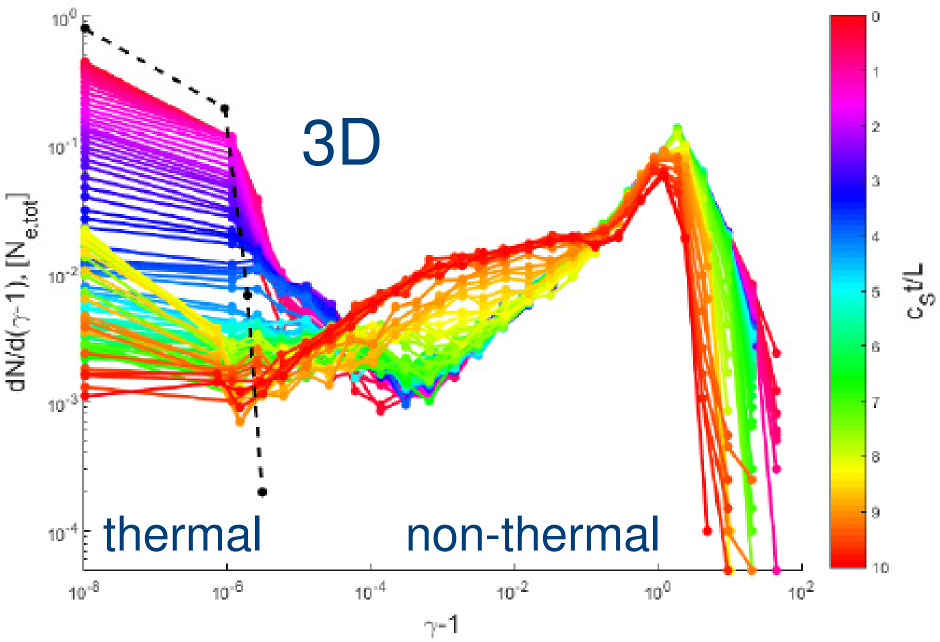
3D MHD effects

- Reconnection everywhere

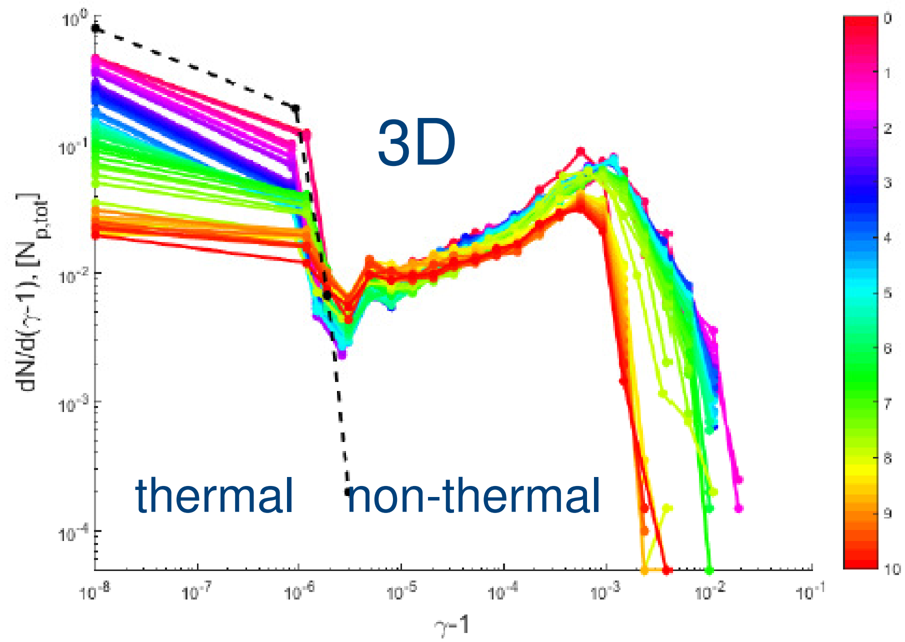


3D results: Energy distribution

20.000 Electrons



20.000 Protons



- Kink adds medium energy tail and redistributes particles in the thermal distribution
- Differences clearly visible for electrons
- Results confirmed for 200.000 electrons

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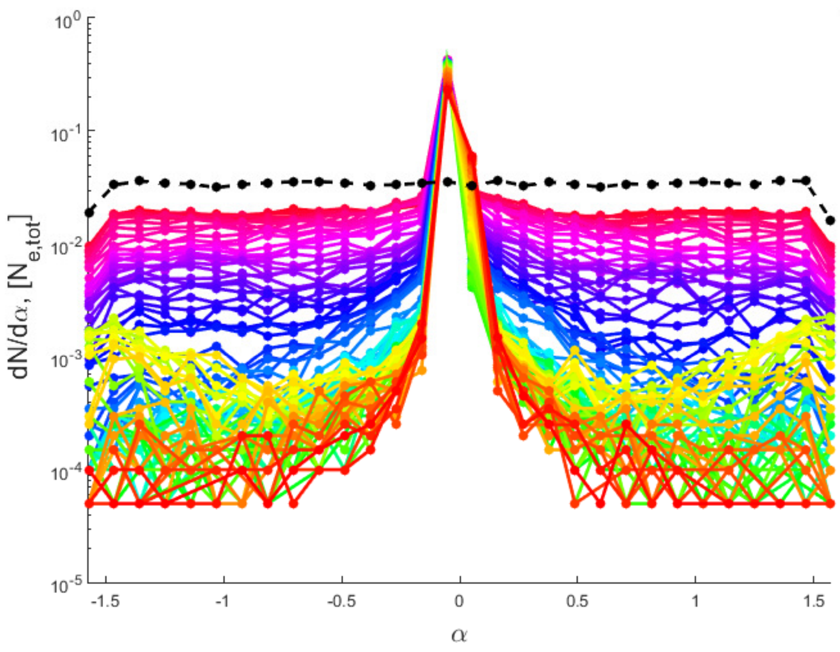
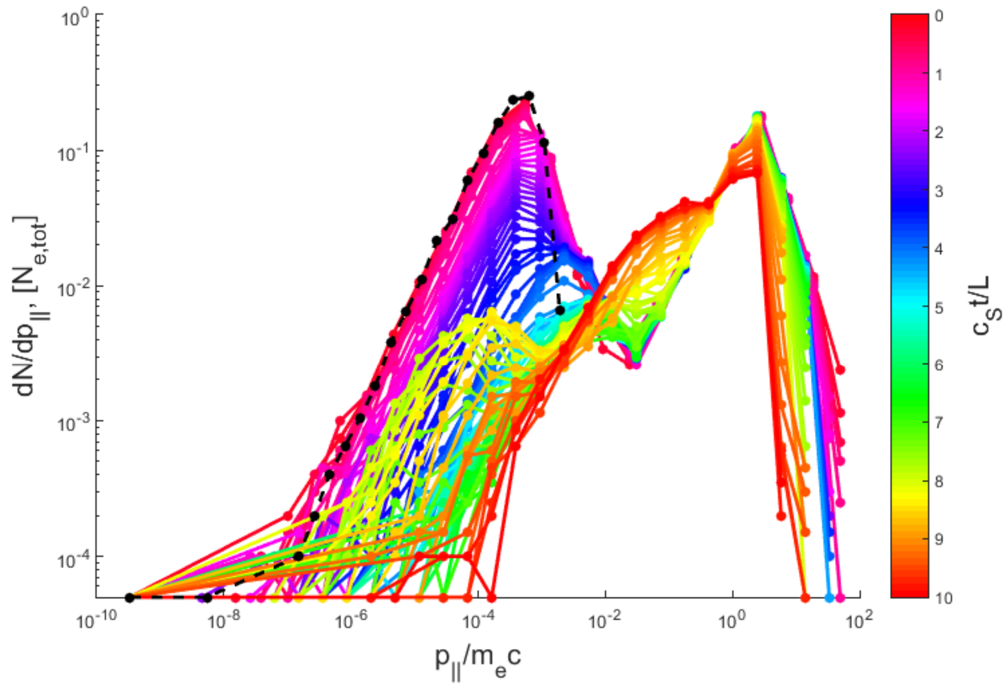
3D results: Some more electron spectra

20.000 Electrons

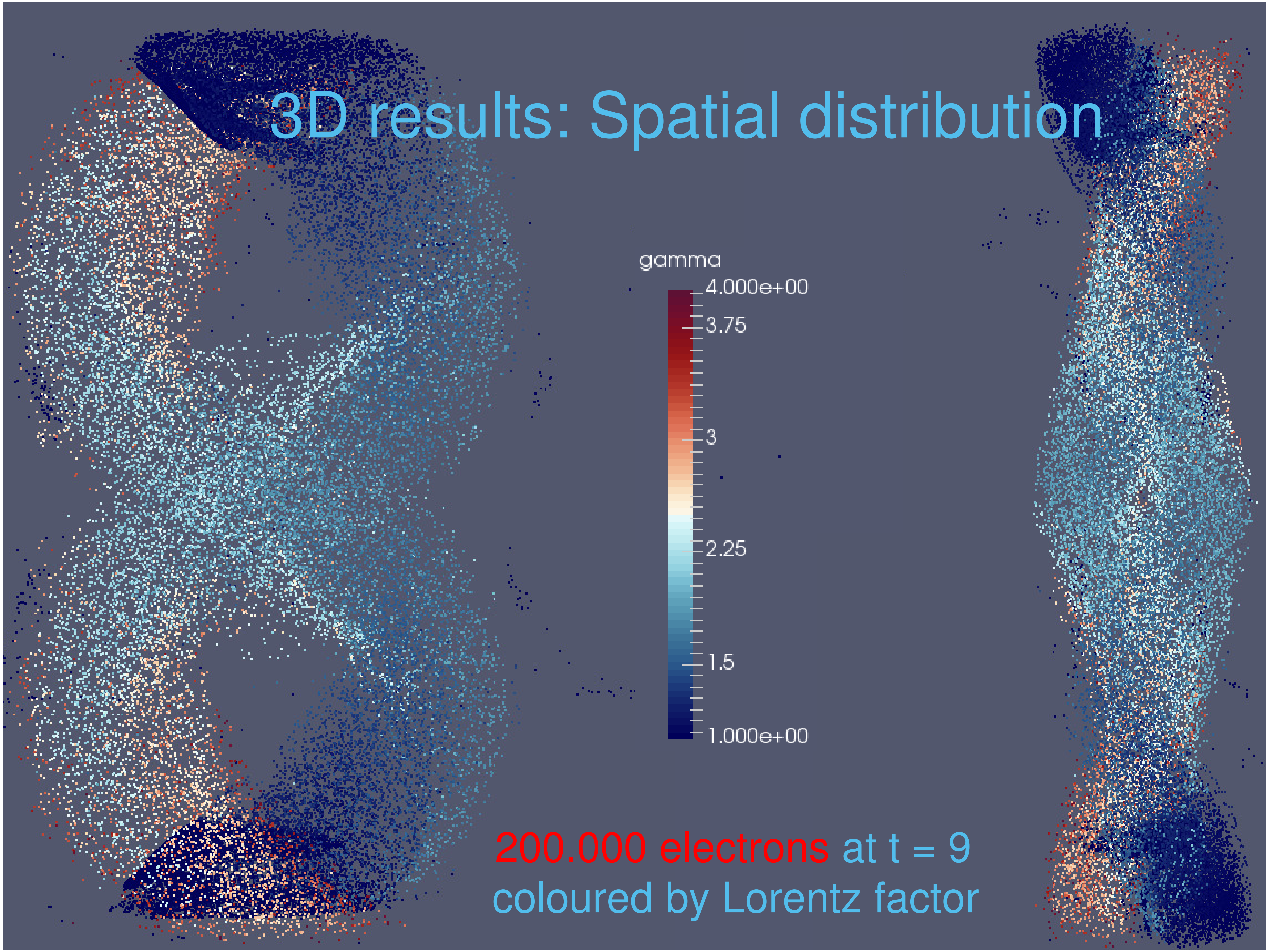




Pitch angle peaked around 0

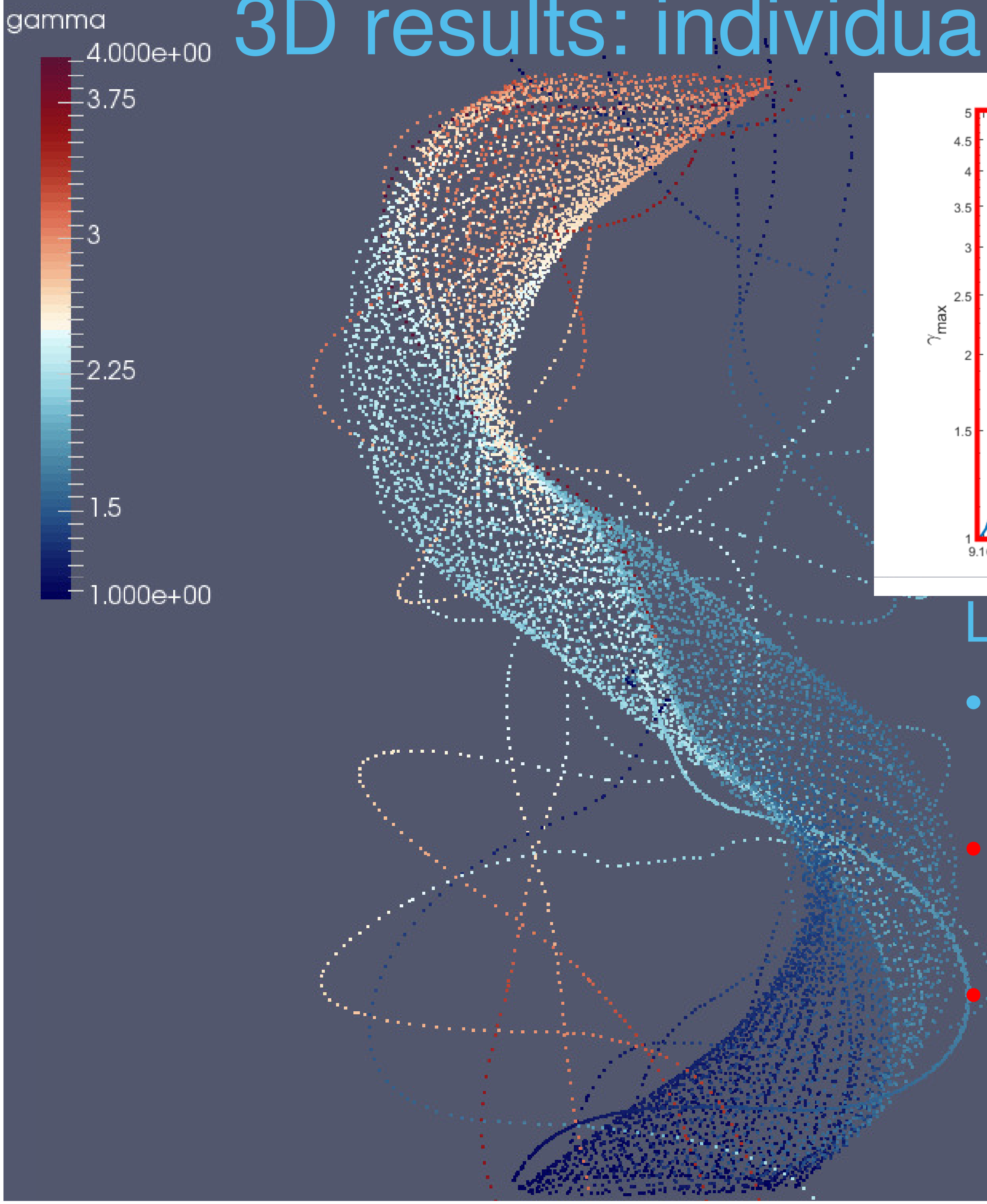
Parallel momentum develops high energy tail in channels and medium tail due to kink

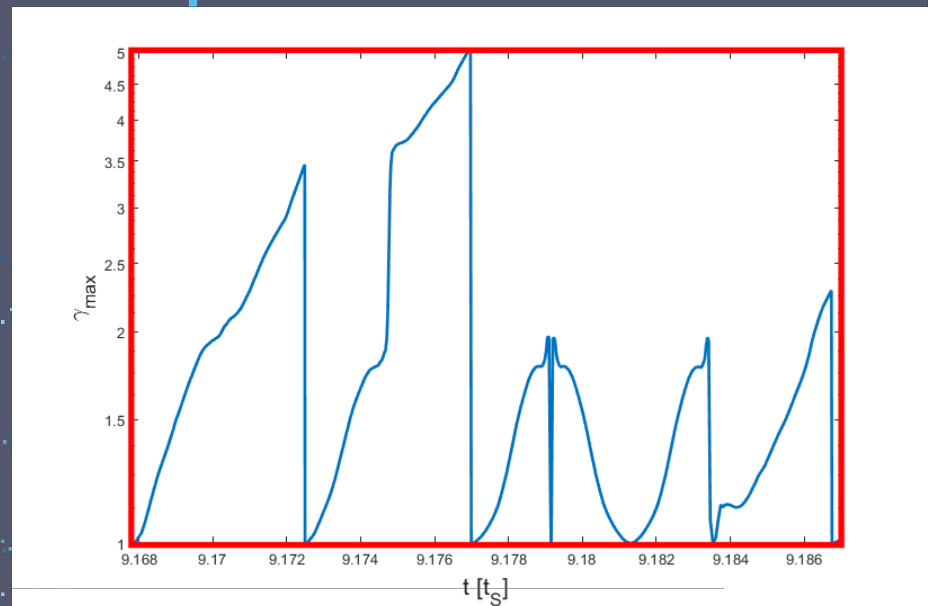
3D results: Spatial distribution



200.000 electrons at $t = 9$
coloured by Lorentz factor

3D results: individual particle orbit

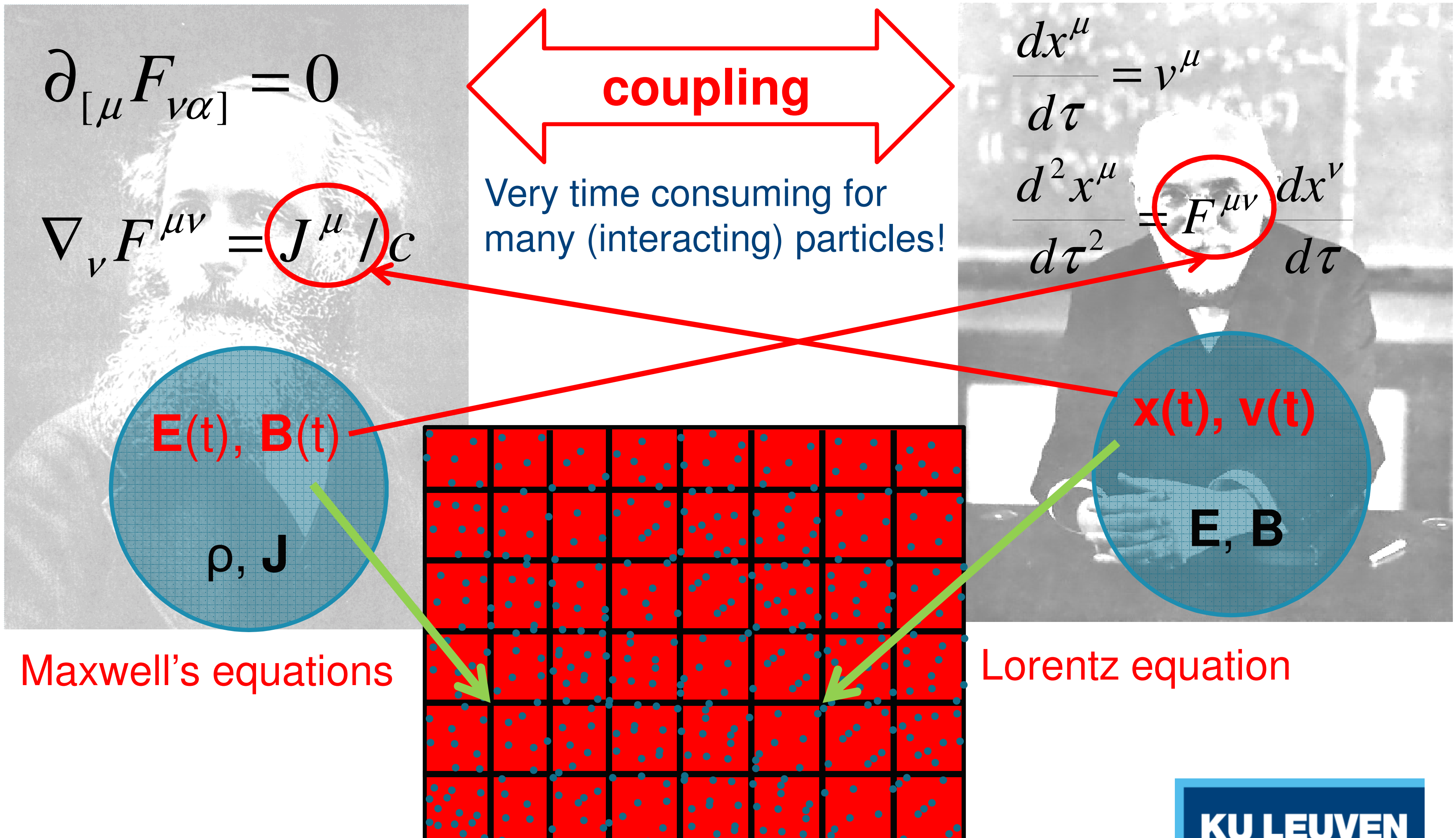




Lorentz factor evolution

- Thermalised after cycle through current channel
- Expelled from current channel by kink
- Decelerates to thermal energy and re-accelerates

... But charged particles do affect fields!



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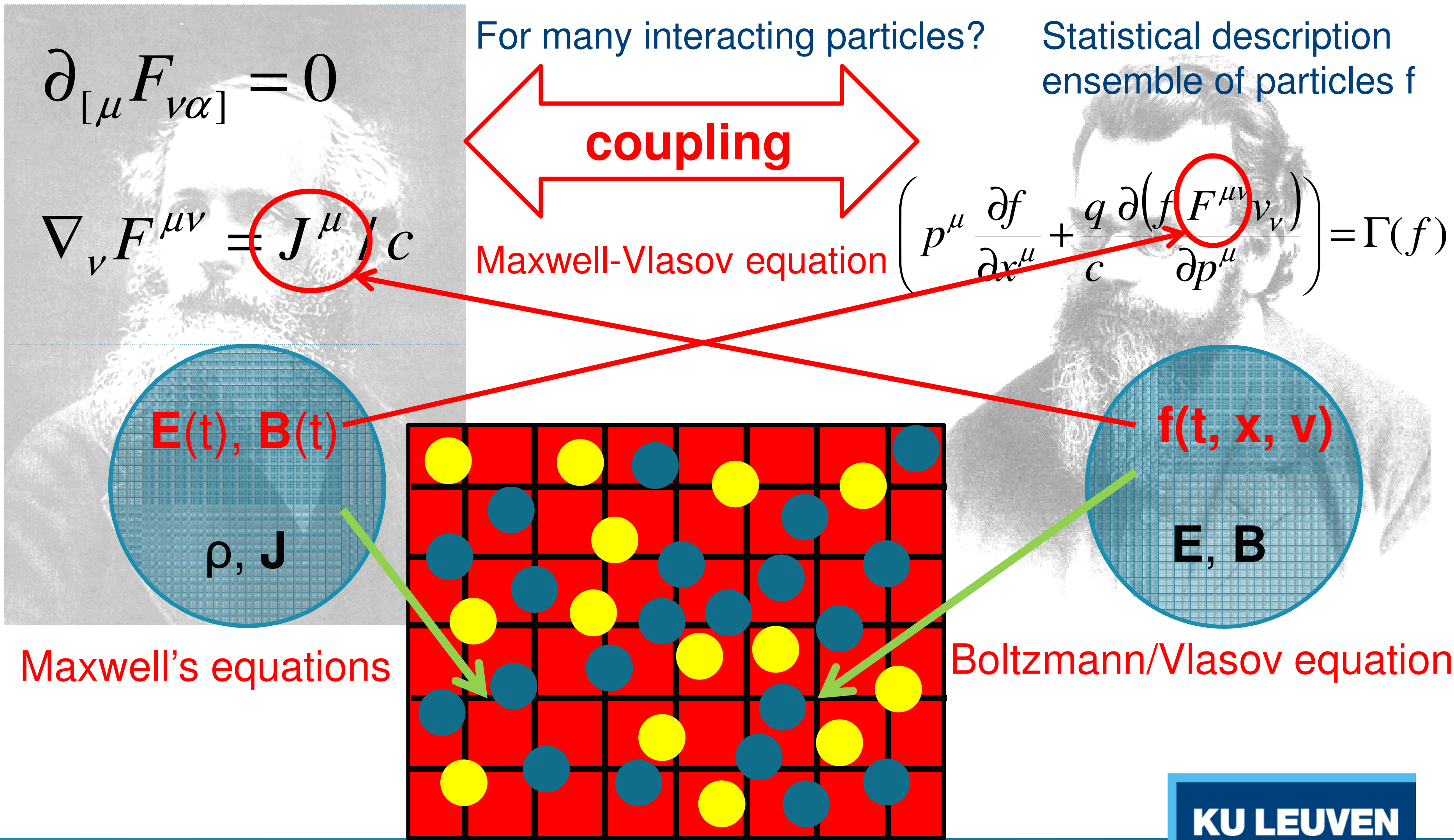
The kinetic picture



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Outlook

- Comparison with kinetic (particle-in-cell) methods to evaluate feedback of the particles on the fields
- Solve a typical test case with several methods, exploring different physics regimes → “Relativistic GEM challenge”
- Coupling two methods with a split based on either energy (treat energetic particles kinetically) or location (treat a certain region kinetically)
- Application to relativistic reconnection (e.g. pulsar winds, black hole flares and magnetar magnetospheres)

→ Resistive relativistic magnetohydrodynamics needed!

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Magnetospheres of compact objects

- Compact objects are described by relativistic MHD (RMHD) on large scales
- Inside the object ideal RMHD is an accurate description (SRMHD module in MPI-AMRVAC)
- In the magnetosphere resistive RMHD is needed
- Resistivity can change several orders across the flow
- Even with low resistivity, large second derivatives of the fields make non-ideal effects significant

→ We need to be able to solve MHD in both regimes for a range of resistivities

Resistive relativistic MHD

Ideal RMHD

$$\nabla_\mu \left(T_f^{\mu\nu} + T_{EM}^{\mu\nu}\right) = 0$$

$$\nabla_\mu \left(\rho_0 u^\mu\right) = 0$$

$$\nabla_\nu F^{\mu\nu} = J^\mu / c$$

$$\partial_{[\mu} F_{\nu\alpha]} = 0$$

3 + 1 split comoving frame

- Vanishing proper electric field $F^{\mu\nu} u_\nu \neq 0 \quad \forall \mu$
- Field is magnetic $F^{\mu\nu} F_{\mu\nu} \not\geq 0 \quad (B^2 - E^2 \geq 0)$
- No Ohmic heating $u_\mu F^{\mu\nu} J_\nu \neq 0$
- Add resistivity to Ohm's law $J^\mu = \left(J^\nu u_\nu\right) u^\mu + \frac{1}{\eta} F^{\mu\nu} u_\nu$

Solver needed for resistive relativistic MHD

Augmented resistive relativistic MHD

3+1 split (parallel and orthogonal to $g^{\mu\nu}$) gives Maxwell equations (and energy, momentum and current conservation)

$$\partial_t \Phi + \nabla \cdot \boldsymbol{B} = -\kappa \Phi,$$

$$\partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} + \nabla \Phi = 0,$$

$$\partial_t \Psi + \nabla \cdot \boldsymbol{E} = q - \kappa \Psi,$$

$$-\partial_t \boldsymbol{E} + \nabla \times \boldsymbol{B} - \nabla \Psi = \boldsymbol{J},$$

$$\partial_t q + \nabla \cdot \boldsymbol{J} = 0.$$

And Ohm's law

$$\boldsymbol{J} = \sigma \gamma [\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - (\boldsymbol{E} \cdot \boldsymbol{v})\boldsymbol{v}] + q \boldsymbol{v}$$

Generalised Lagrange Multiplier method → Augmented system for divergence cleaning

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Problems with resistive RMHD

- Non-ideal processes take place on short time-scale
- RMHD becomes hyperbolic + stiff relaxation terms (newtonian MHD becomes mixed hyperbolic/parabolic)
- Stiff terms dominate and restrict time-step severely

→ Implicit-Explicit Runge-Kutta (ImEx) for resistive RMHD

→ Solve fast dynamics implicitly and slow dynamics explicitly

Implicit-Explicit Runge-Kutta method

- Prototypical stiff system $\partial_t \mathbf{U} = F(\mathbf{U}) + \frac{1}{\varepsilon} R(\mathbf{U})$
- with ε the relaxation time
- For $\varepsilon \to \infty$ hyperbolic (*i.e.* ideal MHD)
- For $\varepsilon \to 0$ stiff system $R(\mathbf{U})$ (and $F(\mathbf{U})$ negligible)
- Treat stiff terms $R(\mathbf{U})$ implicitly and non-stiff $F(\mathbf{U})$ explicitly

$$\boldsymbol{U}^{(i)} = \boldsymbol{U}^n + \Delta t \sum_{j=1}^{i-1} \tilde{a}_{ij} F[\boldsymbol{U}^{(j)}] + \Delta t \sum_{j=1}^{\nu} a_{ij} \frac{1}{\epsilon} R[\boldsymbol{U}^{(j)}] ,$$

$$\boldsymbol{U}^{n+1} = \boldsymbol{U}^n + \Delta t \sum_{i=1}^{\nu} \tilde{\omega}_i F[\boldsymbol{U}^{(i)}] + \Delta t \sum_{i=1}^{\nu} \omega_i \frac{1}{\epsilon} R[\boldsymbol{U}^{(i)}] ,$$

- With coefficients from Butcher tableau $\begin{array}{c|c} c & A \\ \hline & \omega^T \end{array}$ $\quad A = (a_{ij})$

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Simple example

2[nd] order Butcher tableau (left explicit, right implicit)

0	0	0
1	1	0
	1/2	1/2

α	α	0
$1-\alpha$	$1-2\alpha$	α
	1/2	1/2

$$\alpha \equiv 1 - \frac{1}{\sqrt{2}}$$

Gives intermediate and final steps

$$\boldsymbol{U}^{(1)} = \boldsymbol{U}^n + \frac{\Delta t}{\epsilon}\alpha R[\boldsymbol{U}^{(1)}]$$

$$\boldsymbol{U}^{(2)} = \boldsymbol{U}^n + \Delta t\, F[\boldsymbol{U}^{(1)}] + \frac{\Delta t}{\epsilon}\{(1-2\alpha)R[\boldsymbol{U}^{(1)}] + \alpha\, R[\boldsymbol{U}^{(2)}]\}$$

$$\boldsymbol{U}^{n+1} = \boldsymbol{U}^n + \frac{\Delta t}{2}[F(\boldsymbol{U}^{(1)}) + F(\boldsymbol{U}^{(2)})] + \frac{\Delta t}{2\epsilon}\{R[\boldsymbol{U}^{(1)}] + R[\boldsymbol{U}^{(2)}]\}.$$

higher order schemes readily available

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Implementation for SRRMHD I

- Conserved variables are split into set $\mathbf{U} = \{\mathbf{X}, \mathbf{Y}\}$, with
- stiff $\boldsymbol{X} = \{\boldsymbol{E}\}$ and non-stiff $\boldsymbol{Y} = \{\boldsymbol{B}, \psi, \phi, q, \tau, \boldsymbol{S}, D\}$
- Rewrite the system as

$$\begin{cases} \partial_t \mathbf{Y} = F_{\mathbf{Y}}(\mathbf{X}, \mathbf{Y}) \\ \partial_t \mathbf{X} = F_{\mathbf{X}}(\mathbf{X}, \mathbf{Y}) + \dfrac{1}{\varepsilon} R_{\mathbf{X}}(\mathbf{X}, \mathbf{Y}) \end{cases}$$

- $F_{\mathbf{Y}}$ contains first-order spatial derivatives of $\mathbf{Y}$ and the non-stiff source terms, $F_{\mathbf{X}}$ similarly for $\mathbf{X}$
- $R_{\mathbf{X}}(\mathbf{X}, \mathbf{Y}) = A(\mathbf{Y})\mathbf{X} = S_{\mathbf{X}}(\mathbf{Y})$ contains the stiff source terms

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Implementation for SRRMHD II

- Compute the explicit intermediate values $\{X^*, Y^*\}$

$$Y^* = Y^n + \Delta t \sum_{j=1}^{i-1} \tilde{a}_{ij} F_Y[U^{(j)}]$$

$$X^* = X^n + \Delta t \sum_{j=1}^{i-1} \tilde{a}_{ij} F_X[U^{(j)}] + \Delta t \sum_{j=1}^{i-1} \frac{a_{ij}}{\epsilon^{(j)}} R_X[U^{(j)}]$$

- And the implicit part

$$Y^{(i)} = Y^*$$

$$X^{(i)} = X^* + \Delta t \frac{a_{ii}}{\epsilon^{(i)}} R_X[U^{(i)}]$$

- With $R_X(X, Y) = A(Y)X + S_X(Y)$ we then get

$$X^{(i)} = M(Y^*) \left[X^* + a_{ii} \frac{\Delta t}{\epsilon^{(i)}} S_X(Y^*) \right]$$ with $$M(Y^*) = \left[I - a_{ii} \frac{\Delta t}{\epsilon^{(i)}} A(Y^*) \right]^{-1}$$

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Implementation for SRRMHD III

- These are all known for the SRRMHD equations

$$\boldsymbol{R}_E = -\gamma\,\boldsymbol{E} + \gamma\,(\boldsymbol{E}\cdot\boldsymbol{v})\boldsymbol{v} - \gamma\,\boldsymbol{v}\times\boldsymbol{B}$$

$$\boldsymbol{S}_E = -\gamma\,\boldsymbol{v}\times\boldsymbol{B}\,,$$

$$A \equiv \gamma \begin{pmatrix} -1+v_x^2 & v_x v_y & v_x v_z \\ v_x v_y & -1+v_y^2 & v_y v_z \\ v_z v_x & v_z v_y & -1+v_z^2 \end{pmatrix}$$

- Giving the matrix M acting on the intermediate state of the electric field found through the non-stiff part
- Obtaining the final $\boldsymbol{E}$ through the evolution of the stiff part
- Primitive variables need to be reconstructed

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Primitive variables

- Conserved variables $\{D, \tau, \boldsymbol{S}, \boldsymbol{B}\}$ are known at n+1
- However, only intermediate $\{\boldsymbol{E}^*\}$ is known
- Solution found by $\boldsymbol{E} = M(\boldsymbol{v})\,[\boldsymbol{E}^* + a_{ii}\Delta t \sigma^{(i)} \boldsymbol{S}_E(\boldsymbol{v}, \boldsymbol{B})]$

1) Primitive variable $\boldsymbol{v} = \boldsymbol{v}^n$ at previous step n to find $\boldsymbol{E}$
2) Primitive variable $p = p^n$ taken at step n, to compute

$$\boldsymbol{v} = \frac{\boldsymbol{S} - \boldsymbol{E} \times \boldsymbol{B}}{\tau - (E^2 + B^2)/2 + p},$$
$$\gamma = \frac{1}{\sqrt{1 - v^2}},$$
$$\rho = \frac{D}{\gamma},$$
$$\epsilon = \frac{\tau - (E^2 + B^2)/2 - D\gamma + p(1 - \gamma^2)}{D\gamma}.$$

3) Solve $p_{m+1} = p_m - \frac{f(p_m)}{f'(p_m)}$ with $\begin{cases} f(\bar{p}) = p(\rho, \epsilon) - \bar{p} \\ f'(p) = v^2 c_s^2 - 1 \end{cases}$
4) Iterate

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Force-free magnetodynamics

- In magnetospheres magnetic stresses are much larger than pressure gradients
- The magnetic field adjusts itself such that the tension vanishes → Force-free
- Force-free magnetodynamics resembles low-inertia limit of ideal RMHD (Komissarov 2002)

→ Useful comparative test for resistive RMHD and already implemented in FFMD module of MPI-AMRVAC (several tests available in 1D and 2D, e.g. coalescence instability, x-point collapse, reconecting flux tubes, slow and fast stationary shock, cylindrical explosion, self-similarly decaying current sheet)

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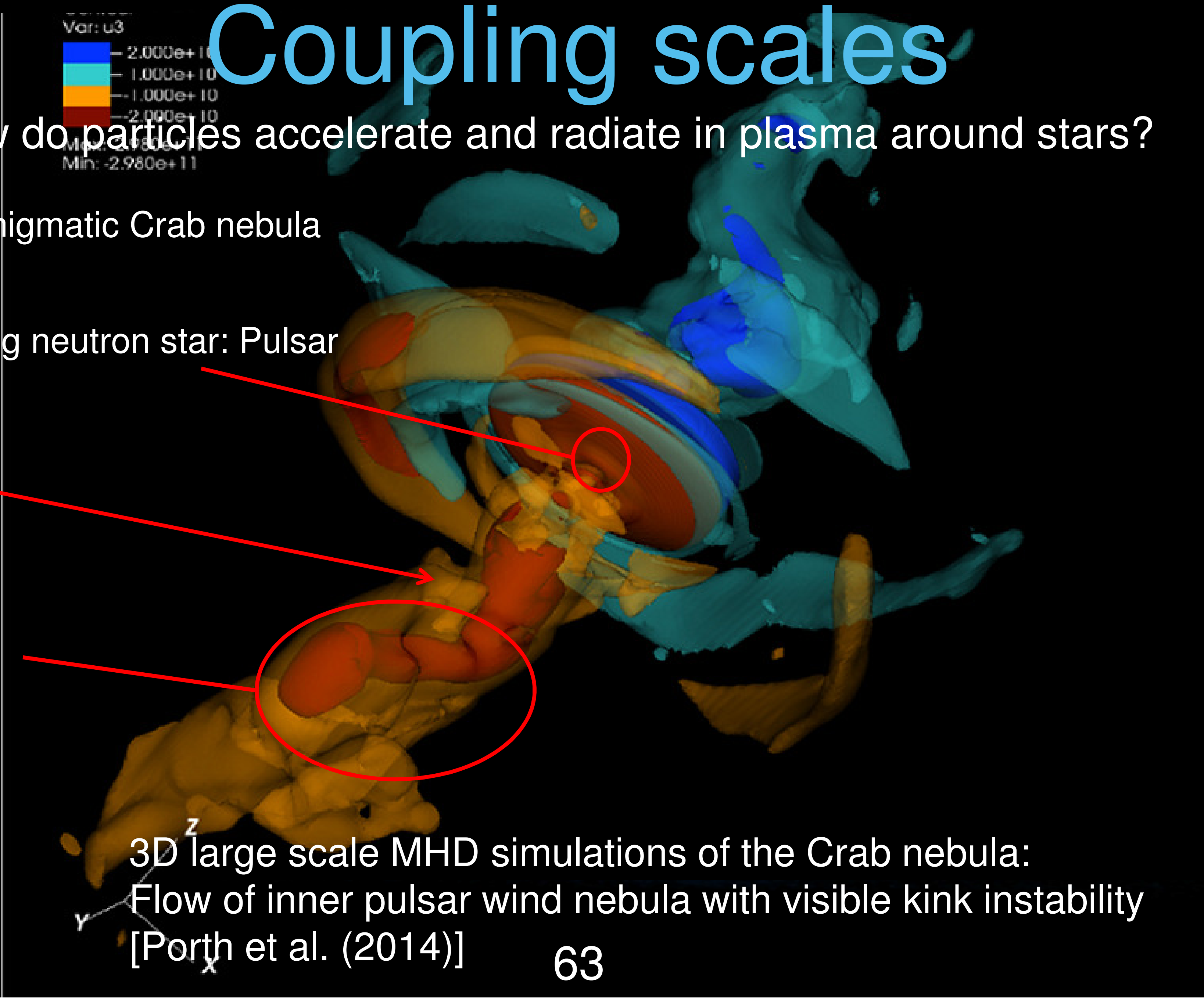
Other options

- Strang splitting (Komissarov, 2007)
  - Unstable for low resistivity and sharp discontinuities
  - Used for tearing instability (Baty, 2013), electron-positron MHD (Barkov, 2014) and in MPI-AMRVAC (SRRMHD module with aforementioned tests)
- Strang splitting, with characteristic speeds < c (Takamoto, 2011)
  - Used for MRI in accretion disks (Takamoto, 2011) and tearing instability (Petri, 2015)
  - Valid for characteristic speeds smaller than speed of light
- ImEx for GRMHD (Dionysopoulou, 2013)
  - Used for neutron star mergers (Dionysopoulou, 2014)

Coupling scales

How do particles accelerate and radiate in plasma around stars?

The enigmatic Crab nebula



3D large scale MHD simulations of the Crab nebula:
Flow of inner pulsar wind nebula with visible kink instability
[Porth et al. (2014)]

Ideal MHD Equilibrium (non-force-free)

- Initial magnetic field:

[Keppens et al., Interacting Tilt and Kink Instabilities in Repelling Current Channels (2014)]

$$\psi_0(x,y) = \begin{cases} \dfrac{2}{j_0^1 J_0(j_0^1)} J_1(j_0^1 r)\cos(\theta) & for\ r < 1, \\ (r - \dfrac{1}{r})\cos(\theta) & for\ r \geq 1, \end{cases} \Rightarrow \mathbf{B} = \hat{\mathbf{z}} \times \nabla \psi_0 = \begin{cases} B_x = +\partial \psi_0 / \partial y \\ B_y = -\partial \psi_0 / \partial x \\ B_z = B_{z0} \end{cases}$$

- Two antiparallel current channels in unit circle $\mathbf{J} = \nabla \times \mathbf{B} = -\nabla^2 \psi_0 \hat{\mathbf{z}}$
- MHD equilibrium satisfied by

$$\nabla p = \mathbf{J} \times \mathbf{B} = -\nabla^2 \psi_0 \nabla \psi_0 = \left(j_0^1\right)^2 \psi_0 \nabla \psi_0 = \frac{\left(j_0^1\right)^2}{2} \nabla\left(\psi_0{}^2\right)$$

$$\Rightarrow \quad p(x,y) = \begin{cases} p_0 + \dfrac{(j_0^1)^2}{2}(\psi_0(x,y))^2 & \text{for } r < 1. \\ p_0 & \text{for } r \geq 1. \end{cases}$$

[Richard et al., Magnetic Reconnection Driven by Current Repulsion (1990)]

Ideal MHD Equilibrium (force-free)

- Initial magnetic field:

[Keppens et al., Interacting Tilt and Kink Instabilities in Repelling Current Channels (2014)]

$$\psi_0(x,y) = \begin{cases} \dfrac{2}{j_0^1 J_0(j_0^1)} J_1(j_0^1 r)\cos(\theta) & for\ r<1, \\ (r-\dfrac{1}{r})\cos(\theta) & for\ r \geq 1, \end{cases} \Rightarrow \quad \mathbf{B} = \hat{\mathbf{z}} \times \nabla \psi_0 = \begin{cases} B_x = +\partial \psi_0 / \partial y \\ B_y = -\partial \psi_0 / \partial x \end{cases}$$

- Two antiparallel current channels in unit circle $\mathbf{J} = \nabla \times \mathbf{B} = -\nabla^2 \psi_0 \hat{\mathbf{z}}$
- MHD equilibrium satisfied by

$$\nabla p = \mathbf{J} \times \mathbf{B} = 0 \quad \rightarrow \quad p = p_0$$

$$\Rightarrow \quad B_z(x,y) = \begin{cases} (j_0^1)(\psi_0(x,y)) & \text{for } r < 1. \\ 0 & \text{for } r \geq 1. \end{cases}$$

[Richard et al., Magnetic Reconnection Driven by Current Repulsion (1990)]

Perturbed equilibrium

- Perturb equilibrium by velocity field in (x,y)-plane

$$\begin{cases} v_x = \dfrac{\partial \phi_0}{\partial y}\sin(k_z z) \\ v_y = -\dfrac{\partial \phi_0}{\partial x}\sin(k_z z) \\ v_z = 0 \end{cases}$$

$$\phi_0(x,y) = \varepsilon \exp\left(-x^2 - y^2\right), \quad \varepsilon = 0.0001$$

$$k_z = 2\pi / L_z$$

- Unstable to ideal MHD instability with Alfvénic growth rates (with variational principle) → Tilt instability [Richard et al. (1990)]
- Instability facilitates nonlinear (reconnection) phase
- Resistivity has little effect on linear phase, allows reconnection
- What is the effect on reconnection and particle acceleration for low plasma beta?

Guiding centre equation of motion

$$m\ddot{\mathbf{R}} = m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v}\wedge\mathbf{B}) - \mu\nabla B + O(\varepsilon)$$

$$\mathbf{u}_{\mathbf{E}} = \frac{c\mathbf{E}\times\mathbf{B}/B}{B}$$

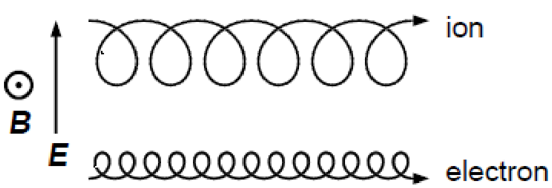
- Guiding centre motion (parallel + drifts):

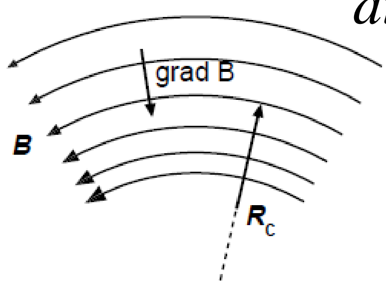
$$\frac{d\mathbf{R}}{dt} = v_{//}\frac{\mathbf{B}}{B} + \frac{\mathbf{B}/B}{B}\times\left\{-c\mathbf{E} + \frac{\mu c}{e}\nabla B + \frac{mc}{e}\left(v_{//}\frac{d\frac{\mathbf{B}}{B}}{dt} + \frac{d\mathbf{u}_{\mathbf{E}}}{dt}\right)\right\} + O\left(\varepsilon^2\right)$$

$\mathbf{E}\times\mathbf{B}$ drift

$\mathbf{B}\times\nabla B$ drift

$$\mathbf{B}\times\frac{d\mathbf{v}}{dt}\begin{cases}\text{Inertial drift (including curvature drift)}\\ \text{Polarisation drift (only for } E_\perp = O(1)\text{)}\\ \text{Non-static fields drift (neglected)}\end{cases}$$





[sketch from De Blank, Guiding Center Motion]

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Relativistic equation of motion

- Relativistic effects modify classical drifts $m \to m_0\gamma,\ \gamma = \gamma(t),\ \mu \to \mu_r$
- Assume non-relativistic flow (Alfvén velocities << c)
- Temporal variations field << variations due to particle motion
- Purely relativistic correction → Drift terms in $\mathbf{E}_\perp$ direction, of order v^2/c^2

[Vandervoort, The Relativistic Motion of a Charged Particle in an Inhomogeneous Electromagnetic Field, (1960)]

$$\frac{d\mathbf{R}}{dt} = \frac{(\gamma v_{//})}{\gamma}\frac{\mathbf{B}}{B} + \frac{\frac{\mathbf{B}}{B}}{B\left(1-\frac{E_\perp^2}{B^2}\right)} \times$$

$$\left\{ -\left(1-\frac{E_\perp^2}{B^2}\right)c\mathbf{E} + \frac{\mu_r c}{\gamma e}\nabla\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right] + \frac{cm_0\gamma}{e}\left(v_{//}\frac{d\frac{\mathbf{B}}{B}}{dt} + \frac{d\mathbf{u}_\mathbf{E}}{dt}\right) + \frac{v_{//}E_{//}}{c}\mathbf{u}_\mathbf{E} + \frac{\mu_r}{\gamma e}\frac{\mathbf{u}_\mathbf{E}}{c}\frac{\partial}{\partial t}\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right]\right\} + O(\varepsilon^2)$$

$\mathbf{E}\times\mathbf{B}$ drift $\mathbf{B}\times\nabla B$ drift curvature drift

$$\left(v_{//}\frac{\partial \frac{\mathbf{B}}{B}}{\partial t} + v_{//}^2\left(\frac{\mathbf{B}}{B}\bullet\nabla\right)\frac{\mathbf{B}}{B} + v_{//}(\mathbf{u}_\mathbf{E}\bullet\nabla)\frac{\mathbf{B}}{B} + \frac{\partial \mathbf{u}_\mathbf{E}}{\partial t} + v_{//}\left(\frac{\mathbf{B}}{B}\bullet\nabla\right)\mathbf{u}_\mathbf{E} + (\mathbf{u}_\mathbf{E}\bullet\nabla)\mathbf{u}_\mathbf{E} \right)$$

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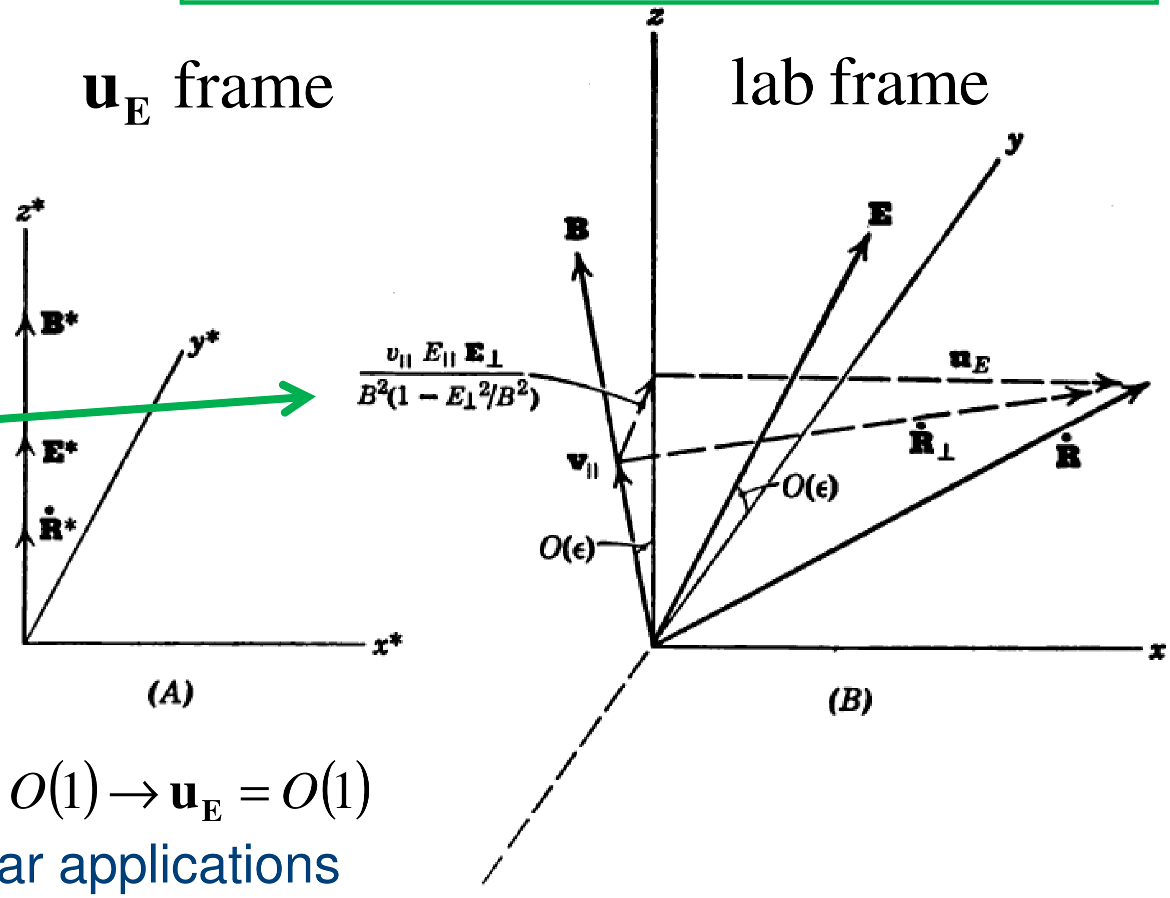
Relativistic drift

- Purely relativistic correction → Drift terms in $\mathbf{E}_\perp$ direction, of order v^2/c^2

$$\frac{\frac{\mathbf{B}}{B}}{B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}} \times \frac{v_{//}E_{//}}{c}\mathbf{u}_\mathrm{E}$$



- Only important if $\mathbf{E}_\perp = O(1) \rightarrow \mathbf{u}_\mathrm{E} = O(1)$
- Often neglected for solar applications

Figure 1.8. The explanation of the drift proportional to $v_\| E_\| \mathbf{E}_\perp$.

[sketch from Northtop, The Adiabatic Motion of Charged Particles (1963)]

Guiding centre momentum and energy

- Guiding Centre parallel acceleration

$$m\frac{dv_{//}}{dt} = eE_{//} + m\mathbf{u}_{\mathbf{E}} \bullet \frac{d\frac{\mathbf{B}}{B}}{dt} - \mu\frac{\mathbf{B}}{B}\bullet\nabla B + O\left(\varepsilon^2\right)$$

electric acceleration — change of direction **B** — mirror deceleration

- An (uninteresting) energy equation

$$\frac{d\left(\frac{mv_{//}^{\;2}}{2} + \frac{u_E^2}{2} + \mu B\right)}{dt} = e\dot{\mathbf{R}}\bullet\mathbf{E}(\mathbf{R},t) + \mu\frac{\partial B(\mathbf{R},t)}{\partial t} + O\left(\varepsilon^2\right)$$

$$\left.\begin{array}{l}\mathbf{E}(\mathbf{R},t)\cong\mathbf{E}(\mathbf{R})\\ \mathbf{B}(\mathbf{R},t)\cong\mathbf{B}(\mathbf{R})\end{array}\right\}\text{ slowly varying fields}$$

- Magnetic moment

$$\mu = \frac{v_\perp^2}{2B} = \text{constant}$$

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Relativistic momentum and energy

- Relativistic effects modify parallel momentum
- Temporal variations field << variations due to particle motion

$$\frac{m_0 d\gamma v_{//}}{dt} = m_0\gamma \mathbf{u_E} \bullet \frac{d\frac{\mathbf{B}}{B}}{dt} + eE_{//} - \frac{\mu_r}{\gamma}\frac{\mathbf{B}}{B}\bullet\nabla\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right] + O(\varepsilon^2)$$

$$\left(\frac{\partial\frac{\mathbf{B}}{B}}{\partial t} + v_{//}\left(\frac{\mathbf{B}}{B}\bullet\nabla\right)\frac{\mathbf{B}}{B} + (\mathbf{u_E}\bullet\nabla)\frac{\mathbf{B}}{B}\right)$$

- Energy equation

$$\frac{dm_0c^2\gamma}{dt} = e\dot{\mathbf{R}}\bullet\mathbf{E} + \frac{\mu}{\gamma}\frac{\partial}{\partial t}\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right] + O(\varepsilon^2)$$

- Magnetic moment (adiabatic invariant, collisions neglected)

$$\mu_r = \frac{m_0\gamma^2 v_\perp^2}{2B} = \text{constant}$$

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