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On the semiclassical limit of the cosmological perturbations

based on

P. Dona & A. Marciano, arXiv:1605.09337

Standard Big-Bang Cosmology

i) Universe's expansion and Hubble's law

ii) Black-body spectrum CMB radiation

iii) BBN and primordial elements

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein equations

Separate Universe assumption

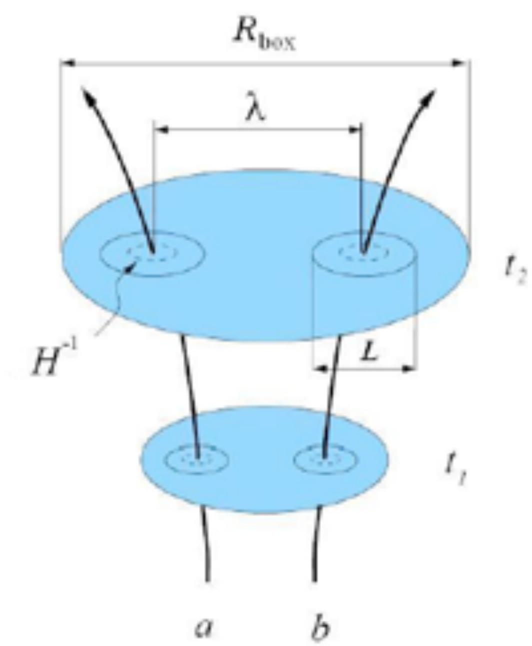
$$ds^2 = N^2 dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

ADM decomposition

$$K_{ij} = -\nabla_{(j}n_{i)} = \frac{1}{2N}\left(-\partial_t \gamma_{ij} + {}^{(3)}\nabla_{(i}N_{j)} + {}^{(3)}\nabla_{(j}N_{i)}\right)$$

$$K_{ij} = -\frac{\theta}{3}\gamma_{ij} + A_{ij} \qquad \theta = \frac{3}{N}\left(\frac{\dot{a}(t)}{a(t)} + \dot{\psi}\right)$$

$$N(t_1, t_2; x_j) = \frac{1}{3}\int_{\gamma(\tau)} \theta d\tau, \qquad n_\mu = (N, 0)$$



Perturbations

$$\epsilon = k/(a\,H)$$

$$\gamma_{ij} = a(t, x_i)\,\tilde{\gamma}_{ij}, \quad \gamma_{ij} = (e^h)_{ij}, \quad a(t, x_i) = a(t)\,e^{\psi(t, x_i)}$$

Curvature perturbation variable

Lyth, Malik & Sasaki, JCAP 2015

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longrightarrow$$

$$\frac{d\rho(t,x_i)}{dt} + 3\tilde{H}(t,x_i)[\rho(t,x_i) + p(t,x_i)] = 0$$

$$\frac{d\rho(t)}{dt} + 3\frac{\dot{a}(t)}{a(t)}[\rho(t) + p(t)] = \dot{\psi}(t)$$

Uniform density slicing & adiabatic pressure

$$\psi(t_2,x^i) - \psi(t_1,x^i) = -\ln\left[\frac{a(t_2)}{a(t_1)}\right] - \frac{1}{3}\int_{\rho(t_1,x^i)}^{\rho(t_2,x^i)} \frac{d\rho}{\rho+p}$$

$$\downarrow$$

$$-\zeta(x^i) = \psi(t,x^i) + \frac{\delta\rho}{3(\rho+p)}$$

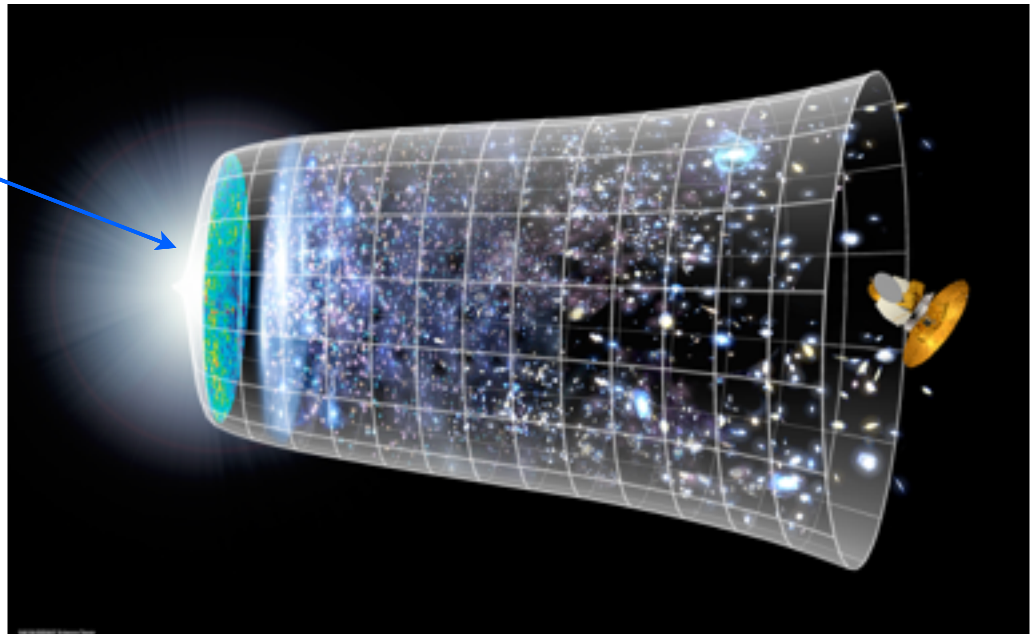
Problems of SBB Cosmology

i) Horizon problem

ii) Flatness problem

iii) Size/entropy problem

Inflation

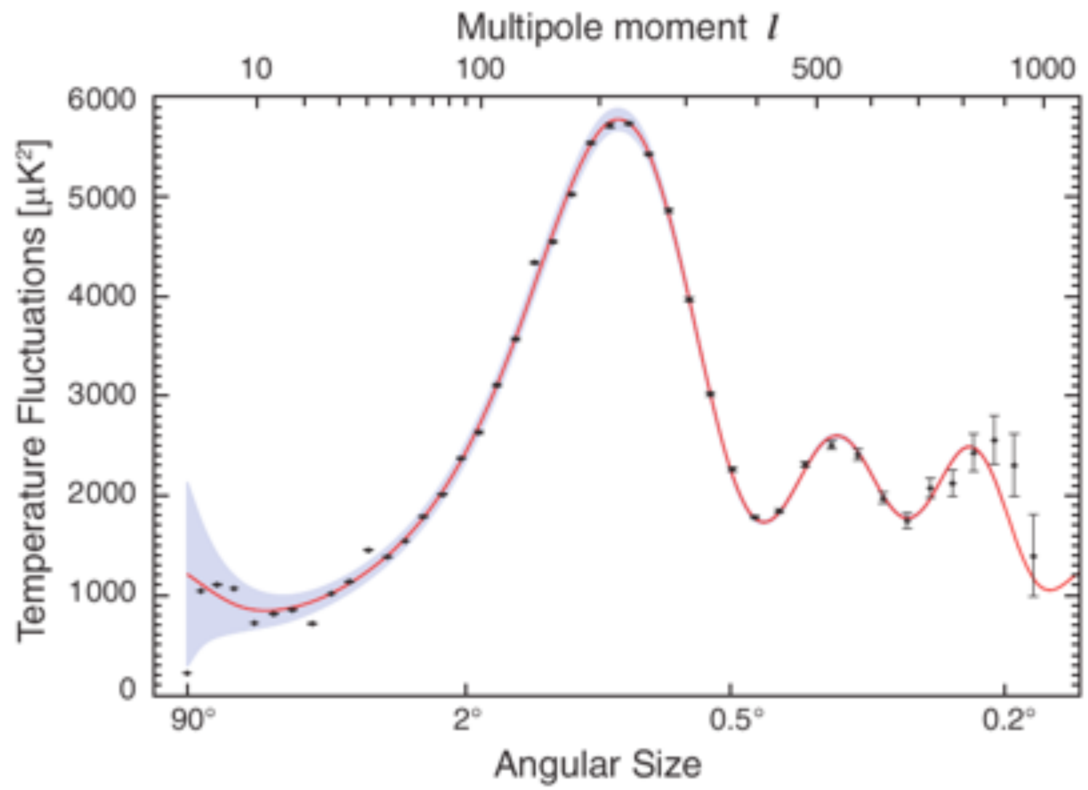


Inflation

i) $\varrho \simeq \varrho_\phi \simeq \text{const} \rightarrow a(t) = e^{Ht}$

ii) $\varrho_K / \varrho_{\text{rad}} \sim a(t)^2 \qquad \varrho_K / \varrho_\phi = 1/a(t)^2$

iii) Universe empty, then $\delta\phi$



$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \qquad \frac{\dot{a}^2}{a^2} \equiv H^2 = \frac{8\pi G}{3c^4}\varrho \qquad \varrho = \varrho_\phi + \varrho_{\text{rad}} + \varrho_{\text{matt.}} + \varrho_K$$

with a bonus!

Causal mechanism for generating primordial cosmological [Chibisov & Mukhanov 1981]

perturbations originate as quantum vacuum fluctuations!

Cosmological perturbations/structure formations

Cosmological fluctuations links early Universe theories to observations

Fluctuations of matter $\longrightarrow$ large$-$scale structure

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Fluctuations of metric $\longrightarrow$ CMB anisotropies

Matter and metric fluctuations coupled though the Einstein equations!

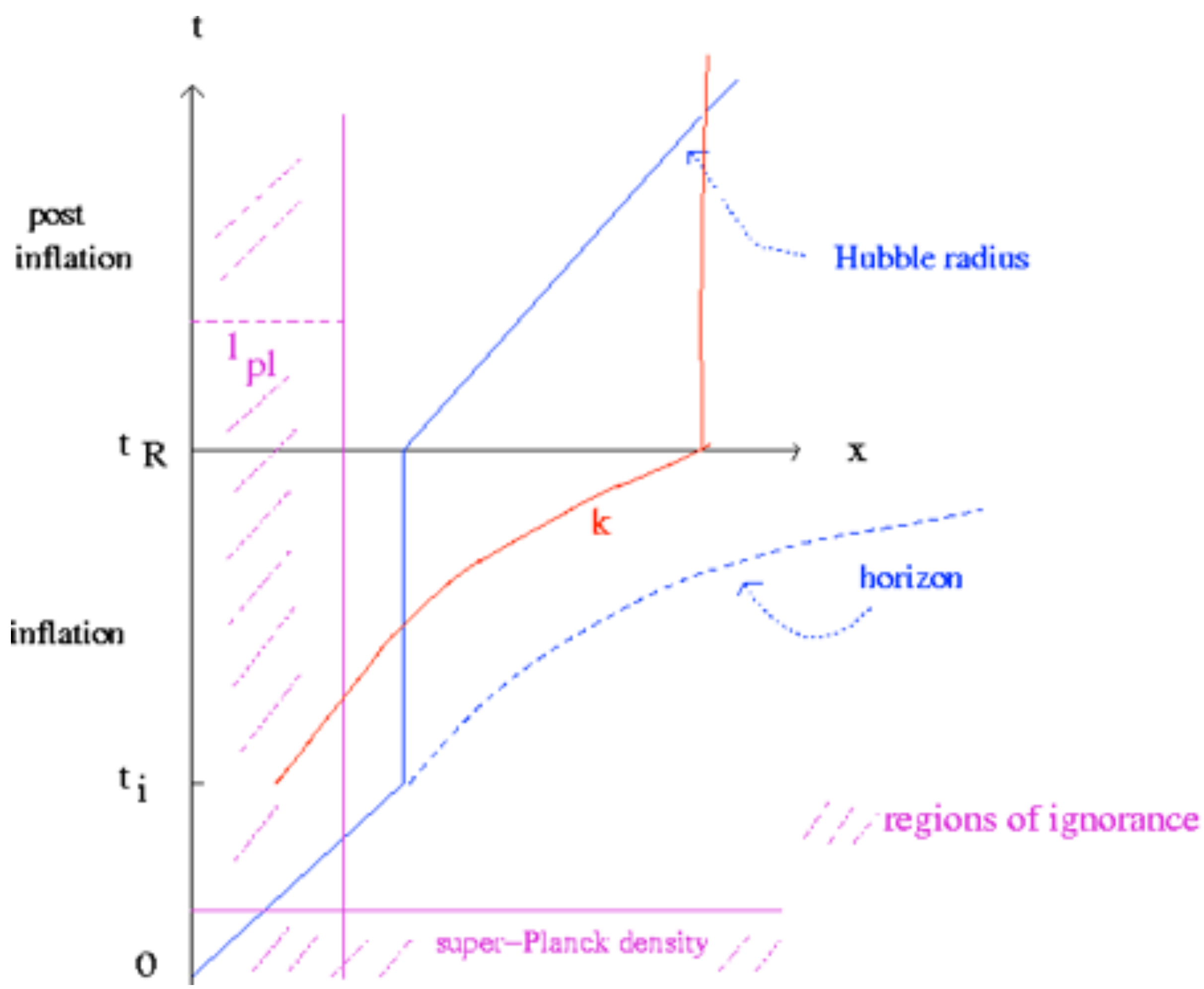
Fluctuations are small today, and were small in the early Universe: linear perturbations

Structure formation at work: heuristics

Brandenberger, 2008

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]$$

$$\phi(t, \vec{x}) = \phi_{\vec{k}}(t)\, e^{\imath \vec{k}\cdot\vec{x}}, \qquad \frac{d^2\phi_{\vec{k}}}{dt^2} + 3H\,\frac{d\phi_{\vec{k}}}{dt} + \frac{k^2}{a^2}\phi_{\vec{k}} = 0$$



$\frac{a}{k} \ll \frac{1}{H}$ **harmonic oscillator behavior**

$\frac{a}{k} \gg \frac{1}{H}$ **overdamped oscillator**

Inflation and scalar perturbations

$$S[\phi] = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Slow-roll approximation

$$\rho \simeq V(\phi), \qquad 3H\dot{\phi} \simeq V'$$

Flat slicing

$$\zeta = \frac{1}{3}\frac{V'}{\dot{\phi}^2}\delta\phi = \frac{1}{m_{\rm Pl}^2}\frac{V}{V'}\delta\phi$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{1}{a^2}\nabla^2\delta\phi + V''(\bar{\phi})\delta\phi = 0$$

well after horizon crossing

$$\delta\chi = a(\eta)\delta\phi \qquad \delta\chi_{\vec{k}}(\eta) = \frac{e^{\imath\frac{\pi}{4}(2\nu-1)}}{\sqrt{2k}}\frac{\Gamma(\nu)}{\sqrt{\pi}}\left(\frac{-k\eta}{2}\right)^{\frac{1}{2}-\nu}$$

Chaotic Inflation

Linde, PLB 1983

$$V(\phi) = \frac{1}{2}m^2\phi^2 \,, \qquad \nu \simeq \frac{3}{2} - \frac{m^2}{3H^2}$$

Light-mass case

Spectral index

$$\mathcal{P}_{\delta\phi} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{\frac{2}{3}\left(\frac{m}{H}\right)^2}$$

$$n - 1 \equiv \frac{d\ln\mathcal{P}}{d\ln k}$$

Scale-invariant scalar perturbations

$$\mathcal{P}_\zeta = \left(\frac{1}{m_{\rm Pl}^2}\frac{V}{V'}\right)^2 \left(\frac{H_k}{2\pi}\right)^2 \simeq \frac{1}{24 m_{\rm Pl}^4 \pi^2}\,\frac{V}{\epsilon_{\rm s.r.}}$$

$$\nu = \frac{3}{2}$$

$$\mathcal{P}_\zeta(k_0) = (2.445 \pm 0.096) \times 10^{-9} \qquad k_0 = 0.002 \mathrm{Mpc}^{-1}$$

Anatomy of cosmological perturbations

Criteria to bear in mind

Horizon $\gg$ Hubble radius

Fluctuations mode have $\lambda \gg H^{-1}$ for a long period (squeezing)

Mechanism accounting for scale−invariant primordial spectrum

Matter perturbation

Perturbations of matter fields are treated classically

Curvature perturbation is conserved when pressure is adiabatic

The classical analysis does not extend to fermion fields perturbations

Fermion fields and linear perturbations

Alexander, Brandenberger, Calcagni, Hui, Nicolis, Piazza, Sasaki, etc...

I **Pressure perturbations (non adiabatic) and conservation of curvature perturbations**

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{\rm na}$$

II **A no-go argument:**

$$\delta\phi \to \delta(\bar{\psi}\psi) = \delta\bar{\psi}\,\psi + \bar{\psi}\,\delta\psi$$

$$\psi(t) = \langle\alpha|\hat{\psi}|\alpha\rangle = \langle\alpha|R^\dagger(\varphi)\,R(\varphi)\hat{\psi}R^\dagger(\varphi)\,R(\varphi)|\alpha\rangle|_{\varphi=2\pi} = -\psi(t)$$

Macroscopic quantum states of matter I

Dona & Marciano, arXiv:1605.09337

I **Classical background fields correspond to expectation values on macroscopic (condensed) states**

$$\phi(x) := \langle \alpha | \hat{\phi} | \alpha \rangle$$

II **Matter perturbations are evaluated as the the first order expansion of the expectation values on perturbed macroscopic states**

$$\delta\phi(x) := \langle \alpha + \delta\alpha | \hat{\phi} | \alpha + \delta\alpha \rangle |_{O(\delta\alpha)}$$

Macroscopic quantum states of matter II

Dona & Marciano, arXiv:1605.09337

III Density matrix and infrared mode of the macroscopic state

$$\rho_{1-\mathrm{p}}(x-x') = \int_{k,k'} e^{-\imath(kx-k'x')}\langle a_k^\dagger a_{k'}\rangle$$

$$\rho_{1-\mathrm{p}}(t-t';\vec{x}-\vec{x}') = \frac{N_0}{V} + \int_k e^{-\imath\vec{k}\cdot(\vec{x}-\vec{x}')}\,n(k)$$

II Off-diagonal long ranged order (ODLRO) and vanishing of correlations at large space-time distances

$$\lim_{||x-x'||\to\infty} \rho_{1-\mathrm{p}}(t-t';\vec{x}-\vec{x}') = \langle\phi(x)\phi(x')\rangle_0 \equiv n_0$$

Bosonic statistics and coherent states

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(a_k e^{-ikx} + a_k^\dagger e^{+ikx} \right)$$

Scalar field

Bosonic Hilbert space and infinite occupation numbers

$$|\alpha\rangle \equiv \prod_k |\alpha(k)\rangle = \prod_k e^{\alpha(k) a_k^\dagger - \alpha^*(k) a_k} |0\rangle = D(\alpha)|0\rangle$$

Displacement operator

$$D(\alpha)^\dagger \phi(x) D(\alpha) = \phi(x) + \phi_\alpha(x)$$

$$\langle\alpha| \mathcal{O}(\phi(x)) |\alpha\rangle = \langle 0| \mathcal{O}(\phi(x) + \phi_\alpha(x)) |0\rangle$$

Dona & Marciano, arXiv:1605.09337

Matter perturbations at linear order

Dona & Marciano, arXiv:1605.09337

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \alpha + \delta\alpha | \widehat{T_{\mu\nu}(\phi)} | \alpha + \delta\alpha \rangle \Big|_{\mathrm{O}(\delta\alpha)}$$

Expanding perturbations in the conservation equation

$$3(\zeta + \psi)\langle \alpha | \widehat{\rho} + \widehat{p} \, | \alpha \rangle = -\langle \alpha + \delta\alpha | \widehat{\rho} | \alpha + \delta\alpha \rangle \Big|_{\mathrm{O}(\delta\alpha)}$$

Example: Chaotic Inflation

$$\langle \alpha + \delta\alpha | \, \widehat{\rho} \, | \alpha + \delta\alpha \rangle = \lim_{x \to y} \frac{1}{2} m^2 \, \langle \alpha + \delta\alpha | \, \widehat{\phi}(x) \, \widehat{\phi}(y) \, | \alpha + \delta\alpha \rangle = \frac{1}{2} m^2 \left[\phi_{\alpha + \delta\alpha}(x) \right]^2$$

Power spectrum of scalar perturbations

Dona & Marciano, arXiv:1605.09337

$$-\widehat{\Xi} = \widehat{\mathbb{1}}\,\psi(t,x^i) + \frac{\widehat{\rho}}{3\langle\alpha|\widehat{\rho}+\widehat{p}\,|\alpha\rangle}$$

Slow-roll condition

$$\langle\alpha|\ddot{\widehat{\phi}} + 3H\dot{\widehat{\phi}} + \widehat{V'(\phi)}|\alpha\rangle = 0 \quad \longrightarrow \quad 3H\phi_\alpha \simeq -V(\phi_\alpha)$$

Power spectrum

$$\mathcal{P}_\zeta = \lim_{x\to y}\langle\alpha+\delta\alpha|\,\widehat{\Xi}(x)\,\widehat{\Xi}(y)\,|\alpha+\delta\alpha\rangle\Big|_{O(\delta\alpha^2)}$$

Fermion fields & macroscopic coherent states I

Dona & Marciano, arXiv:1605.09337

Pauli exclusion principle and quasi particles

$$a_k^\dagger \quad \rightarrow \quad c_k^\dagger = a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger$$

BCS states as macroscopic coherent states

$$|\alpha\rangle \equiv e^{\int d^3k\, \alpha(k) c_k^\dagger - \alpha^*(k) c_k} |0\rangle = D(\alpha)|0\rangle$$

$$J_1 = \frac{1}{2}\left(a^\dagger b^\dagger + h.c.\right), \quad J_2 = -\frac{i}{2}\left(a^\dagger b^\dagger - h.c.\right), \quad J_3 = \frac{1}{2}\left(a^\dagger a + b^\dagger b - 1\right), \quad [J_i,\ J_j] = i\epsilon_{ij}{}^k J_k$$

BCS states are SU(2) coherent states

$$|\hat{n}\rangle = D(\hat{n})|j,-j\rangle = |\xi\rangle = \exp\left(\xi J^+ - \bar{\xi} J^-\right)|j,-j\rangle$$

Fermion fields & macroscopic coherent states II

Dona & Marciano, arXiv:1605.09337

Linear perturbations and SU(2) rotations

$$\langle \hat{n} | \bar{\psi} \psi | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \langle \hat{n} | \vec{J}_k | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \hat{n}_k$$

$$|\hat{n} + \delta\hat{n}\rangle \equiv D(\delta\hat{n}) |\hat{n}\rangle = |R(\hat{z}, \delta\hat{n})\, \hat{n}\rangle$$

$$\langle \hat{n} + \delta\hat{n} | \vec{J} | \hat{n} + \delta\hat{n} \rangle \approx \hat{n} + \delta\hat{n} \times \hat{n} = \hat{n} - \hat{n} \times \langle \delta\hat{n} | \vec{J} | \delta\hat{n} \rangle$$

Macroscopic state & space-time diffeomorphisms

Dona & Marciano, arXiv:1605.09337

$$\hat{\phi}(y) = \hat{\phi}\,(y(x)) \;=\; \int_k \left(\hat{a}_k e^{-iky(x)} + \hat{a}_k^\dagger e^{+iky(x)}\right)$$

The coherent states structure is preserved

$$|\alpha\rangle = D\,(\alpha)\,|0\rangle \;=\; e^{\int d^3k\, \alpha(k)\tilde{a}_k^\dagger - \alpha^*(k)\tilde{a}_k}\,|0\rangle$$
$$\;=\; e^{\int d^3k\, \gamma(k) a_k^\dagger - \gamma^*(k) a_k}\,|0\rangle$$

when Bogolubov transformations for the two set of coordinates are implemented

$$\gamma(k) \;=\; \int_{k'} \left[B^*(k,k')\alpha(k') - A(k,k')\alpha^*(k')\right]$$

Bogolubov transformations & non-BD states

Dona & Marciano, arXiv:1605.09337

Adjoint action of displacement operators

$$D(\alpha)^\dagger a_k D(\alpha) = a_k + \alpha(k)$$

$$D(\alpha)^\dagger a_k^\dagger D(\alpha) = a_k^\dagger + \alpha^*(k)$$

U(1) bosonic case

$$\tilde{a} = \cos(|\xi|)\, a + \frac{\xi}{|\xi|} \sin(|\xi|)\, b^\dagger$$

$$\tilde{b}^\dagger = \cos(|\xi|)\, b \; - \frac{\bar{\xi}}{|\xi|} \sin(|\xi|)\, b^\dagger$$

SU(2) fermionic case

The macroscopic state obtained is the Bogolubov transform of the vacuum

Torsionful connection and fermion fields

Rovelli & Perez, CQG 2005; Freidel & Minic, PRD 2005

$$\mathcal{S}_{\text{Holst}} = \frac{1}{2\kappa}\int_M d^4x\,|e|\,e_I^\mu e_J^\nu P^{IJ}{}_{KL}F_{\mu\nu}{}^{KL}(\omega)$$

$$P^{IJ}{}_{KL} = \delta_K^{[I}\delta_L^{J]} - \epsilon^{IJ}{}_{KL}/(2\gamma)$$

$$\mathcal{S}_{\text{Dirac}} = \int_M d^4x\,|e|\left\{\frac{1}{2}\left[\overline{\psi}\gamma^I e_I^\mu\left(1-\frac{\imath}{\alpha}\gamma_5\right)\imath\nabla_\mu\psi - m\overline{\psi}\psi\right] + \text{h.c.}\right\}$$

Theory with torsion!

[Alexander, Biswas, Magueijo, Kibble, Poplawski...]

One fermion species: integrating out torsion

S. Alexander, Y. Cai & A. Marciano PLB 2015

Theory with torsion

$$e^{\mu}_{I}\, C_{\mu JK} = \frac{\kappa}{4}\,\frac{\gamma}{\gamma^2+1}\,\left(\beta\,\epsilon_{IJKL}\;J^L - 2\theta\,\eta_{I[J}\,J_{K]}\right)$$

$$J^L = \overline{\psi}\gamma^L\gamma_5\psi$$

$$\mathcal{S}_{GR} = \frac{1}{2\kappa}\int_M d^4x|e|e^{\mu}_{I}e^{\nu}_{J}R^{IJ}_{\mu\nu}$$

$$\mathcal{S}_{\rm Dirac} = \frac{1}{2}\int_M d^4x|e|\left(\,\overline{\psi}\gamma^I e^{\mu}_{I}\imath\widetilde{\nabla}_{\mu}\psi - m\overline{\psi}\psi\right) + \text{h.c.}$$

$$\mathcal{S}_{\rm Int} = -\xi\kappa\int_M d^4x|e|\,J^L\,J^M\,\eta_{LM}$$

Fermionic Matter-Bounce

Are Einstein equations rusting?

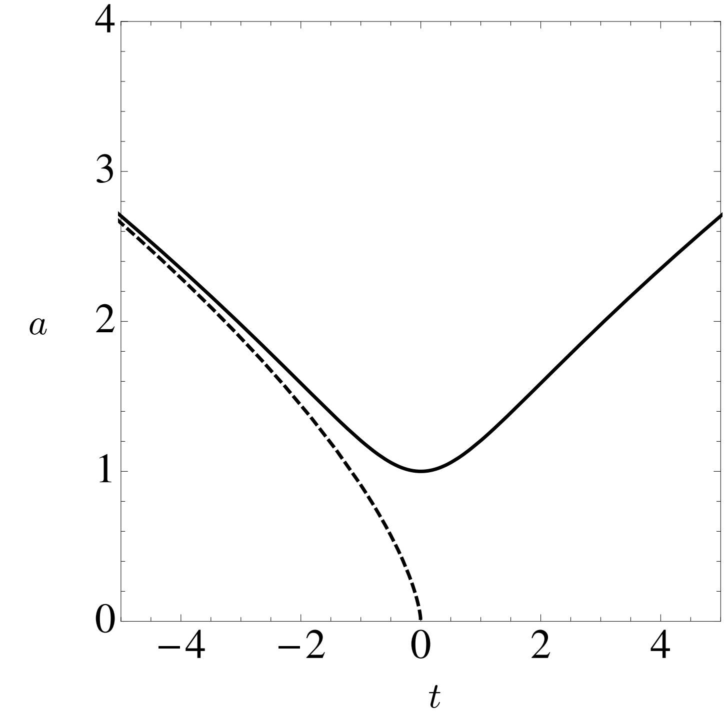
Fermioinc matter may violate the null energy conditions!

Armendariz-Picon, Alexander, Biswas, Brandenberger, Magueijo, Kibble, Poplawski...

$$H^2 = \frac{\kappa}{3}\varrho_{\rm tot} = 0$$

$$\dot{H} - H^2 = \frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\varrho_{\rm tot} + 3p_{\rm tot}) > 0$$

At the bounce

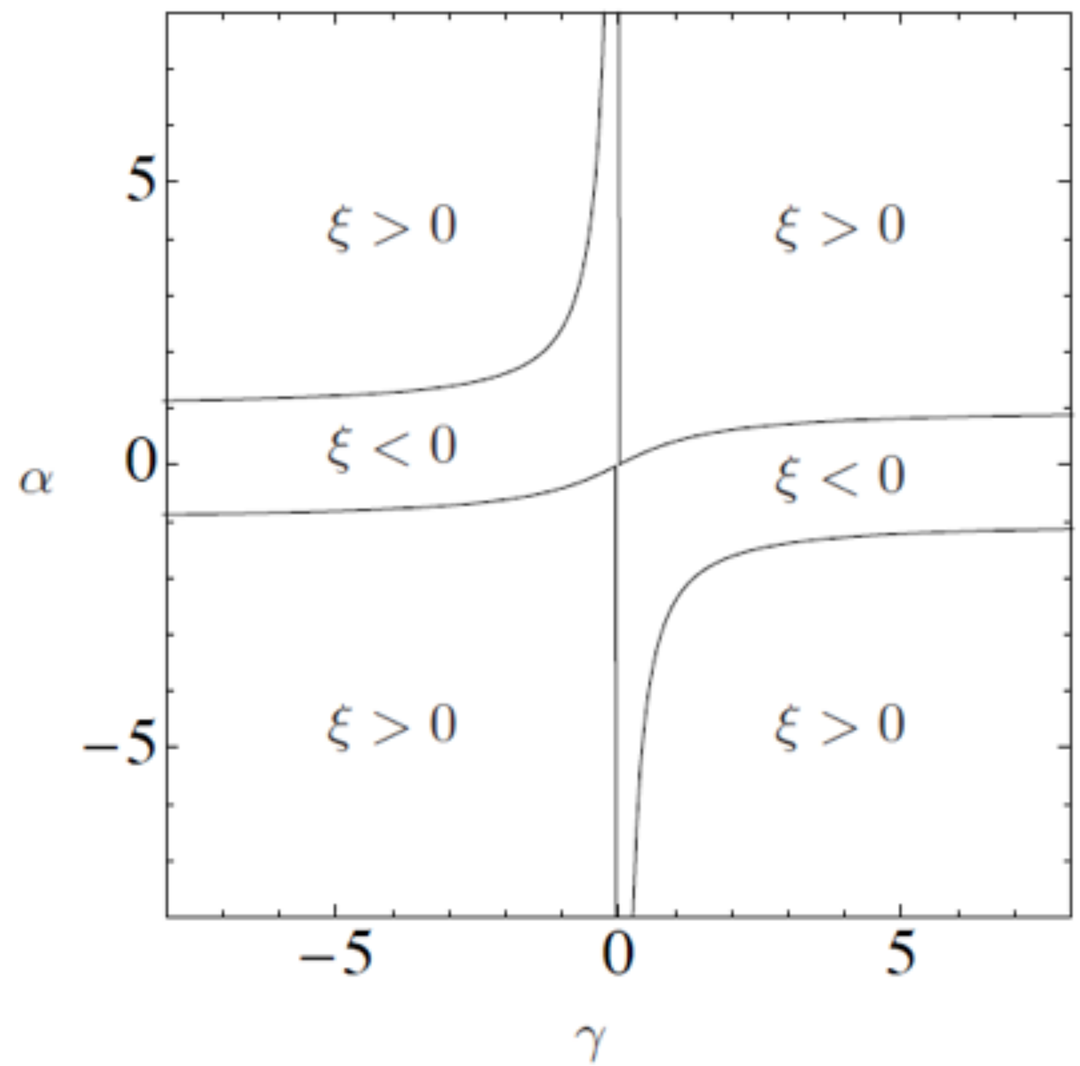


BCS condensation or torsion may provide four fermion contribution to the energy density

$$\varrho_{\rm tot} \simeq m\overline{\psi}\psi + \xi\kappa\,\overline{\psi}\psi\overline{\psi}\psi$$

Parameter space of the theory

S. Alexander, C. Bambi, A. Marciano & L. Modesto, [arXiv:1402.5880] PRD 90 (2014) 123510



$$\xi = \frac{3}{16}\frac{\gamma^2}{\gamma^2+1}\left(1+\frac{2}{\alpha\gamma}-\frac{1}{\alpha^2}\right)$$

Superconductive Inflation

Dona, Marciano & Varlamov, in preparation

An initial macroscopic state of inflation as a superposition of the BD vacuum and BCS

$$|\Omega\rangle = c_0|0\rangle + c_\alpha|\alpha\rangle$$

Energy density on the macroscopic matter states of the Universe

$$\rho_\psi^{(0)} := m\langle\Omega|\overline{\psi}\psi|\Omega\rangle = \left(|c_0|^2 + 2\,\mathfrak{Re}\{c_0\,c_\alpha\}\right)\,\langle 0|\overline{\psi}\psi|0\rangle + |c_\alpha|^2\,\langle\alpha|\overline{\psi}\psi|\alpha\rangle = |c_\alpha|^2\,m\frac{n_0}{a^3}$$

$$\begin{aligned}
\rho_\psi^{\mathrm{tot.}} - \rho_\psi^{(0)} &:= \xi\kappa\,\langle s|\overline{\psi}\gamma^5\gamma_I\psi\;\overline{\psi}\gamma^5\gamma^I\psi|s\rangle = \\
&= \left(|c_0|^2 + 2\,\mathfrak{Re}\{c_0\,c_\sigma\}\right)\,\xi\kappa\,\langle 0|\overline{\psi}\gamma^5\gamma_I\psi|0\rangle\langle 0|\overline{\psi}\gamma^5\gamma^I\psi|0\rangle + |c_\sigma|^2\,\xi\kappa\,\langle\sigma|\overline{\psi}\gamma^5\gamma_I\psi|\sigma\rangle\langle\sigma|\overline{\psi}\gamma^5\gamma^I\psi|\sigma\rangle \\
&= \left(|c_0|^2 + 2\,\mathfrak{Re}\{c_0\,c_\sigma\}\right)\,\xi\kappa\,J_0^2 + |c_\sigma|^2\,\xi\kappa\frac{(n_0^2 + \tilde{n}^2 + n^I n_I)}{a^6}
\end{aligned}$$

The evolution of the initial state define the period for Inflation

Gravitational perturbations I

S. Alexander, R. Brandenberger & A. Marciano, in preparation

$$ds^2 = (1+2\Psi)dt^2 - a^2(1-2\Phi)\delta_{ij}dx^i dx^j$$

Perturbed Einstein equations

$$\delta G^\mu_\nu = 8\pi G \delta T^\mu_\nu$$

$$\nabla^2 \Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2 \delta T^0_0,$$

$$(\Phi' + \mathcal{H}\Psi)_{,i} = 4\pi G a^2 \delta T^0_i,$$

$$\left[\Phi'' + \mathcal{H}(2\Phi' + \Psi') + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{1}{2}\nabla^2(\Psi - \Phi)\right]\delta_{ij} - 2(\Psi - \Phi)_{,ij} = -4\pi G a^2 \delta T^i_j$$

Gravitational perturbations II

S.Alexander, R. Brandenberger & A. Marciano, in preparation

Perturbed energy-momentum tensor from fermion action $V = V(\bar{\psi}\psi)$

$$\delta T^0_0 = \langle V' \rangle_{O(\alpha)} \, \langle \bar{\psi}\psi \rangle|_{O(\delta\alpha)};$$

$$\delta T^0_i = \frac{3\imath}{8}\Phi_{,l}\langle \psi\gamma^0\gamma^i\gamma^l\psi\rangle_{O(\alpha)}, \quad (l \neq i)$$

$$\delta T^i_j = \delta_{ij}V''\langle\bar{\psi}\psi\rangle_{O(\alpha)} \, \langle\bar{\psi}\psi\rangle|_{O(\delta\alpha)}$$

ISW effect can be recosntructed

Anisotropic dof are present

Cross correlation functions

Conclusions

i) Proposal for matter cosmological perturbations

ii) Fermion fields can contribute to linear cosmological perturbations

iii) Spinorial perturbations entail non-isotropic d.o.f. not present in scalar field perturbations

iv) Richer phenomenology available

Danke!

Thank you!

FIAS Frankfurt Institute
for Advanced Studies

谢谢

Grazie!