Modelling of PWN: Magnetohydrodynamics and particle transport (in three dimensions)

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Overview

- PWN: Our favorite relativistic plasma laboratory in the sky
- From the spherical cow to 3D dynamical models
- Electron transport in turbulent PWN
- Test particle simulations and Fokker-Planck
- Back to the spherical cow: Fitting of observations
• electrons are accelerated at the termination shock to relativistic energies according to \( n \propto E^{-2.2} \)

• loose energy due to synchrotron and inverse Compton emission. \( \Rightarrow \) Successful to model spectrum from visible to \( \gamma \)-rays

\[
\begin{align*}
E_{\text{snr}} &= 10^{51}\text{erg} \\
E_{\text{pwn}} &= 10^{49}\text{erg}
\end{align*}
\]

Donnerstag, 19. Mai 2016
\[ E_{\text{snr}} = 10^{51} \text{erg} \]
\[ E_{\text{pwn}} = 10^{49} \text{erg} \]

**Model**

- particle dominated relativistic pulsar wind with purely azimuthal magnetic field terminates at shock
- sub-sonic nebula flow velocity decreases to match speed of remnant
- magnetic field increases towards the outer boundary of the nebula

- electrons are accelerated at the termination shock to relativistic energies according to \( n \propto E^{-2.2} \)
- loose energy due to synchrotron and inverse Compton emission. \( \Rightarrow \) Successful to model spectrum from visible to \( \gamma \)-rays

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The general morphology could already be modeled in 2D: Del Zanna, Komissarov...
\[ \sigma \equiv \frac{f_m}{f_k} = \frac{c}{4\pi} \frac{|\mathbf{E} \times \mathbf{B}|}{|\Gamma^2 \rho v c^2|} \approx \frac{1}{4\pi} \frac{B'^2_\phi}{\rho c^2} \]

- Pulsar magnetospheres: Poynting dominated wind at light cylinder \( \sigma \sim 10^4 \) (high sigma)
  
  e.g. Michel 1982, Arons 2007

- Uncollimated ideal RMHD wind is inefficient at bulk flow acceleration and remains Poynting dominated \( \sigma \approx 1 \) (high sigma)
  

- Yet 1D and 2D dynamical RMHD of the nebula models require particle dominated winds at the termination shock \( \sigma \sim 10^{-3} - 10^{-2} \) (low sigma)
  

(Dynamical) Modelling problem:

Can’t match shock size and expansion speed at same time with high sigma!
Setup of PWN simulation

Numerical details:

Domain:
3D Cartesian box, 20 lightyears
MPI-AMRVAC\textsuperscript{1}
ideal RMHD,
Minkowski spacetime,
ideal gas EOS with $\gamma=4/3$

Adaptive mesh refinement:
Base resolution $64^3$
PWN on level 5-6; hllc lim03
Termination shock on level 8-10; tvdlf minmod

- Initialize relativistic Pulsar Wind inside expanding Supernova remnant
- PW much lighter than SNR => Shock structure (termination shock, contact and forward shock) forms
- We study the dynamics downstream of the termination shock


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Setup of PWN simulation

• Pulsar wind setup

\[ L_{\text{tot}} = 5 \times 10^{38} \text{ erg s}^{-1} \quad \Gamma = 10 \quad \text{Lorentz factor in PW} \]

Anisotropic total energy flux

\[ f_{\text{tot}}(r, \theta) = \frac{1}{r^2} (\sin^2 \theta + b) \quad , b = 0.03 \]

\[ f_m(r, \theta) = \sigma(\theta) \frac{f_{\text{tot}}(\theta, r)}{1 + \sigma(\theta)} \]

\[ f_k(r, \theta) = \frac{f_{\text{tot}}(r, \theta)}{1 + \sigma(\theta)} \]

Assume: stripes are entirely dissipated

Image: Sironi+

\[ \alpha = 45^\circ \]
Distant observer sees a near split-monopole configuration, field-lines near toroidal!

Kalapotharakos, Contopoulos and Kazanas (2012)
Setup of PWN simulation

- Pulsar wind setup

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Assume: stripes are entirely dissipated

\[ \alpha = 45^\circ \]
Magnetization after annihilation:

\[ \sigma(\theta) = \frac{\tilde{\sigma}_0(\theta)\chi_\alpha(\theta)}{1 + \tilde{\sigma}_0(\theta)(1 - \chi_\alpha(\theta))} \]

Limits:

\[ \sigma_1 \rightarrow \chi_\alpha/(1 - \chi_\alpha) \quad (\tilde{\sigma}_0 \rightarrow 0) \]

\[ \sigma_1 \rightarrow \tilde{\sigma}_0 \chi_\alpha \quad (\tilde{\sigma}_0 \rightarrow \infty) \]

Coroniti 1990, Lyubarski 2003, Sironi & Spitkovsky 2011
Simulation results
Total pressure in 2D and 3D

$\log_{10} p_{\text{tot}}$

Total pressure slices for consecutive simulation snapshots 51 years after start of simulation

$\alpha = 45^\circ$, $\sigma_0 = 1$

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Total pressure slices for consecutive simulation snapshots 51 years after start of simulation

\( \alpha = 45^\circ, \ \sigma_0 = 1 \)
• Closeup of the termination shock, velocity magnitude

• Shock “Torus” due to anisotropic wind $\propto \sin^2 \theta$

• Oblique shock gives rise to fast equatorial flow

• Plasma gets diverted into poloidal jet

3D, slice

Plasma re-focused due to hoop-stress
Shock radii

- $H$ is the non-magnetic theory, in self-similar phase: $r_{\text{max}}/r_n = 0.095$
- Self-similar regime after $t \sim 200$
- Observations provide $r_{\text{max}}/r_n = 0.085$
- Shock sizes in 3D:
  - Don’t collapse for high $\sigma_0$
  - Little dependence on $\sigma_0$

\[ z_{\text{max}}/r_n \]

\[ 2 \times r_{\text{max}} \]

\[ 2 \times z_{\text{max}} \]
What happened to the sigma problem?

- Dissipation in the nebula
  - $\alpha = 45^\circ$
  - 2D: thin lines
  - 3D: thick lines

- Observed value from fitting Synchrotron and i.Compton emission: $E_e \approx 30E_m$

- 2D cases are also fairly dissipative!

- Dynamics dominated by gas pressure

Disclaimer: Dissipation entirely numerical!

Lyutikov (2010), Komissarov (2012)
What remains of the sigma problem

- Dissipation in the Nebula

\[ \alpha = 45^\circ, \sigma_0 = 1 \]

Dissipation region in 2D run.

magenta line: magnetic null

- Polarities from opposing hemispheres can mix in Nebula, e.g. Camus+ 2009; Olmi+2014
What happened to the sigma problem?

- Dissipation in the Nebula
  \( \alpha = 45^\circ, \sigma_0 = 1 \)

![Graphs and charts showing dissipation over time in the nebula.](image_url)
The magnetic field is strongest in the vicinity of the termination shock (in contrast to classical models), where it is still predominantly azimuthal. It is disordered further away from the shock.
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Polar beam and jet

$\alpha = 10^\circ \quad \log_{10} \beta$

$\alpha = 45^\circ \quad \log_{10} \beta$

$\sigma_0 = 3$
Injection at striped region of TS, follow adiabatic and radiative losses of cutoff energy:
\[ f(\epsilon) = A n_0 \epsilon^{-p} \quad \text{for} \quad \epsilon < \epsilon_{\infty,0}, \]

Optical \( \nu = 10^{15}\text{Hz} \)

- Sprite
- Knot
- Inner Ring
- Torus

hard Xray \( \nu = 10^{19}\text{Hz} \)

Good resemblance with Hubble observations of Crab

But: No jet in (hard) Xray?
• Highly variable feature at the base of the jet

• Tempting: candidate for γ-ray Flares (Tavani et al 2011, Abdo et al 2011)?

• Not seen in 2D simulations

\[ \log_{10} \sigma \]

\[ y \text{[cm]} \]

\[ z \text{[cm]} \]

square root filtered intensity ~40 days between frames (1 year total)
Variability of Knot

- optical intensity (linear scale) of the knot measured ~ 1 month apart
- Flux as a function of time and as function of displacement
- Unresolved polarisation degree and direction of the Knot
- Consistent with Komissarov & Lyubarsky (2004)
- Significant flux variability ~20%
- Closer in <-> brighter
- Stable polarisation signal at a degree of 60%

\[ \alpha = 45^\circ, \ \sigma_0 = 1 \]
They show that the photon eufield vector of knot et al. agree on the polarization direction in this region. Both the optical emitting region and the outer Crab portant information on the geometry of magnetic field in the horizontal line in the plots. We note a variability of the knot flux kleftl and as function of the displacement between peak intensity and pulsar position. We note a variability of the knot flux kleftl and as function of the displacement between peak intensity and pulsar position.

Figure 23. Figure 23. Variability of Knot.

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- Flux as a function of time and as function of displacement.
- Unresolved polarisation degree and direction of the Knot.
- Consistent with Komissarov & Lyubarsky (2004).
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3D models tell us

- Three mechanisms involved in the solution of the sigma problem:
  - Dissipation in striped Wind (here: assumption)
  - Nebula turbulence (field randomization)
  - We might not get the exact rate correct, but Turbulent magnetic dissipation in the nebula is dynamically important
- 3D RMHD models for Crab with $\sigma_0=3$ (>1)
- MHD model of Crab can explain many observed features: shock variability, jet, torus, wisps, knot 1, robust in 3D!
- The jets form downstream of the termination shock where the magnetic hoop stress causes collimation of the flow lines that pass through the shock at intermediate latitudes.
- Jets don’t drill through the nebula bubble, z-pinch magnetohydrostatic configurations obtained in 2D unphysical! Total nebula pressure mostly uniform.
- Illuminating the jet (v up to 0.7c) might require particle acceleration in addition to the striped wind region at the termination shock.
Particle transport in PWN

With Michael Vorster, Eugene Engelbrecht and Maxim Lyutikov
MNRAS, accepted

arXiv: 1604.03352
Particle transport: Motivation

- PWN are often center filled: relativistic leptons injected centrally, transport and cool

- Spectral index maps and radiative fluxes can help to constrain parameters, depend on:
  - Particle injection and acceleration
  - Dynamics of PWN
  - Particle transport and cooling

- May help to understand particle acceleration: Only at shock or in-situ in the nebula?

- Escape of PWN accelerated particles: Might explain properties of cosmic rays, e.g. positron excess?
The PWN is a turbulent environment

- Turbulence driven by kink-instability of the jet and equatorial shear-flow
- Injection scale: $R_{ts}$
- Less vigorous and decaying further out.
- Hemispherical guide-field still present

Slice through a 3D simulation

$P_m(k) \propto k^{-5/3}$

Driving scale, termination shock
The PWN is a turbulent environment

Turbulence driven by kink-instability of the jet and equatorial shear-flow

Injection scale: \( R_{ts} \)

Less vigorous and decaying further out.

Hemispherical guide-field still present

\[ P_{in}(k) \propto k^{-5/3} \]

Driving scale, termination shock.
Figure 4. Characterisation of the turbulence in magnetic field and velocity fields.

Left: Logarithm of the average fields magnetic field given in Gauss. We also show streamlines of the poloidal velocity vector fields as white lines. Two large-scale counterstreaming vortices form on each hemisphere.

Middle: Magnitude of the fluctuating field component.

Right: Fluctuating components in terms of the average. We average over the azimuthal angle and 21 snapshot corresponding to 2–years. We apply the Taylor-Green-Kubo (TGK) formulation directly to the MHD velocity field:

$$\hat{D}_{xy} = \int_{0}^{\Delta t} \left( \hat{v}_x \hat{v}_y - \hat{v}_y \hat{v}_x \right) dt$$

where we subtract the background flow velocities $\hat{v}_x, \hat{v}_y$. In the following we will only study the radial transport, thus assuming homogeneity in time and purely diffusive behaviour. This definition would be identical to the general expression if the velocities were taken as particle velocities. On the other hand, as long as the magnetic field remains toroidally dominated, the components of particle drift velocity $\mathbf{E} \times \mathbf{B}$ and flow velocity $v_r$ can be interchanged. It is hence instructive to see how this comparison to the direct measurement from test-particles using Eqs. (1).

To obtain convergence of the integral $\hat{D}_{rr}$, we ensure that the upper bound $\Delta t$ is chosen larger than the correlation time following from:

$$\mathcal{R}_{\Delta t} = \int_{0}^{\Delta t} \left( \hat{v}_r \hat{v}_r - \hat{v}_r^2 \right) dt$$

The right-hand panel of Fig. w shows the resulting coefficient for radial diffusion $\hat{D}_{rr}$ after an integration over five years. We overplot white contours of $\mathcal{R}_t$ which is usually rate correlated from uncorrelated regions. At this time, the flow is uncorrelated in most regions which indicates convergence of $\hat{D}_{rr}$. Qualitatively, we obtain good agreement with $2\mathcal{R}_t$. In principle the integration time should be chosen as large as possible. However, we note that convergence is lost for larger integration times, most likely due to finite box effect.
Suppose X-ray emitting particles are injected at the termination shock. *How are they transported through the nebula?*

\[
\frac{du}{dt} = \frac{q}{mc} \left( E + \frac{u \times B}{c\Gamma_p} \right)
\]

Particle Lorentz factors \( \Gamma_p \geq 10^7 \)

\[ D_{rr}(t) = \frac{1}{2} \frac{\langle (\Delta r(t))^2 \rangle_{r,\phi,\theta}}{t} \]
Particle transport in Pulsar Wind Nebulae

Bohm Diffusion:
\[
D^B = \frac{1}{3} r_g^2 \omega_g
= 1.7 \times 10^{24} \left( \frac{\Gamma_p}{10^7} \right) \left( \frac{B}{100 \mu G} \right)^{-1} \text{ cm}^2 \text{ s}^{-1}
\]  

Turbulent Eddy diffusion:
\[
D^E_{Ls} = \frac{1}{3} v_f L_s
= 1 \times 10^{27} \left( \frac{v_f}{0.5 c} \right) \left( \frac{L_s}{0.2 L_y} \right) \text{ cm}^2 \text{ s}^{-1}.
\]

Average profile of the radial diffusion coefficient for increasing particle energies.

\( L_s \): Scale of largest Eddy, termination shock \( \sim 2 \times 10^{17} \) cm. \( v_f \): Velocity at this scale \( \sim 1/2 c \).

\[
r_g = \frac{p_\perp c}{eB} = 1.7 \times 10^{16} \left( \frac{\Gamma_p}{10^9} \right) \left( \frac{B}{100 \mu G} \right)^{-1} \text{ cm}
\]

Diffusion becomes energy dependent when \( r_g \geq L_s \), thus for \( \Gamma_p = 10^{10} \), these particles have too short synchrotron lifetimes however \( \Rightarrow \) Diffusion always energy independent!
lets do some modeling
What do X-ray observations tell us about...

Vela

G21.5-09

3C58

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Particle transport in Pulsar Wind Nebulae

Back to the transport equation:

$$\frac{\partial f}{\partial t} + v \cdot \nabla f = \frac{\partial}{\partial \mu} \left( D_{\mu \mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left( D_{pp} \frac{\partial f}{\partial p} \right)$$

(18)

Look for steady state solutions for the radial transport and including \textit{adiabatic} and \textit{radiative} losses:

$$D_{rr}(r) \frac{\partial^2 f}{\partial r^2} + \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D_{rr}(r) \right) - V(r) \right] \frac{\partial f}{\partial r} + \left[ \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V(r) \right) + z_p p \right] \frac{\partial f}{\partial \ln p} + 4z_p pf = 0,$$

(19)

with the Synchrotron loss term

$$z_p(r) = \frac{4\sigma_T}{3(m_e c)^2} \frac{B^2(r)}{8\pi}$$

(20)

\[\text{Averaged profiles from a fiducial MHD simulation} \quad \text{Fokker-Planck spectral modeling, e.g. Tang & Chevalier (2012)}\]
Particle transport in Pulsar Wind Nebulae

Back to the transport equation:

\[
\frac{1}{p^2} \frac{\partial}{\partial p} \left( D_{pp} \frac{\partial f}{\partial p} \right) + \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V(r) \right) + z_p \rho \frac{\partial f}{\partial \ln \rho} + 4z_p \rho f = 0, \tag{18}
\]

transport and including \textit{adiabatic} and

\[
\frac{1}{8\pi} \frac{\partial}{\partial t} \left( B^2(r) \right) = s \tag{19}
\]

Kennel & Coroniti (1984) laminar model

Averaged profiles from a fiducial MHD simulation

Fokker-Planck spectral modeling, e.g. Tang & Chevalier (2012)

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Particle transport in Pulsar Wind Nebulae: **Vela**

\[ r_n = 40'' \]
\[ r_0 = 21'' \]
\[ v_n = 1000 \text{km/s} \]
Particle transport in Pulsar Wind Nebulae: **G21.5-0.9**

\[ r_n = 40'' \]
\[ L_s = 1''.5 \]
\[ v_n = 910 \text{km/s} \]
Particle tracing and acceleration in MHD evolution

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Particle transport in Pulsar Wind Nebulae: **3C58**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>G21.5-0.9</th>
<th>Vela</th>
<th>3C 58</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s$ (µG)</td>
<td>33</td>
<td>38</td>
<td>300</td>
</tr>
<tr>
<td>$V'_s$ (units of c)</td>
<td>0.36</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>$\kappa_s$ (10^{26} cm^2 s^{-1})</td>
<td>8.5</td>
<td>5.7</td>
<td>1.4</td>
</tr>
<tr>
<td>$\sigma$ (10^{-3})</td>
<td>1.3</td>
<td>142</td>
<td>0.6</td>
</tr>
<tr>
<td>$\eta$ (10^{-2})</td>
<td>8.8</td>
<td>4.5</td>
<td>2.1</td>
</tr>
<tr>
<td>$\dot{B}$ (µG)</td>
<td>158</td>
<td>5.8</td>
<td>63</td>
</tr>
<tr>
<td>$\dot{V}$ (10^{-3}, units of c)</td>
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<td>1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>$\dot{\kappa}$ (10^{26} cm^2 s^{-1})</td>
<td>1.2</td>
<td>0.9</td>
<td>1.4</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2.0</td>
<td>0.9</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- Re-scaled simulation give better fits for photon index maps
- Typically, lower average magnetic field strength and velocities obtained
- Peclet number $\bar{\xi} = \frac{\nu r}{D_{xx}} = O(1)$
  thus diffusion important transport mechanism!

$v_n = 730 \text{km/s} \quad r_0 = 5''.25 \quad r_n = 90''$
Conclusions Particle transport

• Turbulence in PWN gives rise to high diffusive particle transport, most likely dominates over kinetic diffusion. Peclet numbers of ~1

• Propose a scaling of diffusion with the predominant scale: Termination shock

• Particle escape time in Crab: ~300 years

• Test particle simulations find energy independent diffusive regime, not Bohm.

• Performed fits of X-ray flux and spectral index for three young PWN

• Yields acceptable fits also for $\sigma_{\text{wind}} \sim 1$ in particular for the X-ray spectral index

• Don‘t use Kennel-Coroniti model! Its wrong and does not tell you anything about $\sigma_{\text{wind}}$