Anomalous hydrodynamics kicks neutron stars

AstroCoffee, ITP Frankfurt, 15.12.2015

Matthias Kaminski
University of Alabama

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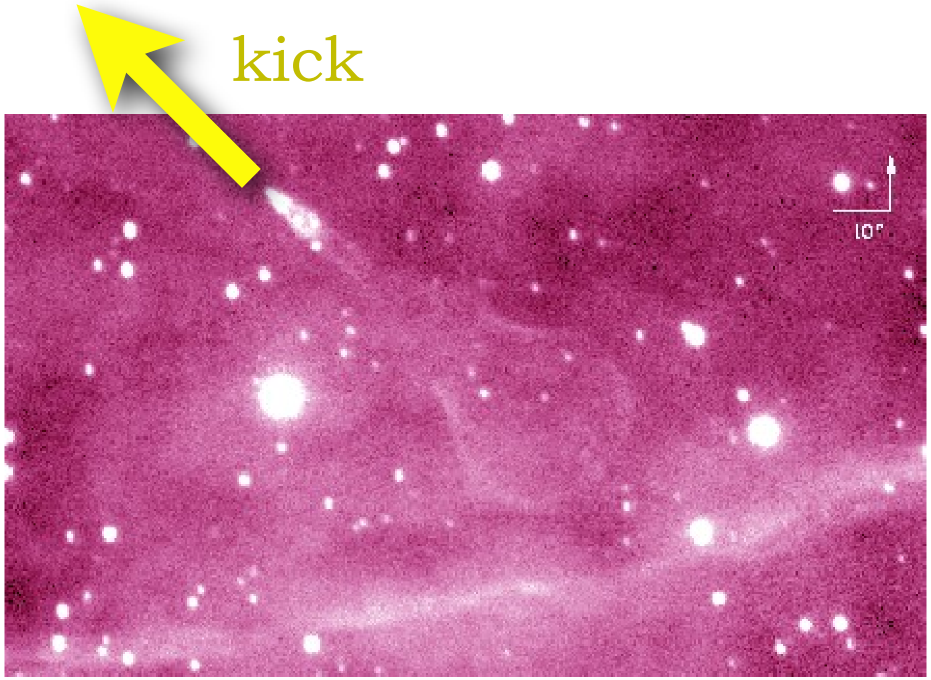
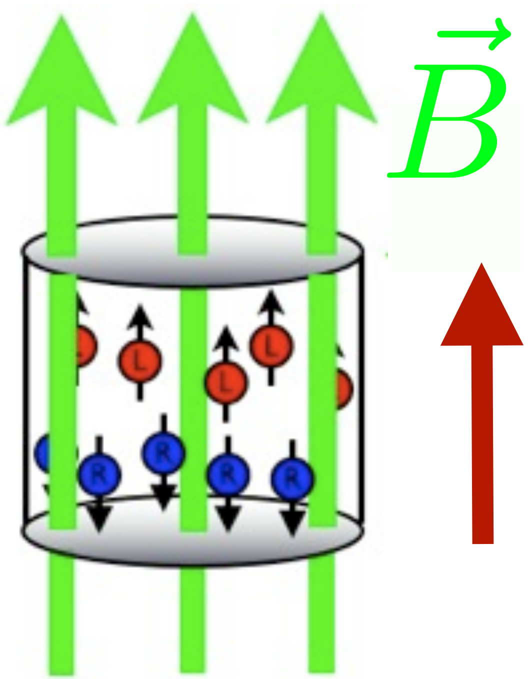
[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

Chiral hydrodynamics & neutron star kicks



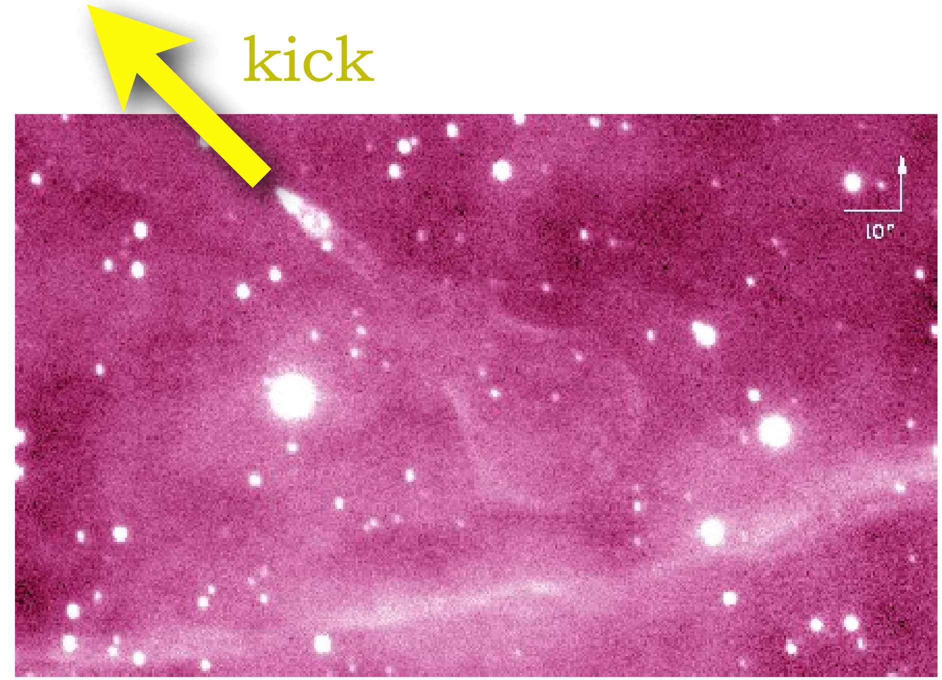
observation: neutron stars undergo a large momentum change (a kick)

Chiral hydrodynamics & neutron star kicks



hydrodynamics: fluids with left-handed and right-handed particles produce a current along magnetic field

e.g. right/left-handed electrons, neutrinos, ...



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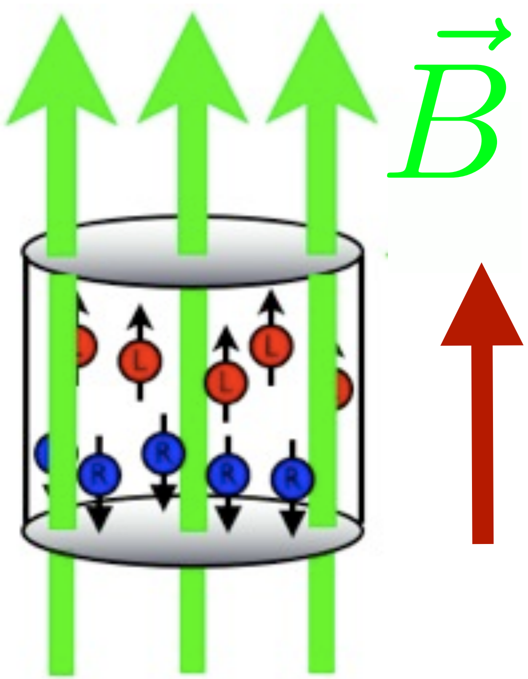

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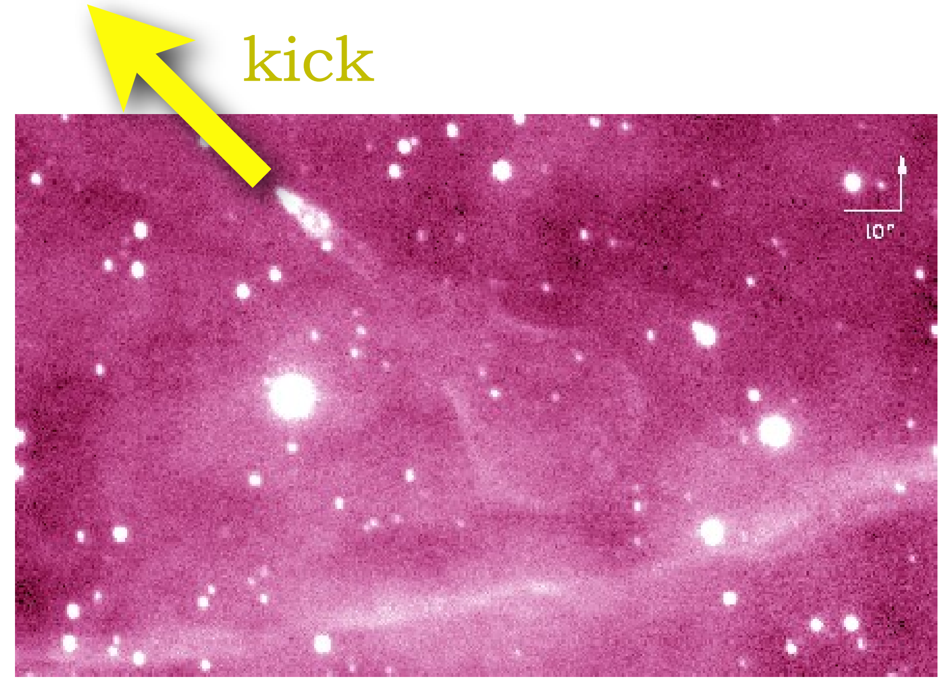

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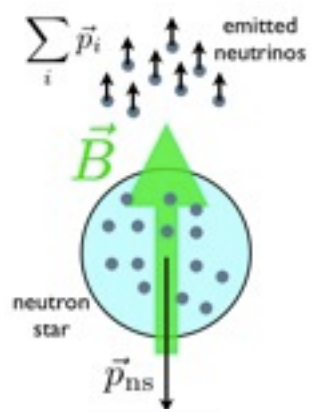

Chiral hydrodynamics leads to neutron star kicks

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

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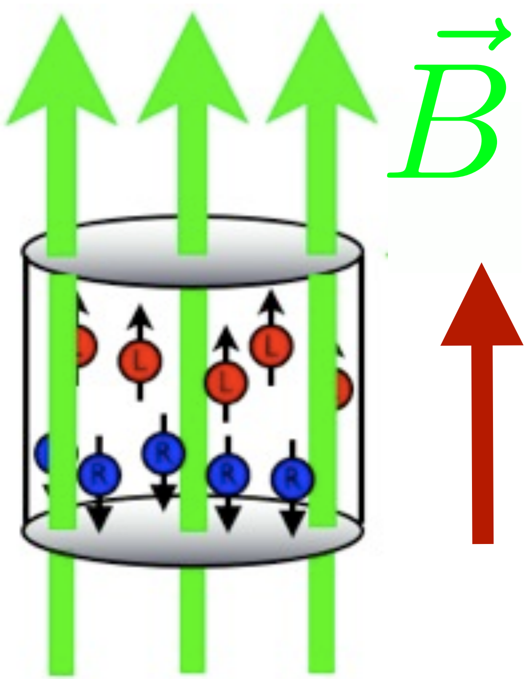

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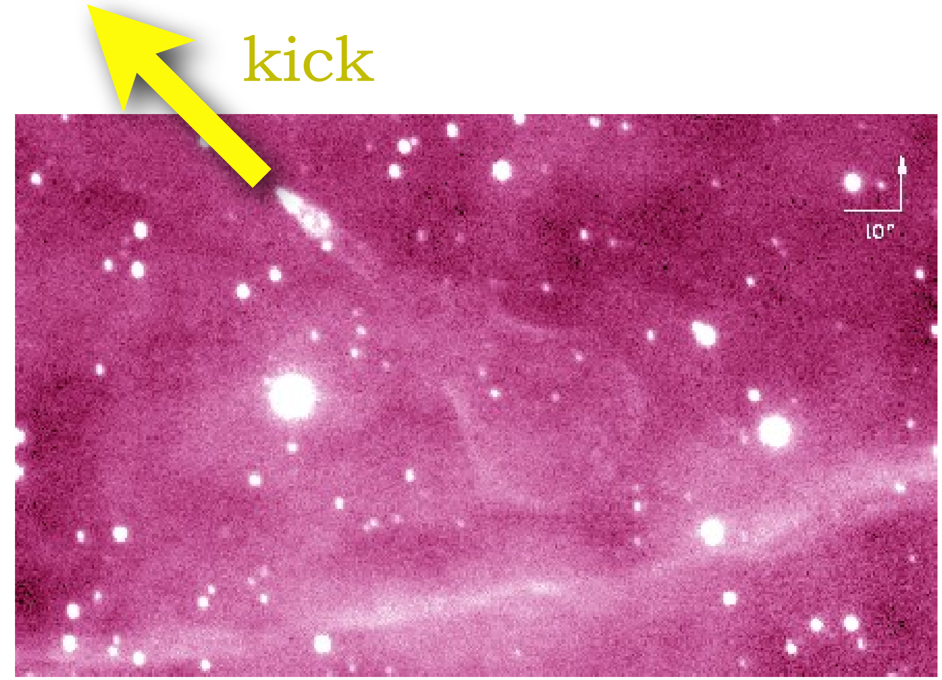

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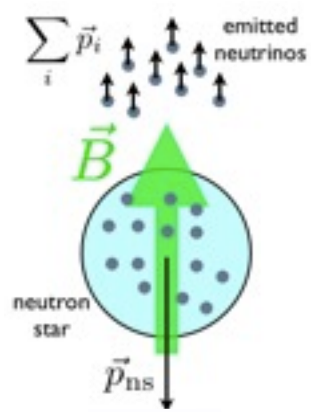

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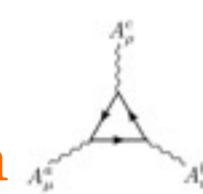
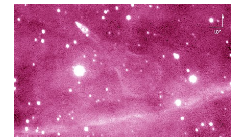


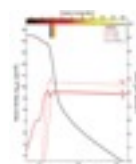
[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

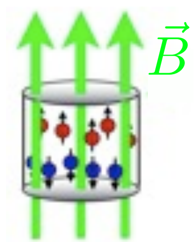
Outline

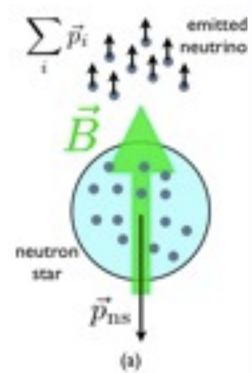
✓ Invitation: chiral hydrodynamics & neutron star kicks

1. Neutron star observations 

2. Previous kick mechanisms

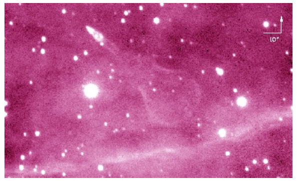
3. Sophisticated simulations 

4. Chiral hydrodynamics 

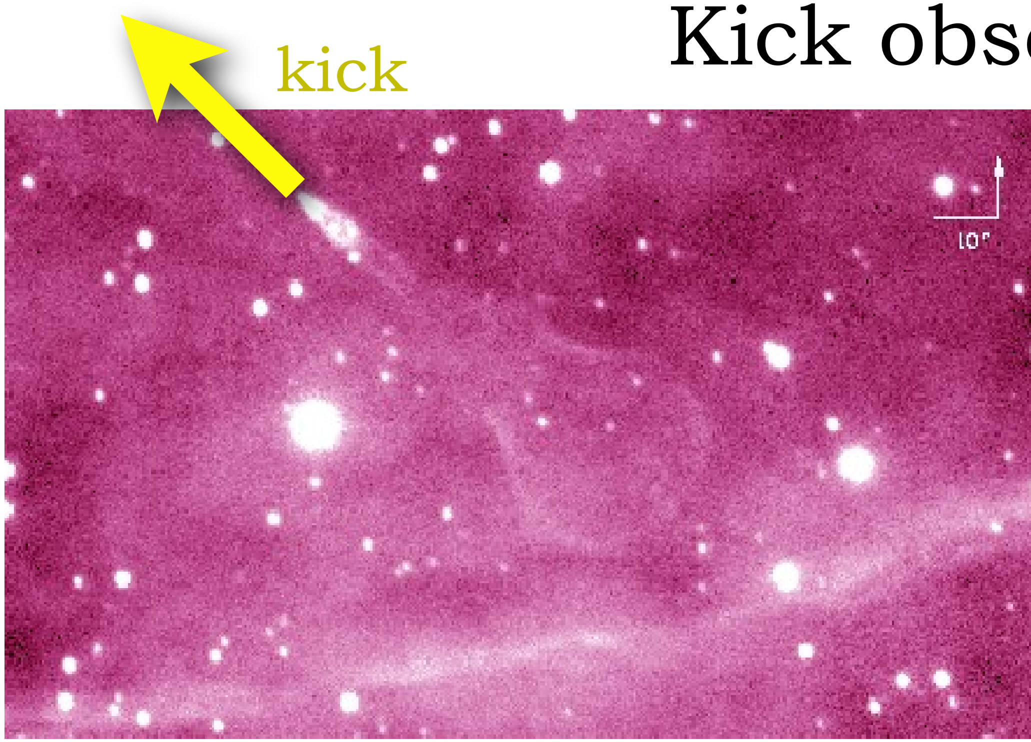
5. Kicks from anomalies 

6. Observable signals?

1. Observations



Kick observations



Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

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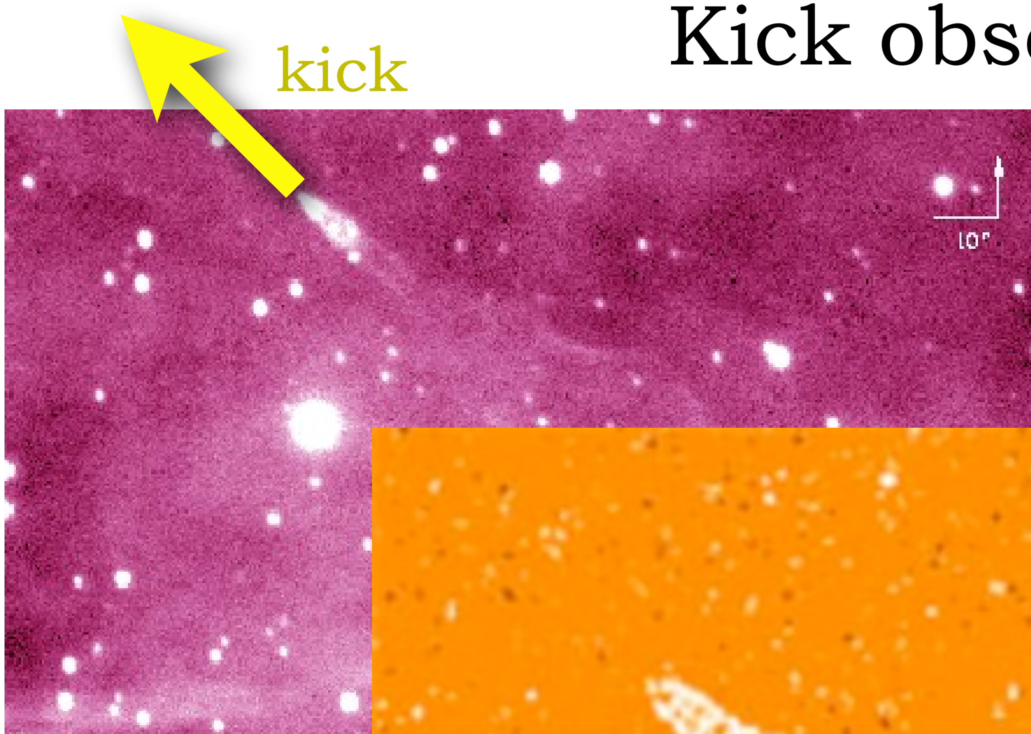
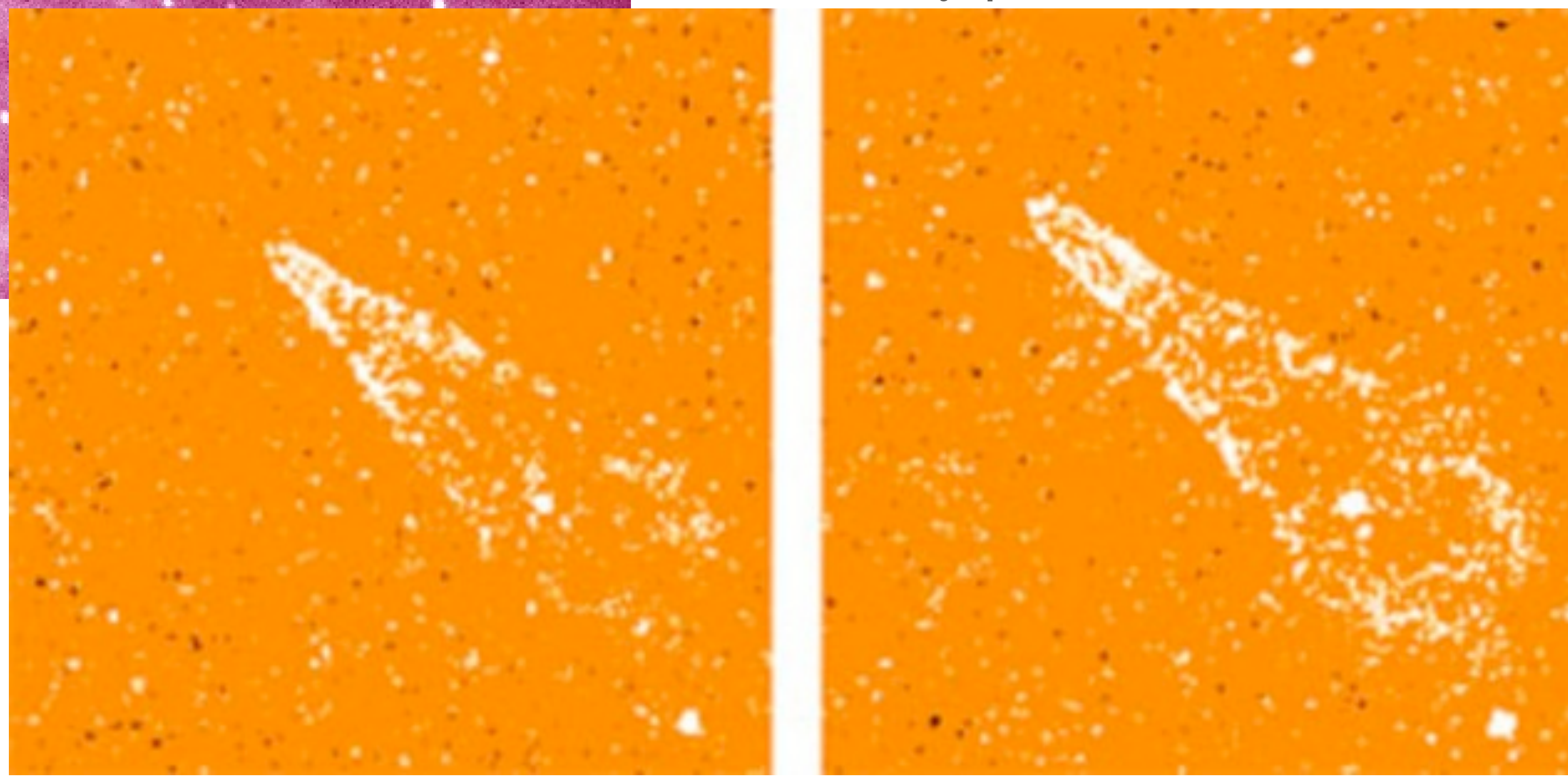


[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]



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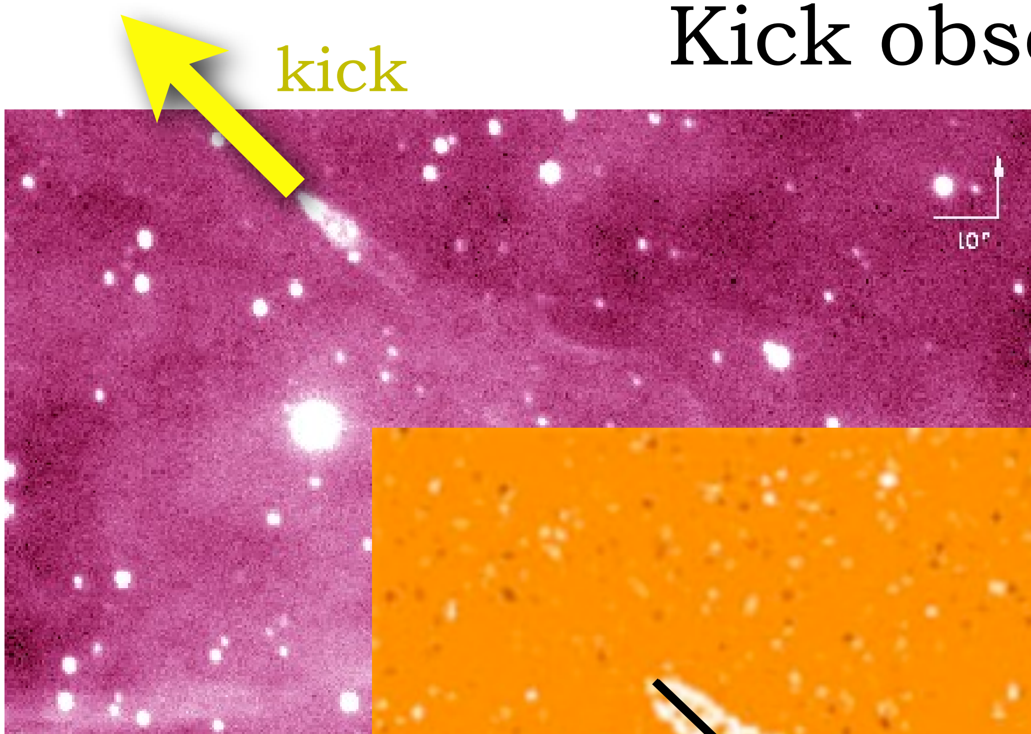

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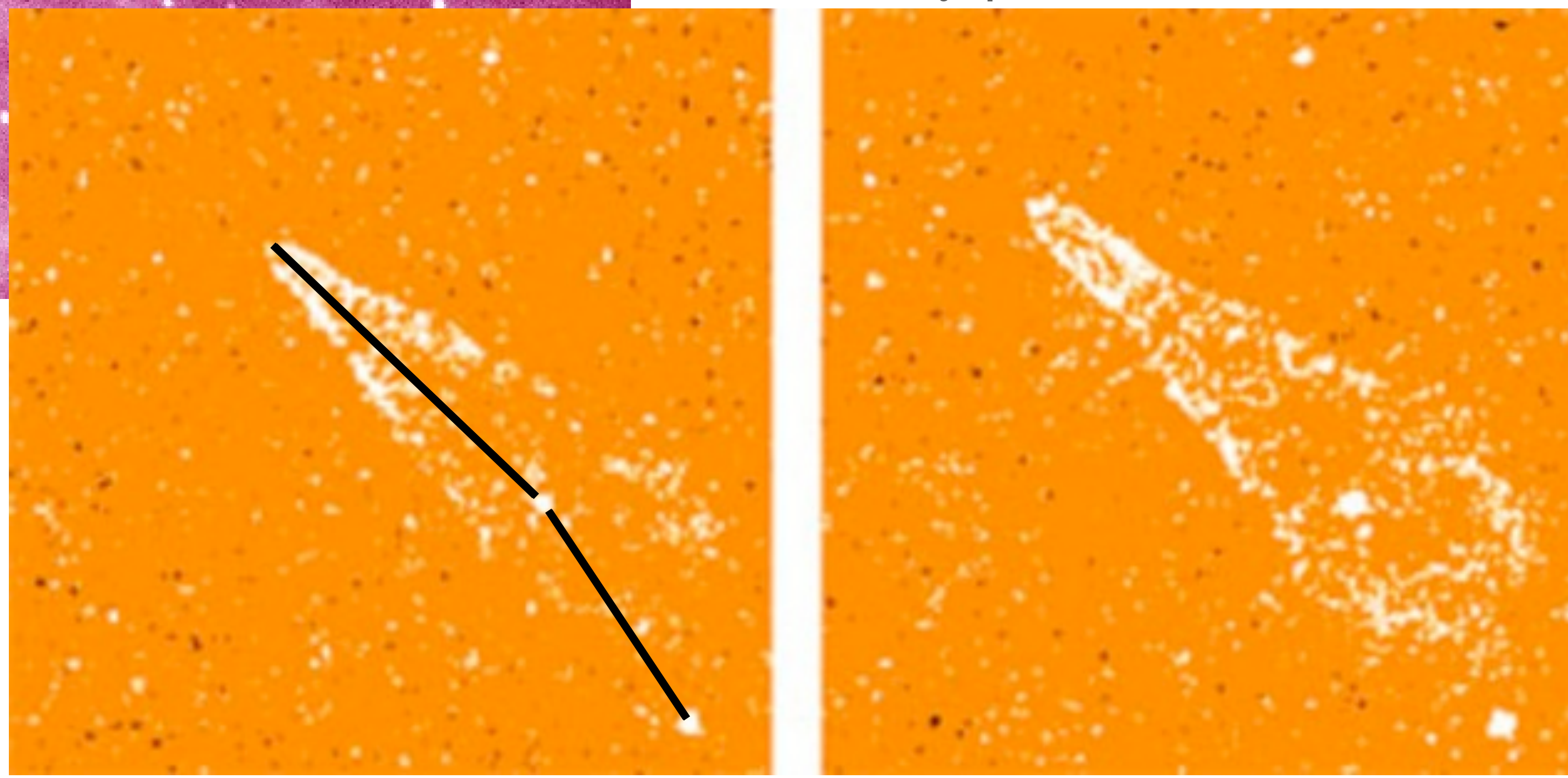

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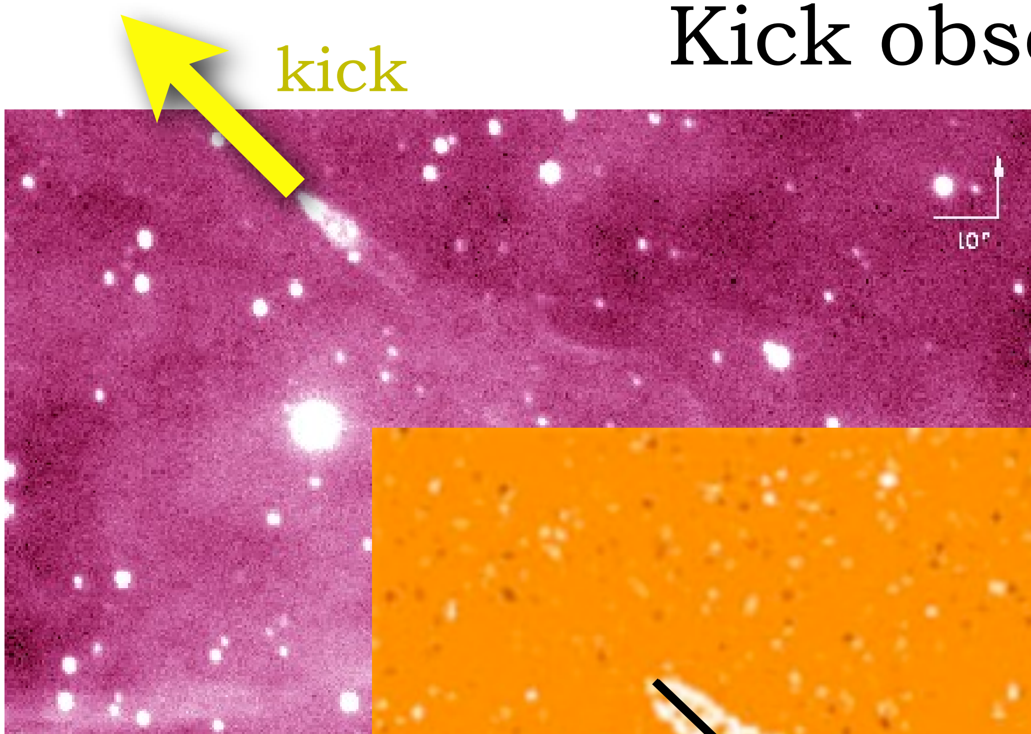

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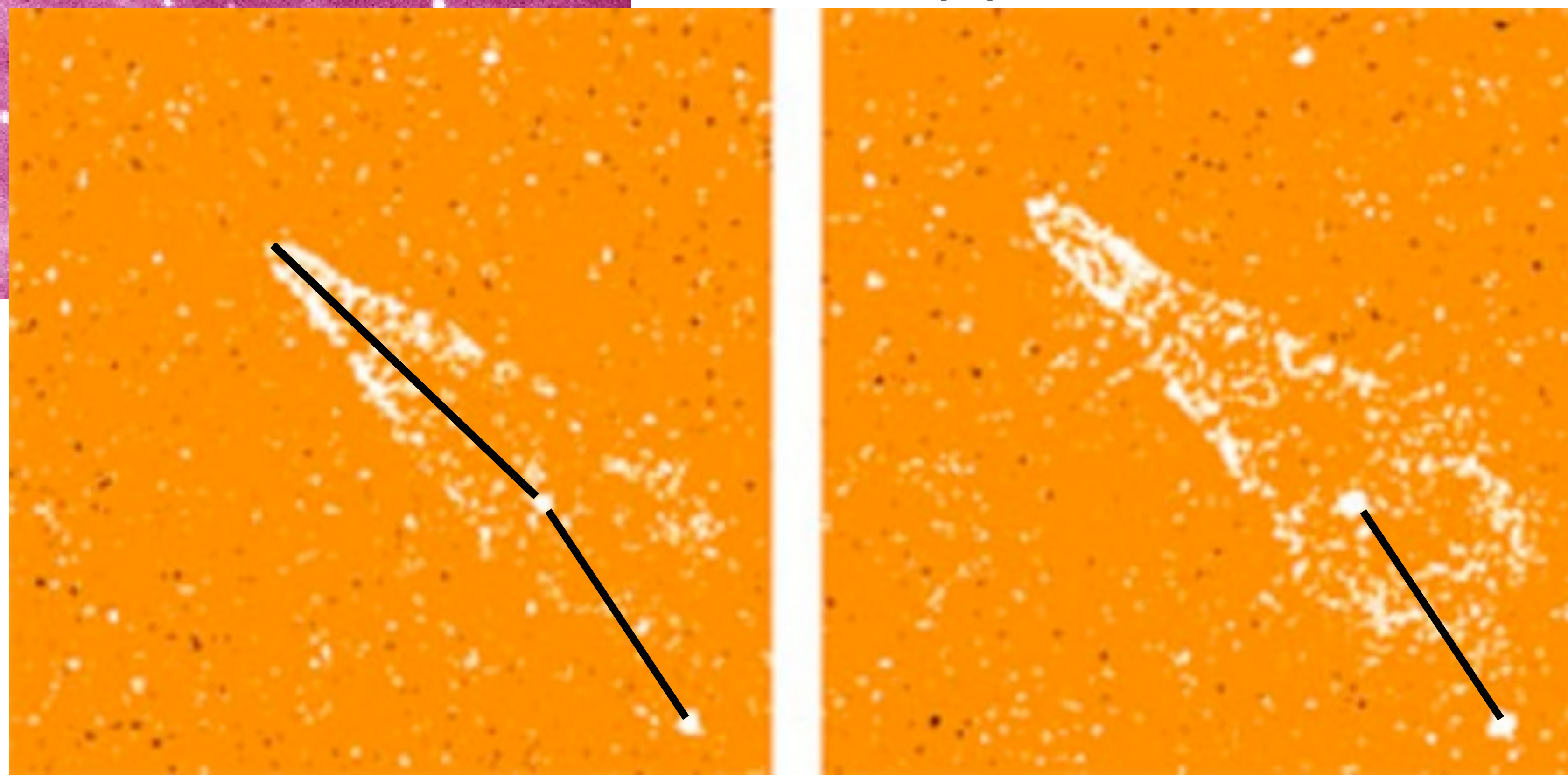

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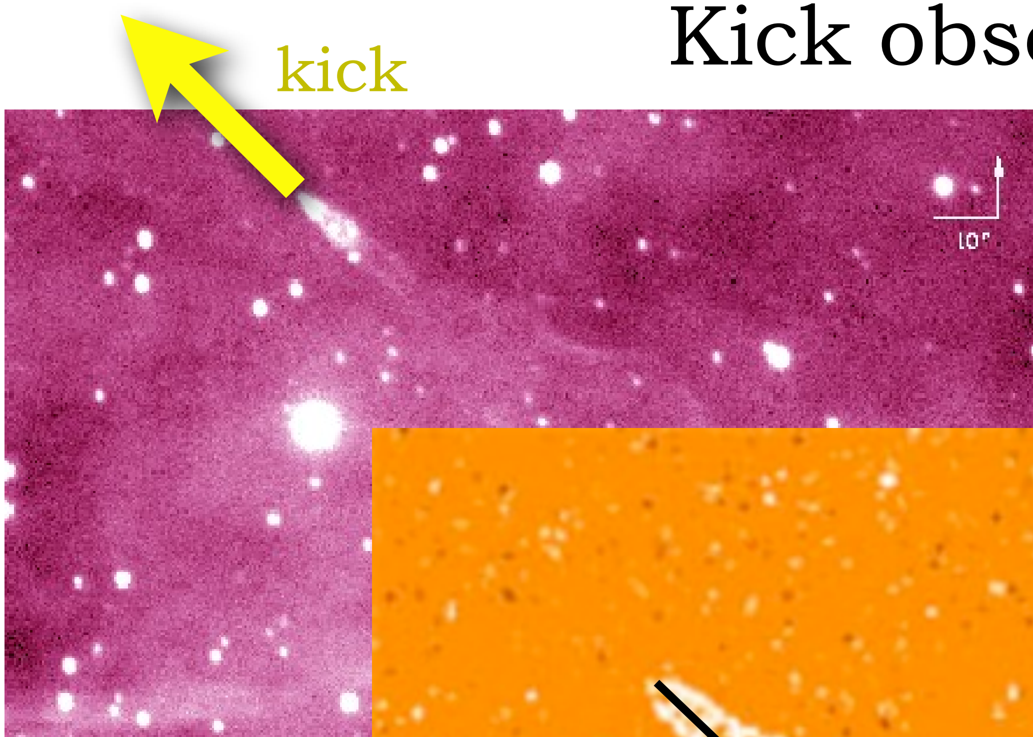

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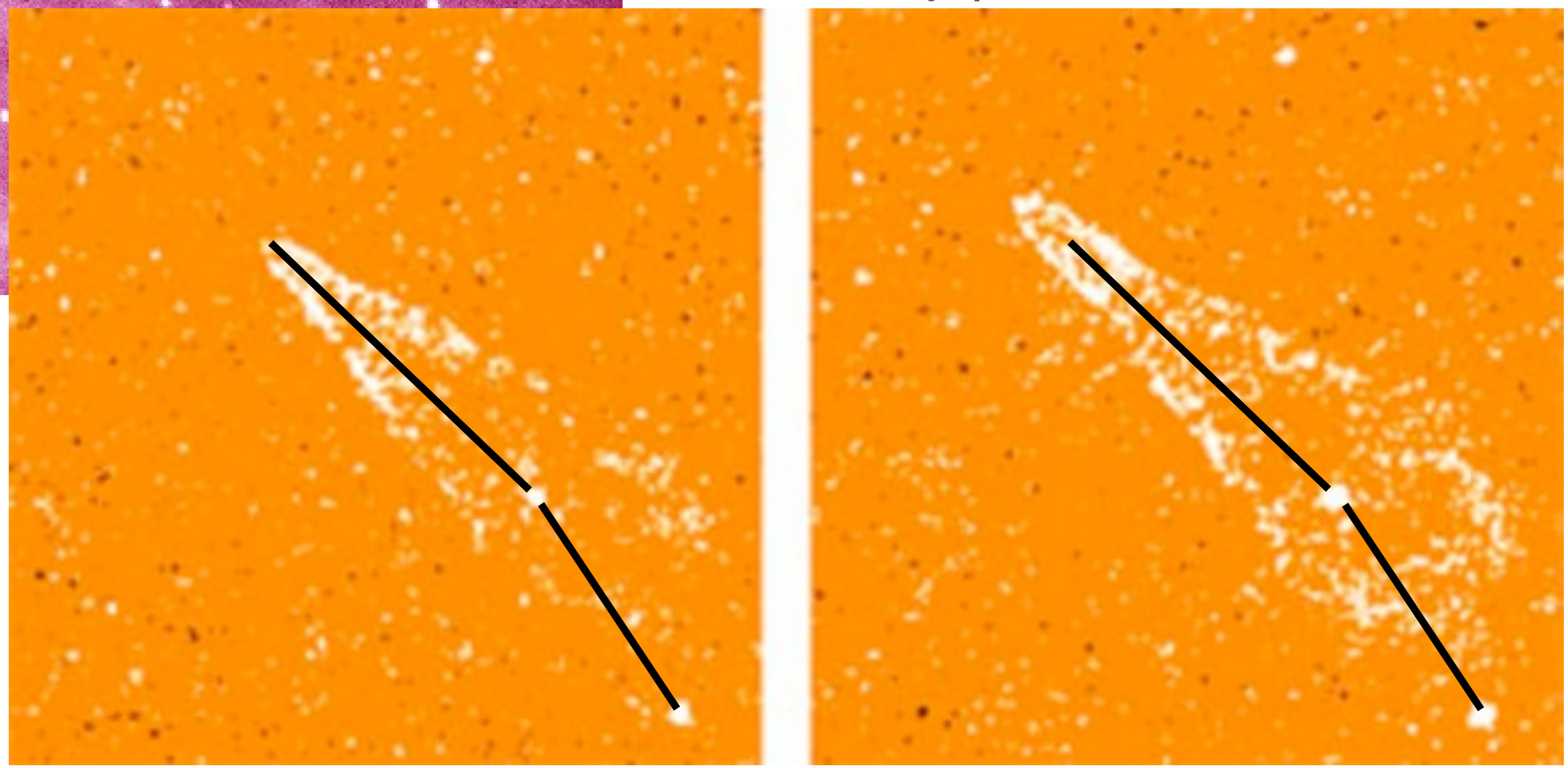

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Neutron star genesis

- compact star
  - * small radius
  - * large mass
  - * high density
- supernova remnant
- quick rotation
- large B field

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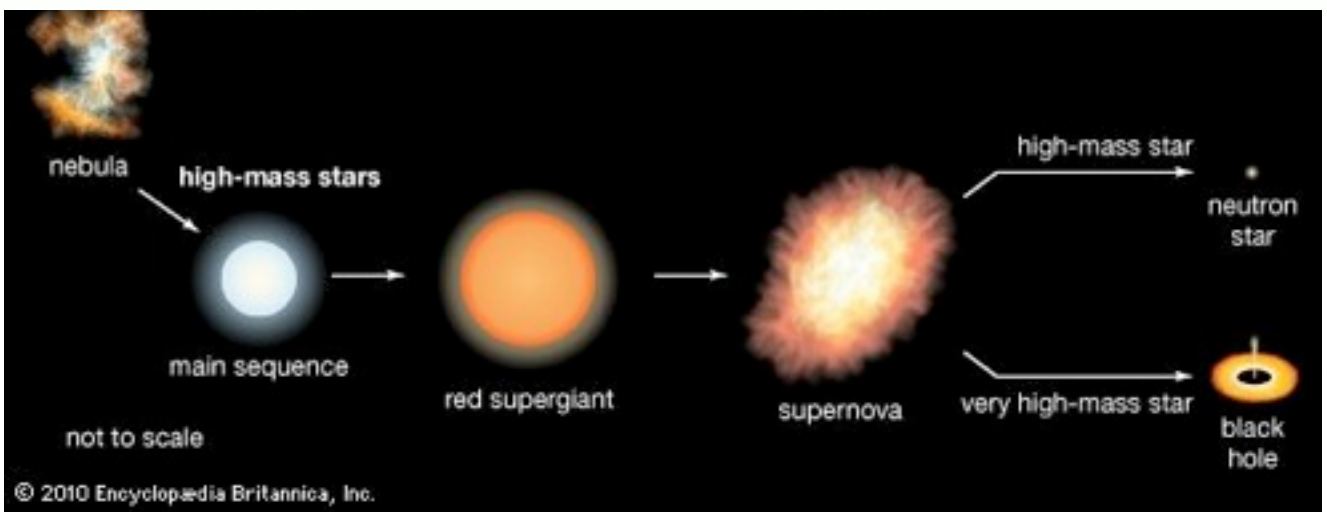

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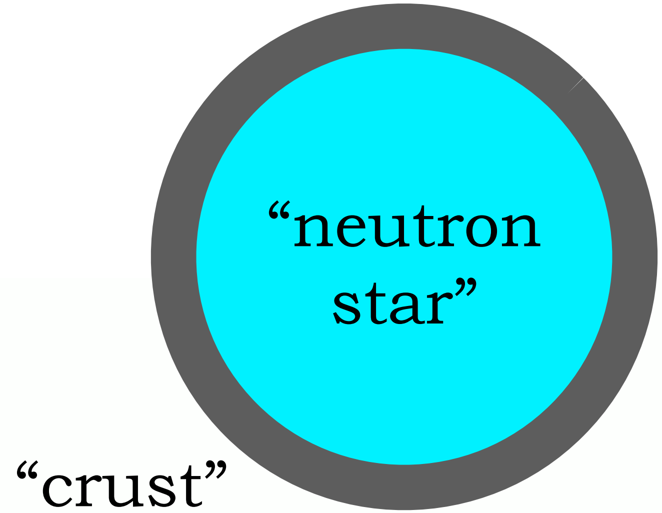

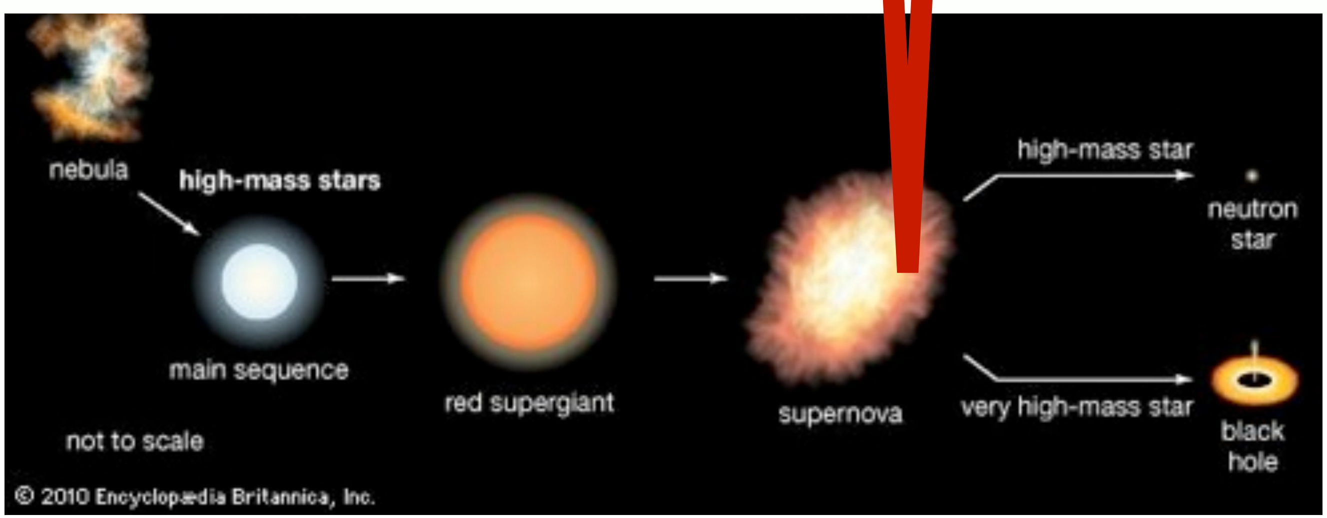

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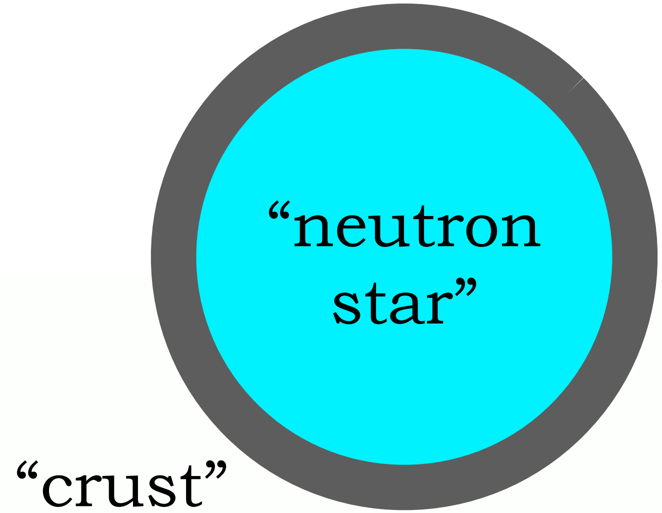

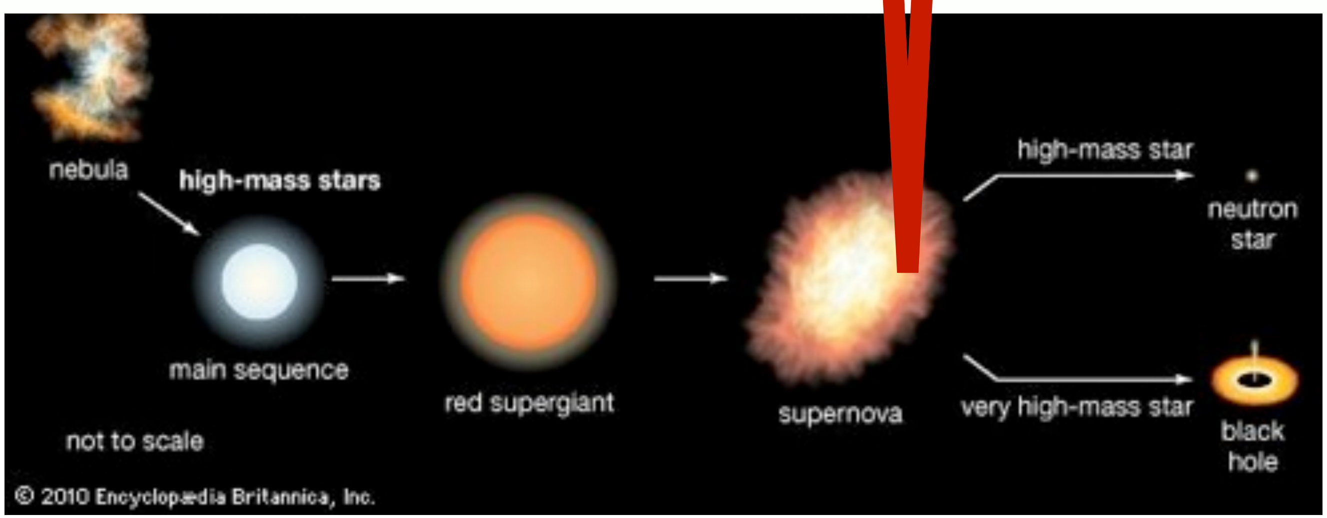

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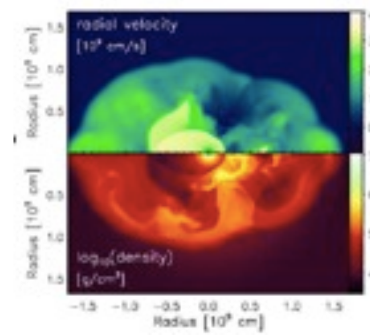
proto-neutron stars are dense objects with “crust” and preferred directions

2. Previous kick mechanisms



Two categories of kick mechanisms

Something has to cause an asymmetry in the momentum distribution.

1.) asymmetric supernova explosion

2.) asymmetric emission of matter

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[Scheck, Kifonidis, Janka, Muller; (2003)]
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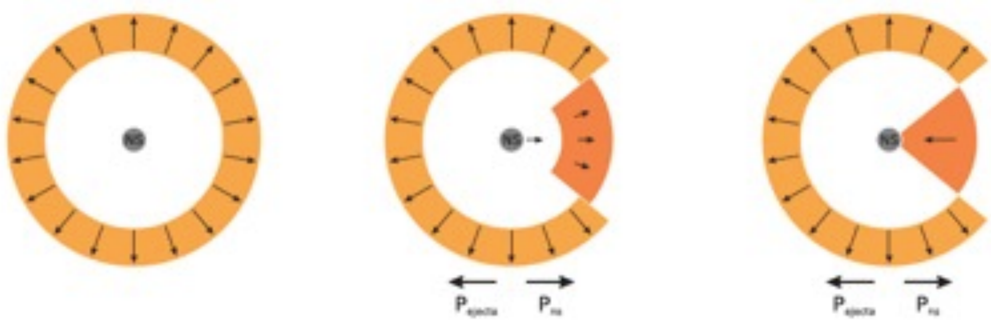
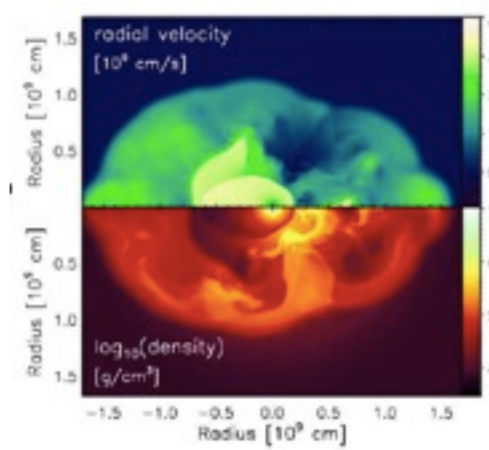
- kicks of about 1000 km/s
- random seed perturbations plus hydro instabilities (SASI)
- hydro model, neutrino transport, boundary cond's [Blondin et al; ('02)]
- timescale: ~5 seconds





2.) asymmetric emission of matter

- neutrino emission [Vilenkin (1978)]
- beyond the standard model [Fuller, Kusenko, Mocioiu, Pascoli (2003)]
- neutrino kicks, nucl-th [Sagert, Schaffner-Bielich (2007)]

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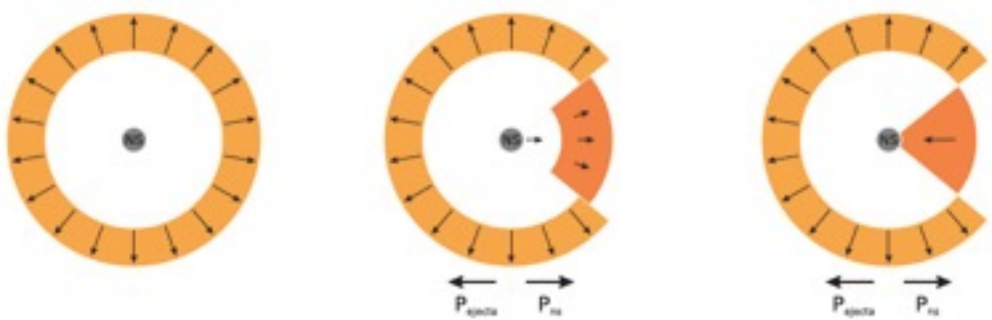

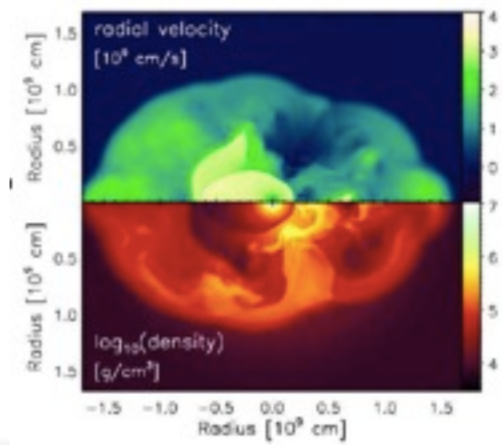

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Problems with previous kick mechanisms

ad 1.) asymmetric supernova explosion

- kick may be too small
- numerical analysis, no analytic understanding
- no magnetic dipole fields, no chiral hydro
- instabilities disturbed by other hydro effects?

ad 2.) asymmetric emission of matter

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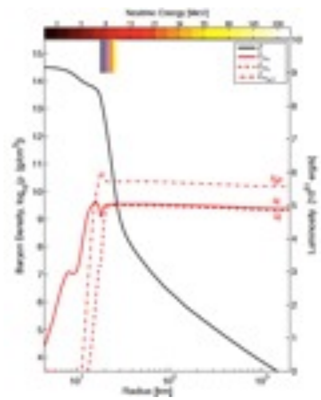
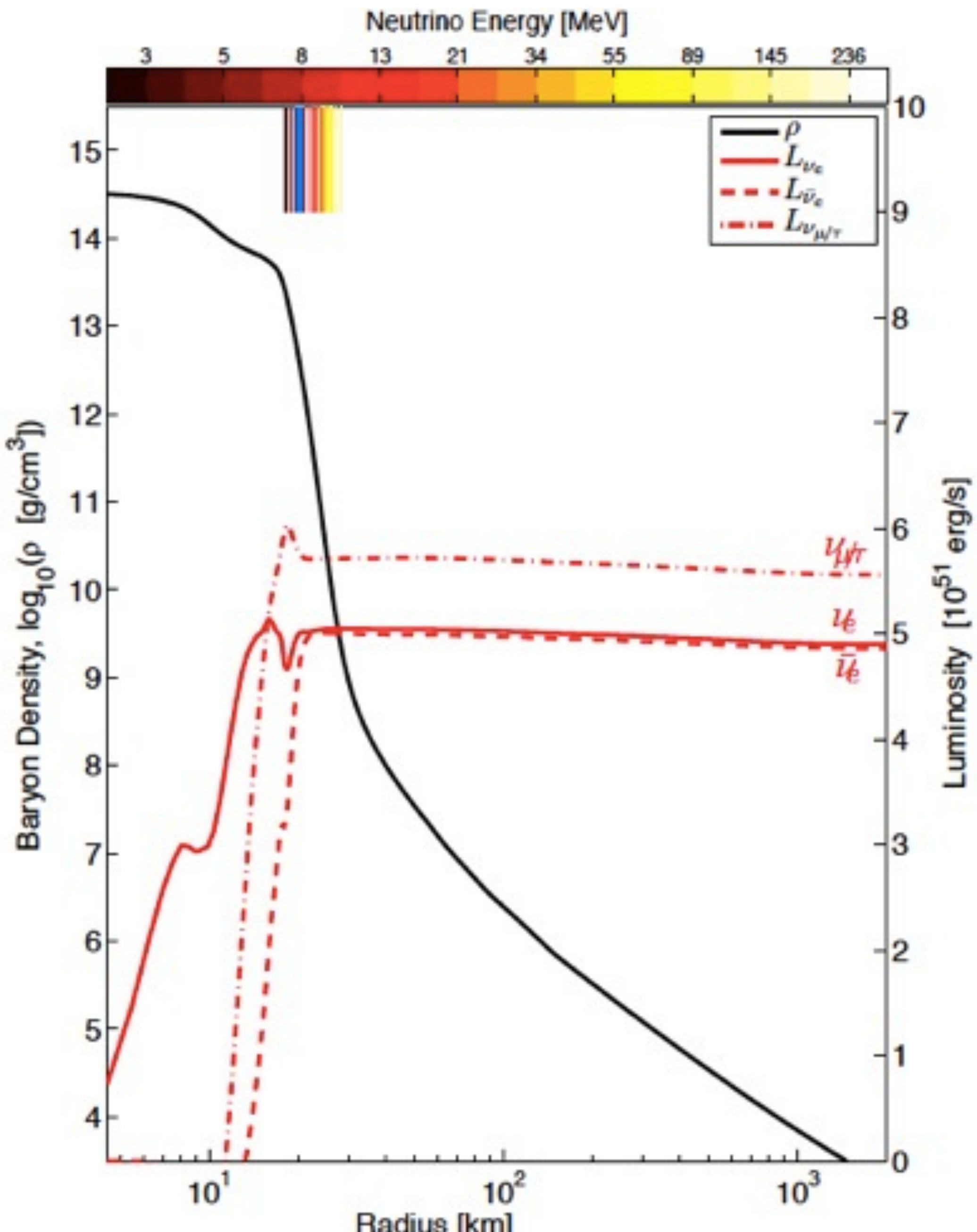
Our anomalous hydrodynamic formalism addresses and overcomes these problems.

3. Sophisticated simulations



First 10 seconds inside proto-neutron stars



cf. [Wongwathanarat, Janka, Muller; (2012)]

- baryonic matter: 10 km radius
- neutrinos: last scattering surfaces around 10 km
- no anti-neutrinos
- only electron flavor inside 10 km
- high densities
- neutrinos trapped! (everything trapped)

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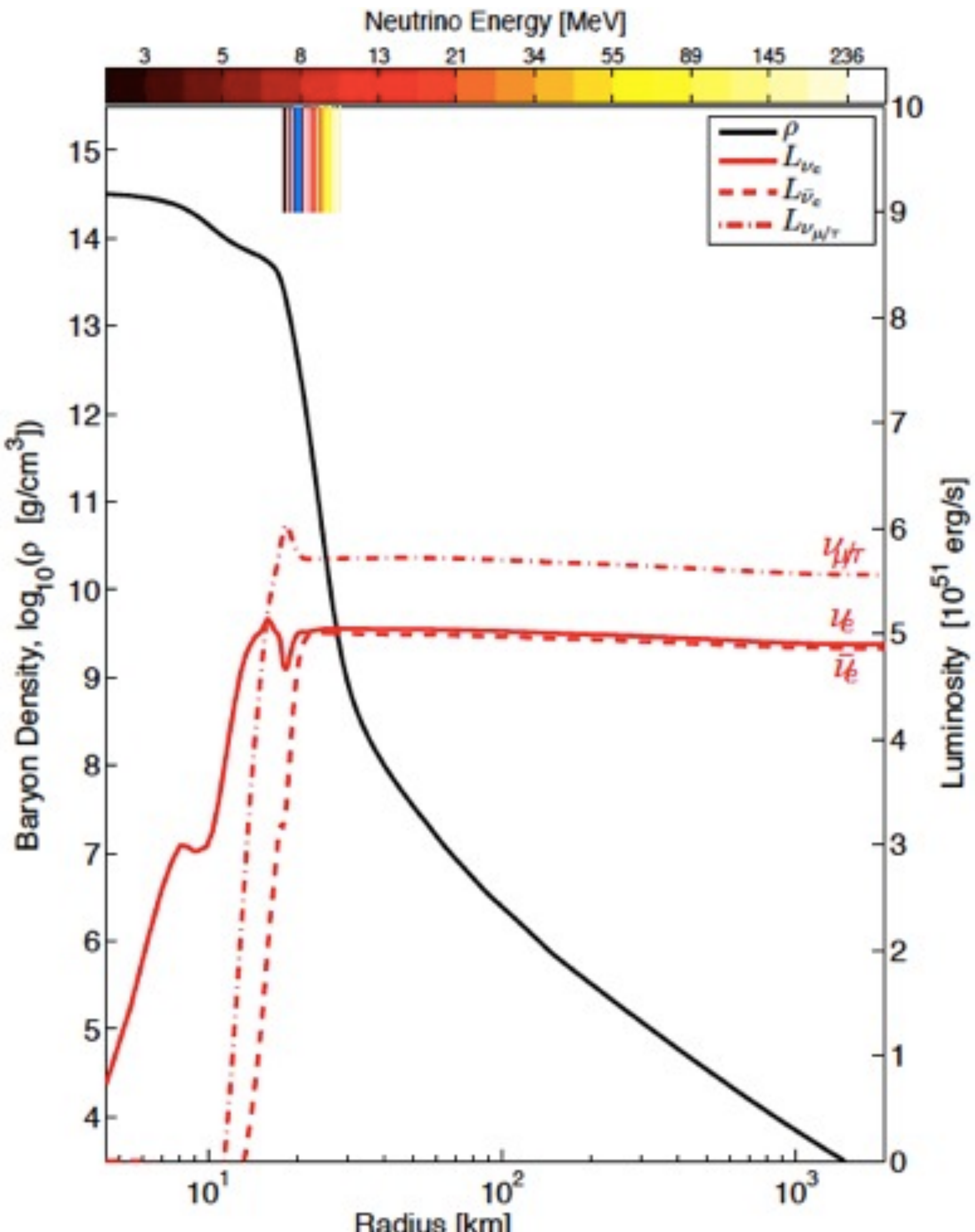

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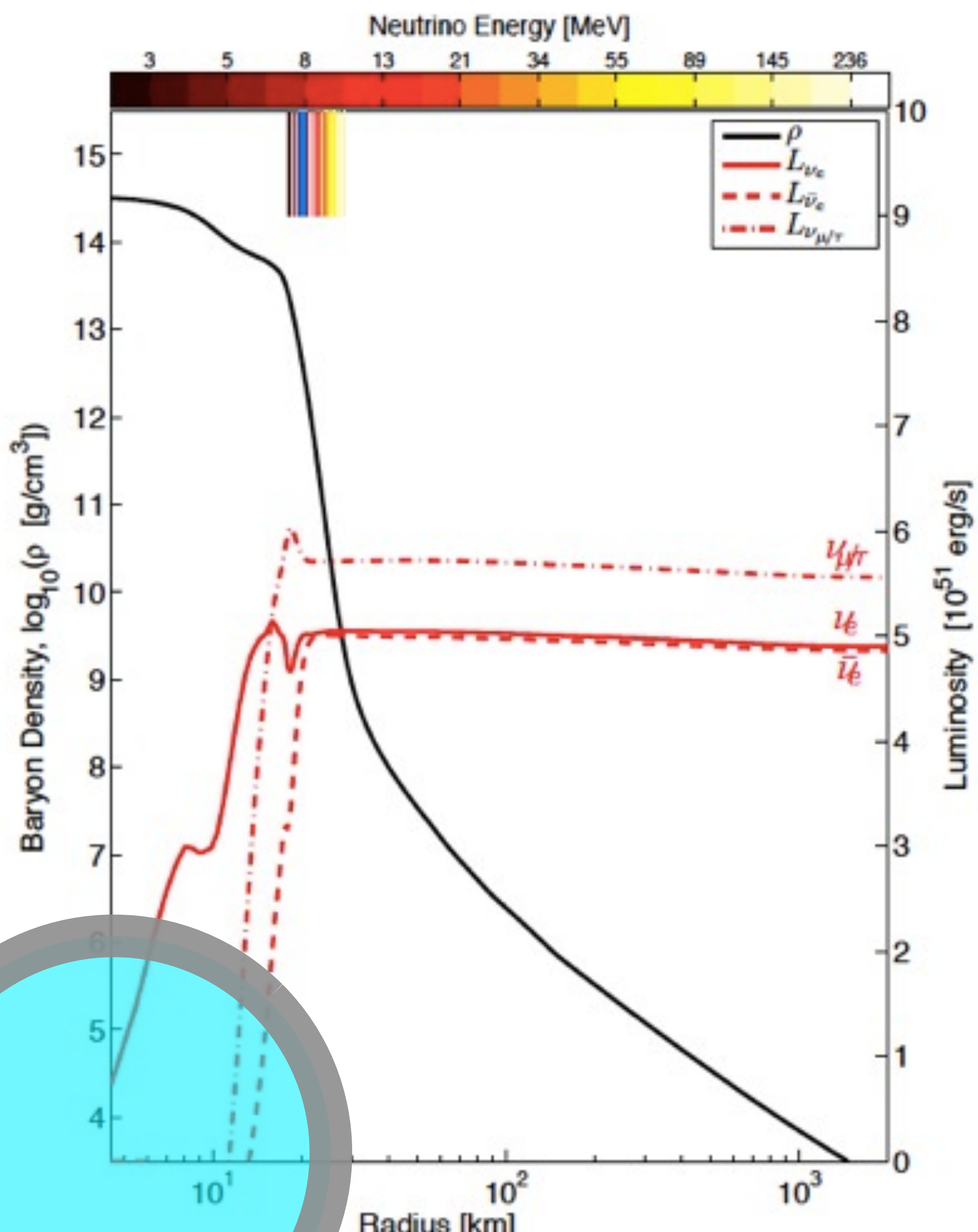

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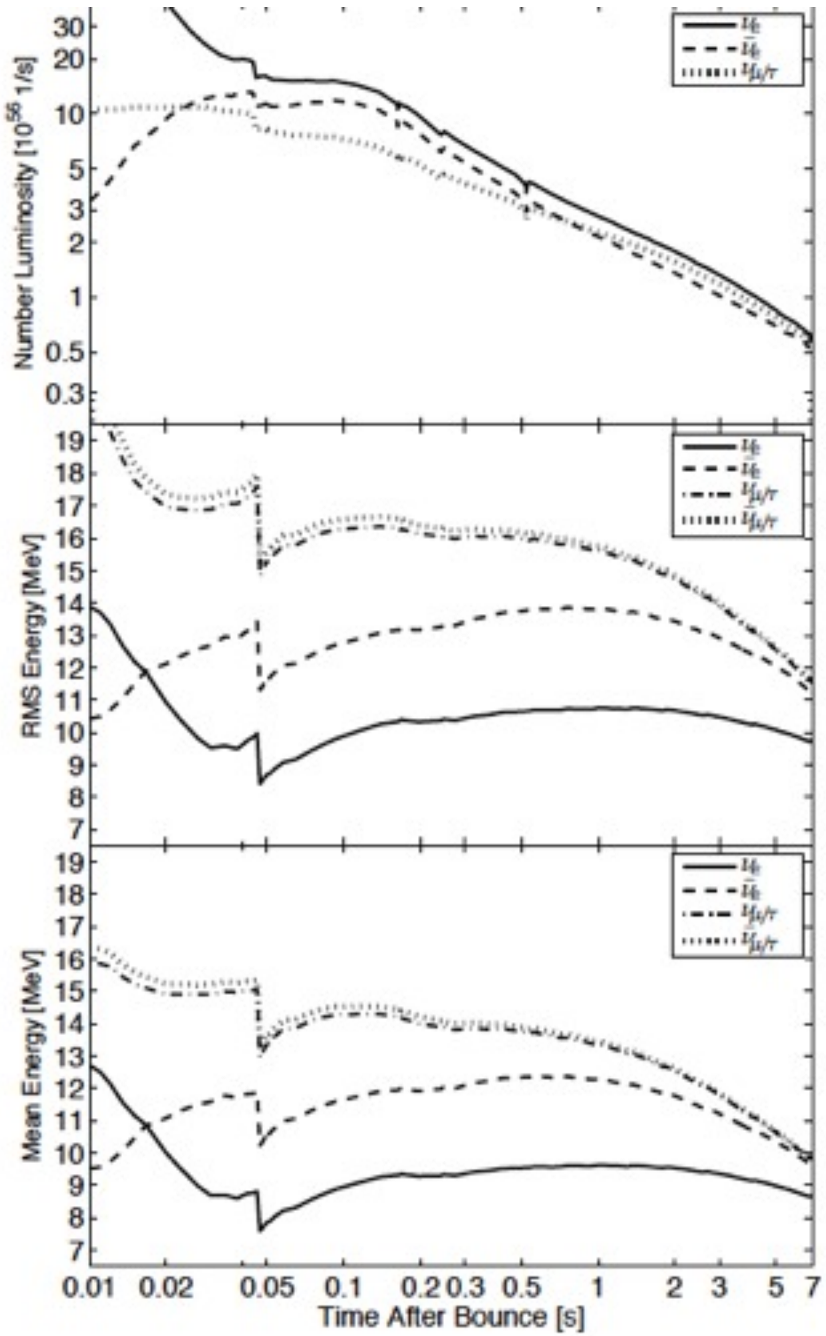
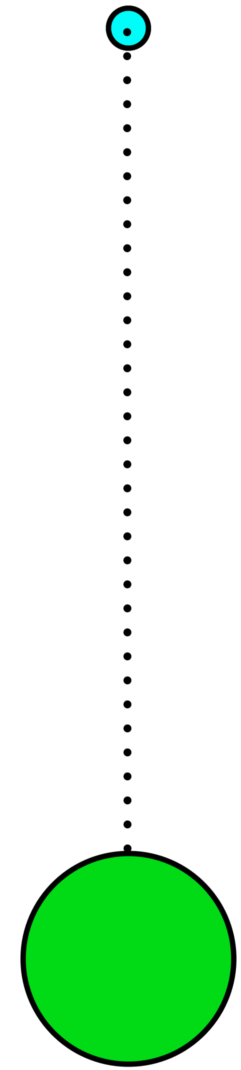
confirms our naive picture;
apply hydrodynamics!

We are not interested in signals at infinity





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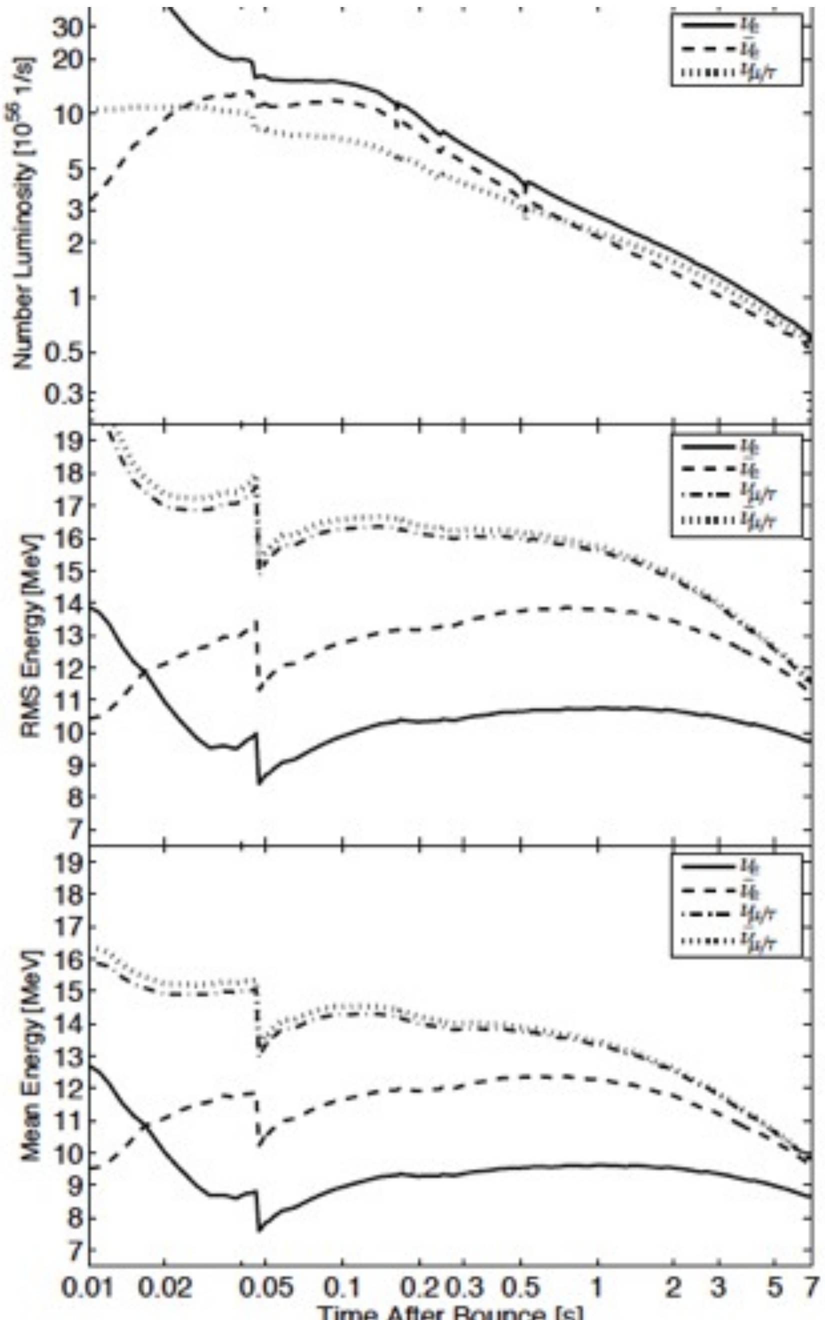
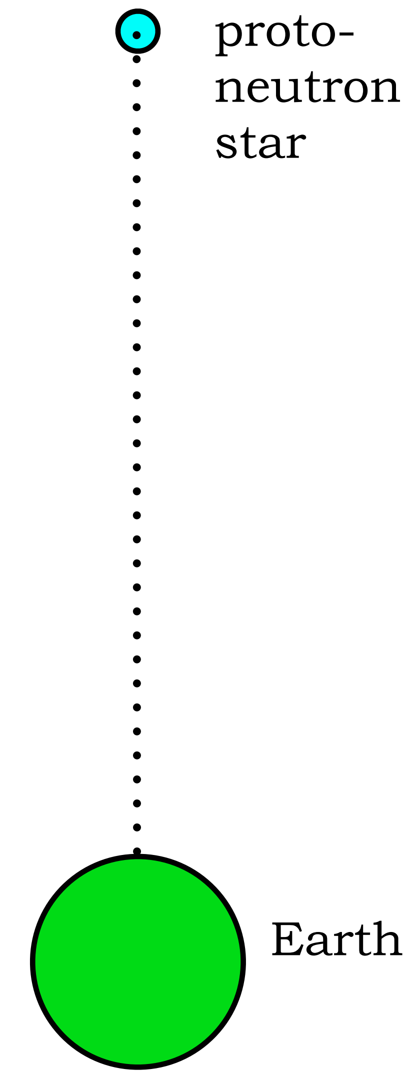


measured
far away
from star

[Fischer et al.; PRD (2011)]



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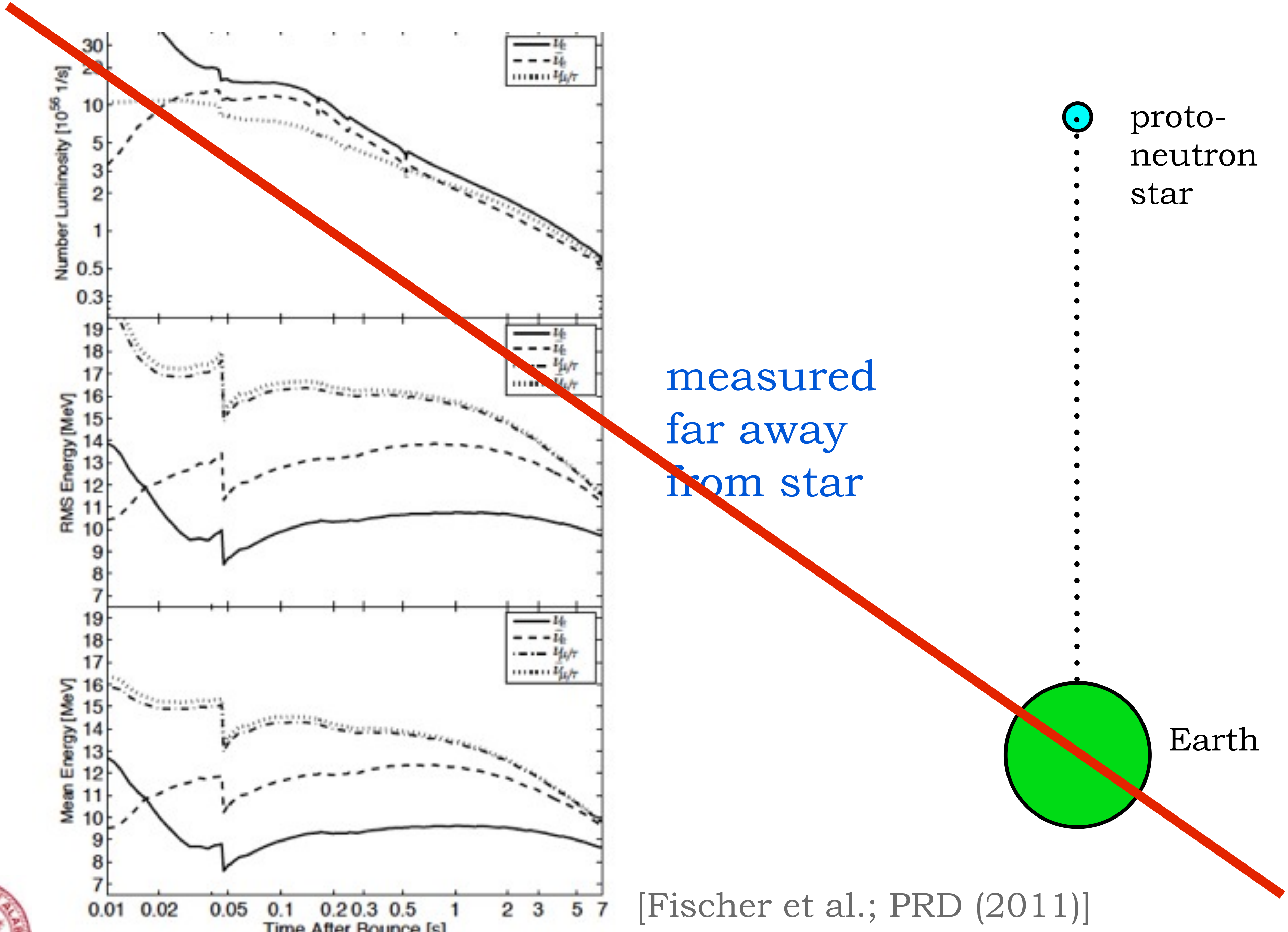

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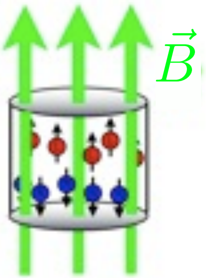

Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$

Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

Hydrodynamic variables

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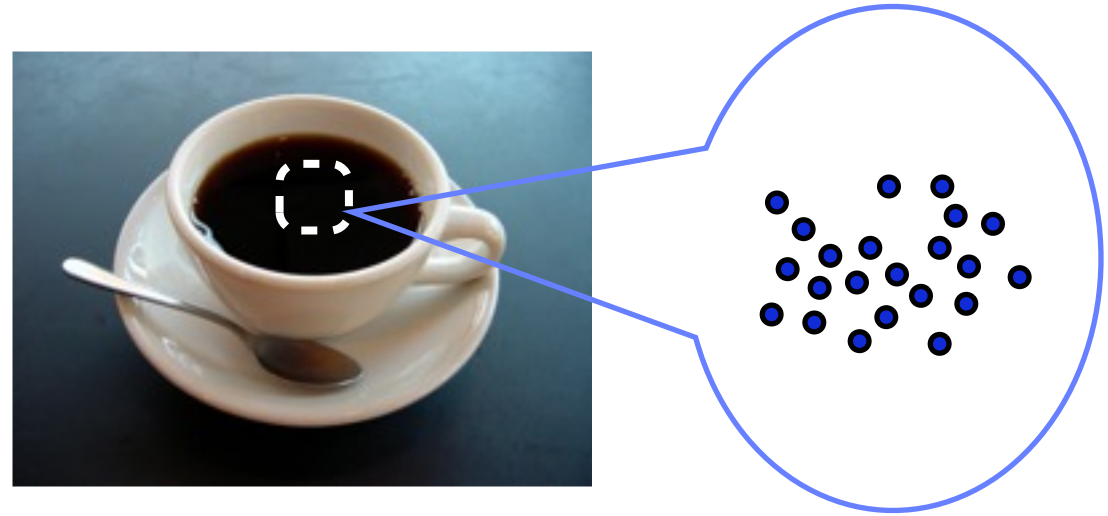

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thermodynamic variables:
temperature, chemical potential,
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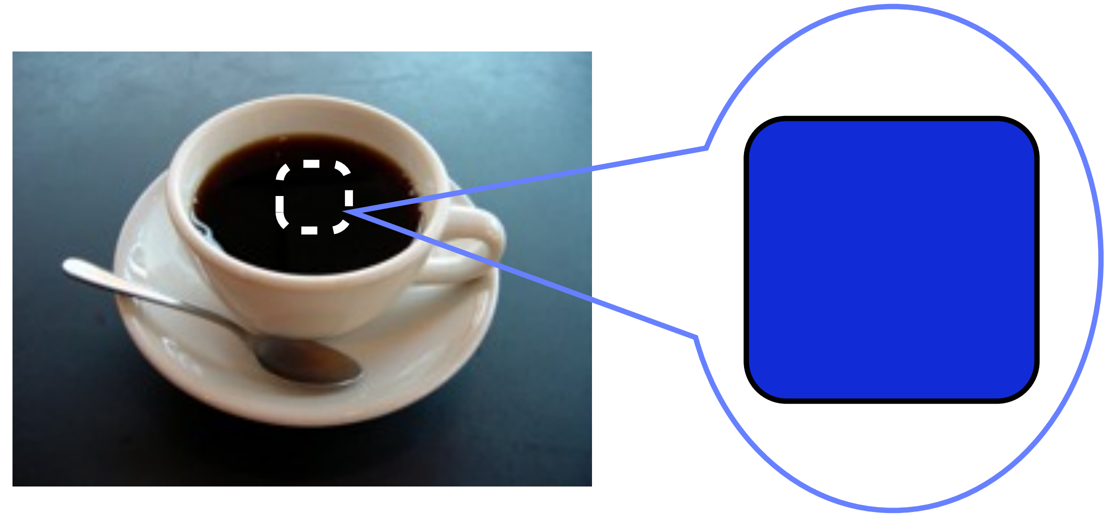

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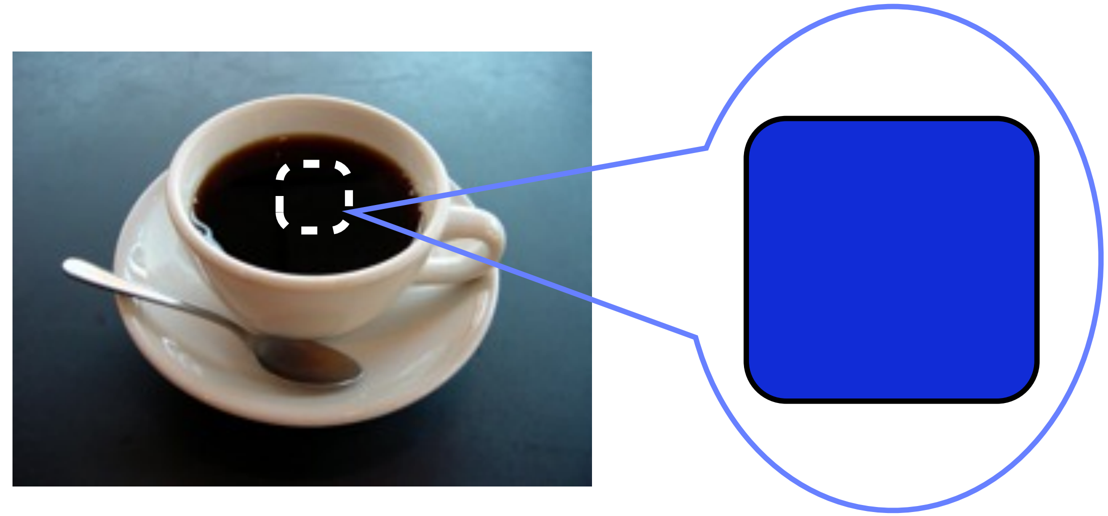

Hydrodynamics

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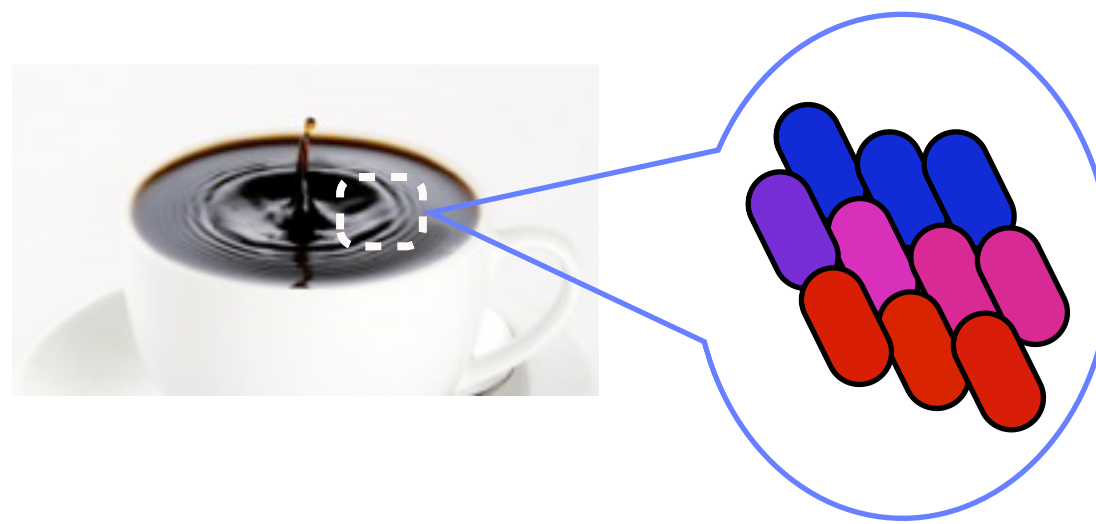
hydrodynamic fields



Hydrodynamics

universal effective field theory, expansion in gradients of temperature, chemical potential and velocity

- fields $T(x),\ \mu(x),\ u^\nu(x)$
- conservation equation

$$\nabla_\nu j^\nu = 0$$

- constitutive equation (Landau frame)

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Conserved current $$j^\mu = n u^\mu + \nu^\mu$$

Form can be derived and restricted from first principles.
[Landau, Lifshitz]

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]
[Loganayagam; arXiv (2011)]

Derived for any theory with chiral anomaly

(e.g. the standard model of particle physics)

$\nabla_\nu j^\nu = 0$ classical theory

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quantum theory

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quantum theory

Generalized constitutive equation with external fields:

$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \dots$$

j^μ	$n u^\mu$	σE^μ	$\sigma^B B^\mu$	$\sigma^V \omega^\mu$
(non) conserved current	(ideal) charge flow	conduc- tivity term	magnetic field term	vorticity term

Agrees with gauge/gravity prediction
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

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quantum theory

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$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \ldots$$

(non) conserved current | (ideal) charge flow | conductivity term | magnetic field term | vorticity term

Agrees with gauge/gravity prediction
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

Chiral magnetic conductivity:

$$\sigma^B = C\,\mu$$

anomaly-coefficient C

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]
[Loganayagam; arXiv (2011)]

Derived for any theory with chiral anomaly (e.g. the standard model of particle physics)

$$\nabla_\mu j^\mu = C\,\epsilon^{\nu\rho\sigma\lambda} F_{\nu\rho} F_{\sigma\lambda}$$

quantum theory

Generalized constitutive equation with external fields:

$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \ldots$$

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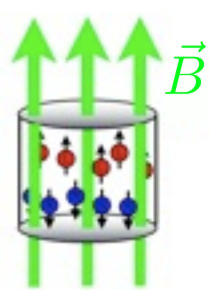


Observable in:
heavy ion collisions?
[Kharzeev, Son.; PRL (2011)]

neutron stars?
[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

Chiral effects in various currents

[Neiman, Oz; JHEP (2010)]

More than one anomalous current $\langle \partial_\mu J^\mu_a \rangle = \frac{1}{8} C_{abc} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma}$

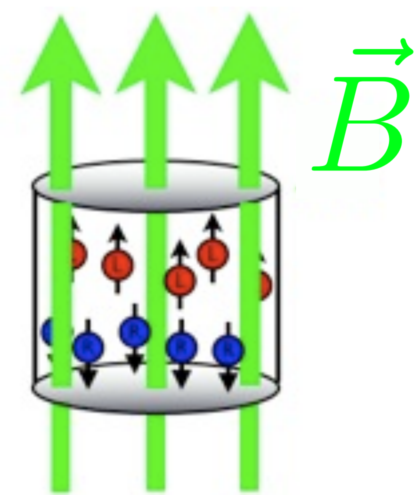
Constitutive relation:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V^\mu_b + \boxed{\sigma^V_a \omega^\mu + \sigma^B_{ab} B^{b\,\mu}} + \mathcal{O}(\partial^2)$$

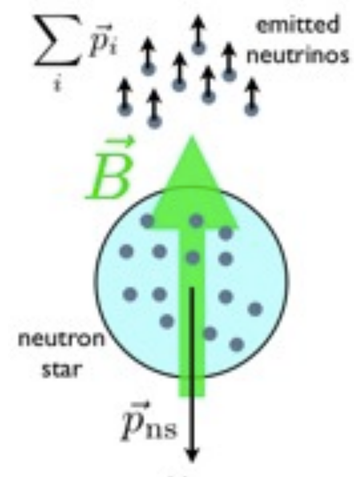
Chiral magnetic conductivity:

$$\sigma^B_{\boxed{ab}} = C_{abc}\, \mu^c$$

various charges
(e.g. lepton number,
electromagnetic charge, ...)

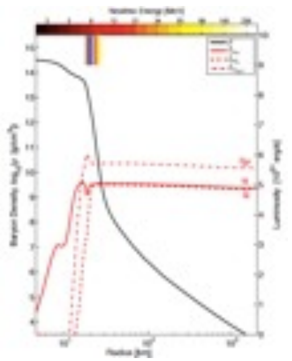


5. Kicks from hydrodynamics

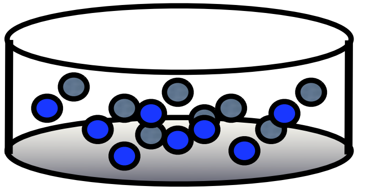


Relevant currents in neutron stars

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]



A bucket full of electrons and electron neutrinos with short mean free path



$B = 0.1\,\mathrm{MeV}^2$

$\mu^\ell \approx 300\,\mathrm{MeV}$

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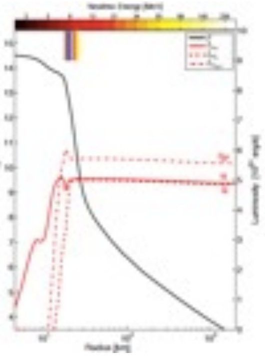

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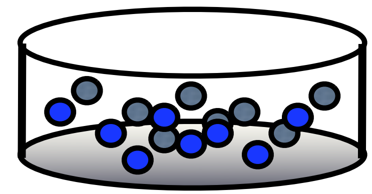

$B = 0.1\,\mathrm{MeV}^2$

$\mu^\ell \approx 300\,\mathrm{MeV}$

Microscopic (standard model) currents: axial/lepton/EM:

$$J^\mu_{\ell 5} = \bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R + \bar{\nu}_L \gamma^\mu \nu_L$$

$$J^\mu_{\ell} = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R + \bar{\nu}_L \gamma^\mu \nu_L$$

$$J^\mu_{EM}$$

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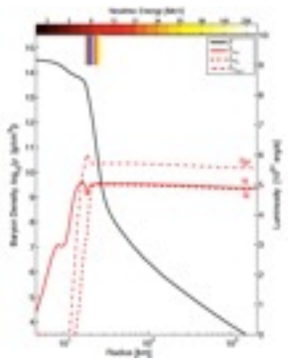

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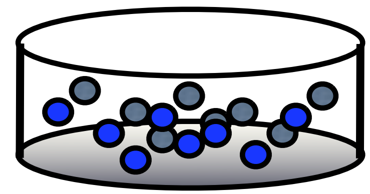

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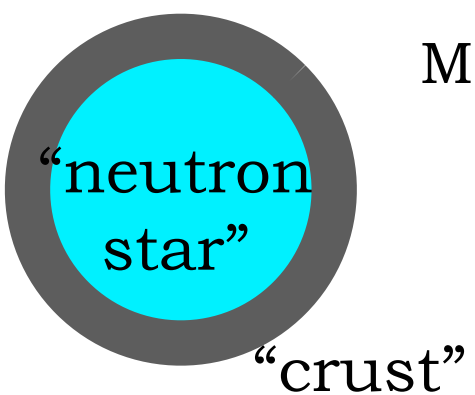
$$J^\mu_{EM}$$



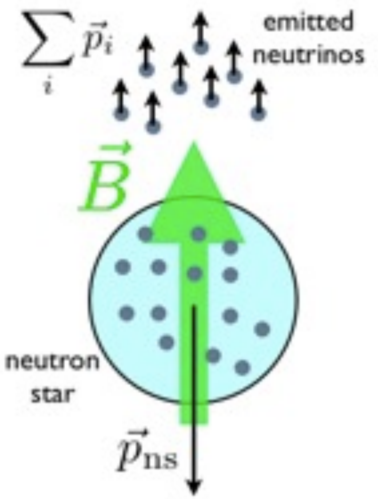
Macroscopic (hydrodynamic) description:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V_b^\mu + \sigma_a^V \omega^\mu + \boxed{\sigma^B_{ab} B^{b\mu}} + \mathcal{O}(\partial^2)$$

$$\sigma^B_{ab} = C_{abc}\,\mu^c$$

Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]



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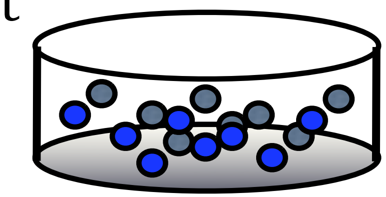

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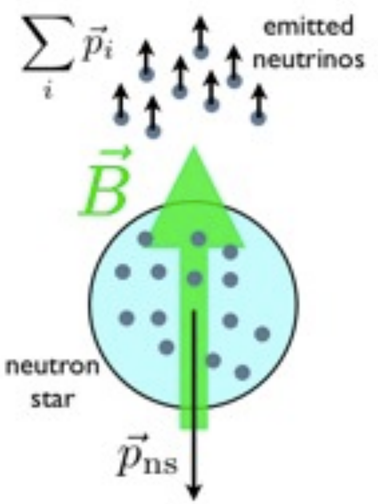

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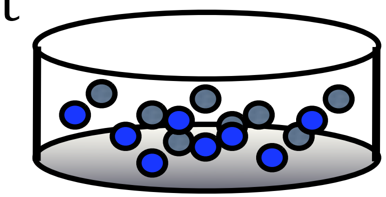

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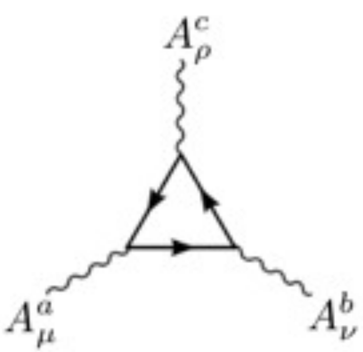

e.g.:

$$C_{\ell,\ell 5,EM} = \frac{1}{2\pi^2}$$

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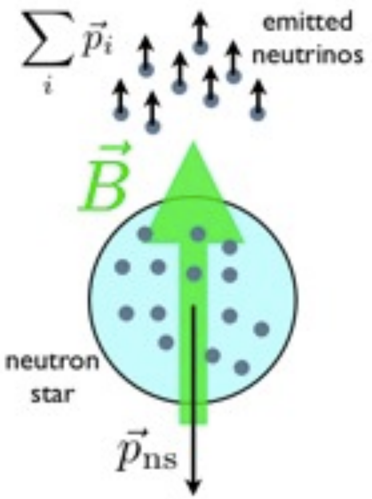

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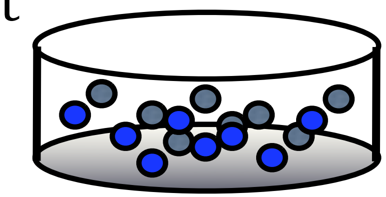

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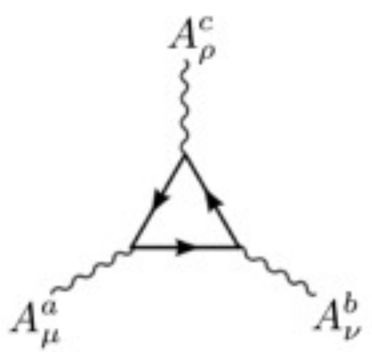

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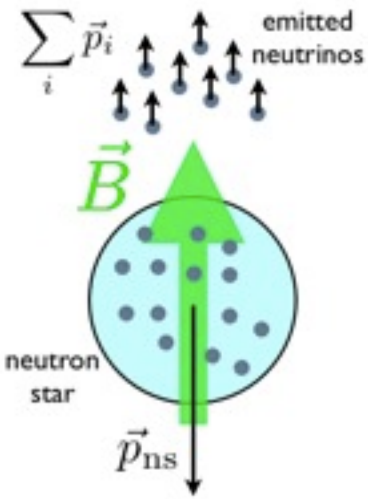

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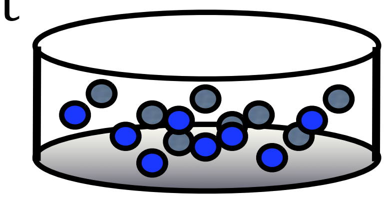

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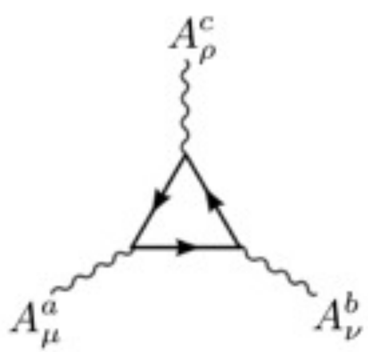

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$$\vec{J}_{\ell 5} = C\mu^\ell \vec{B} \approx \vec{e}_B \cdot 1\,\text{MeV}^3$$

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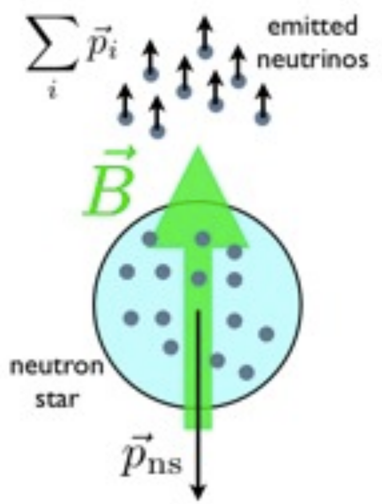

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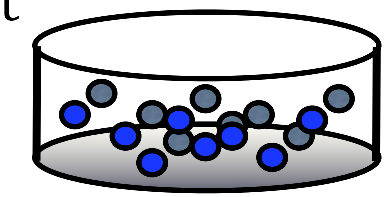

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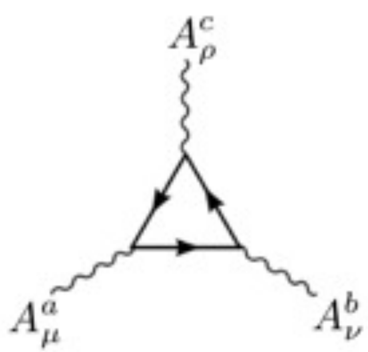

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Particle flux and momentum transfer:

$$\dot{N}_\nu = |\vec{J}|\,A_{\rm surface}$$

$$\Delta P_{\rm NS} = \Delta t \dot{N}_\nu \langle p_\nu \rangle.$$

$$\Delta t \approx 10\,\mathrm{s}$$

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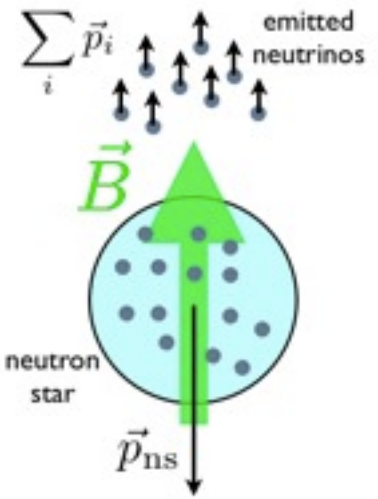
[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]



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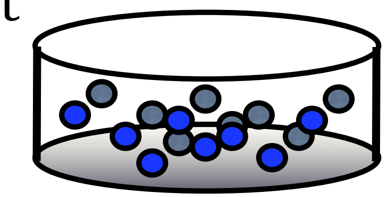

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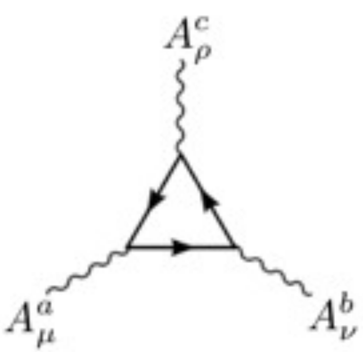

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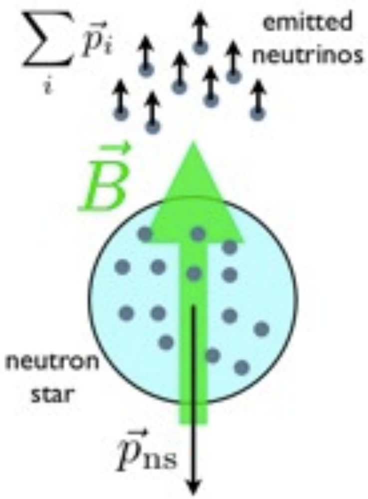

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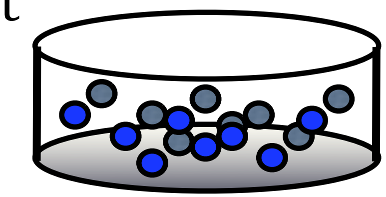

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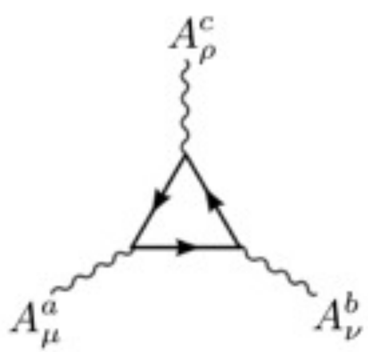

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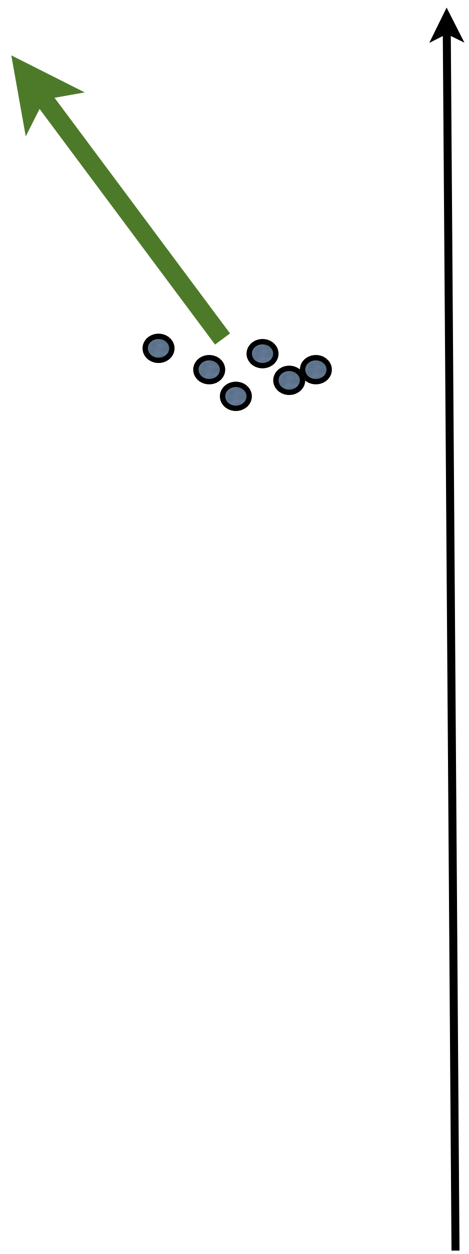
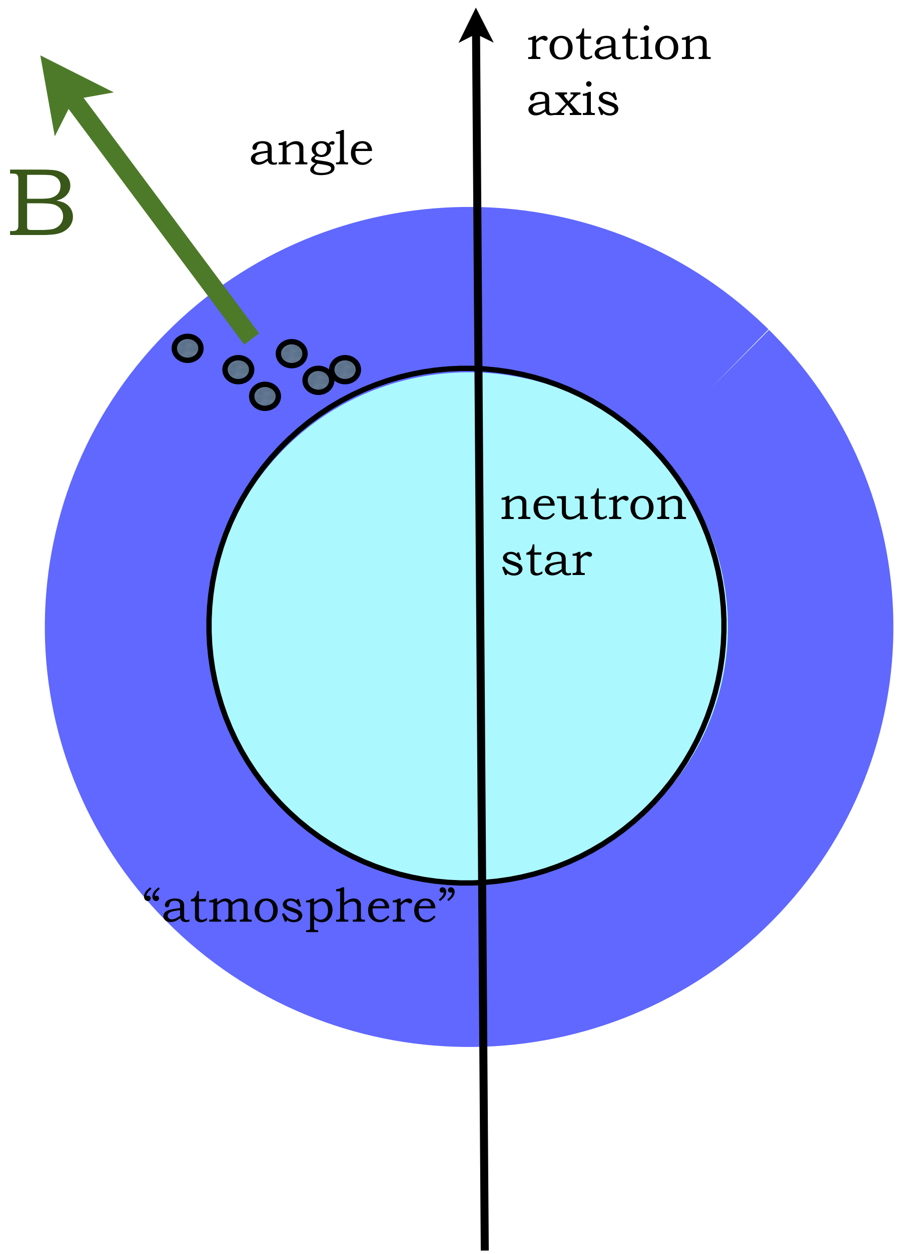
Kick velocity agrees with observations: $\Rightarrow \quad v_{\rm kick} \approx 1000 \dfrac{\rm km}{\rm s}$

6. Observable signals?

Observable signal?



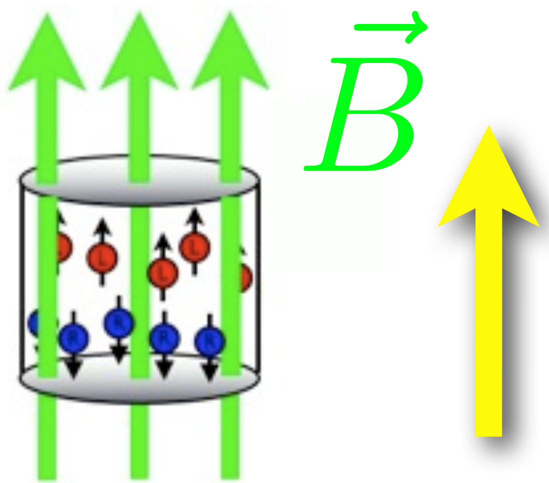
Observable signal?



Prediction: Kick magnitude depends on angle between rotation axis and internal magnetic field axis.

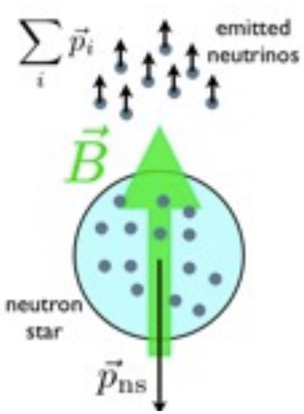
For fast spinning neutron stars, kick directed along rotation axis.

Summary



hydrodynamics: fluids with left-handed and right-handed particles produce a current along magnetic field

observation: neutron stars undergo a large momentum change (a kick)



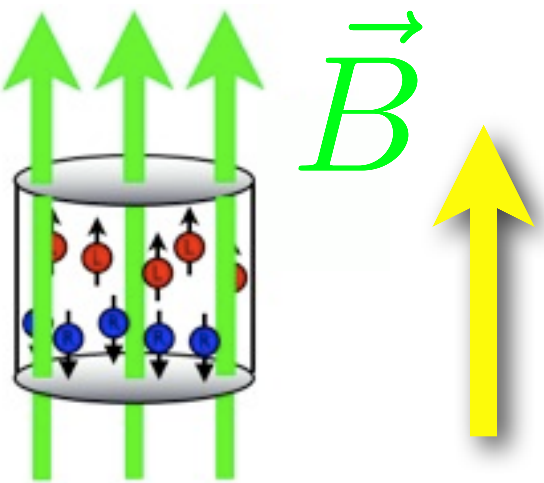
Anomalous hydrodynamics leads to neutron star kicks

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

Outlook

- check with simulation
- kick aligned with spin?
- determine B from kick
- measure gravitational anomaly?

Summary



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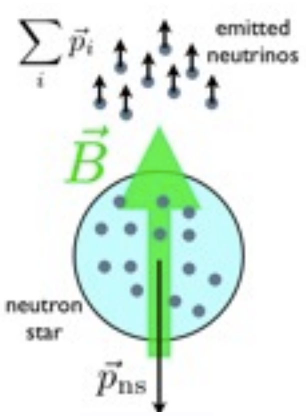

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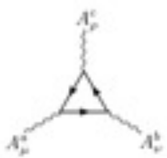

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further implications of chiral effects

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Holography Workshop & Summer School

http://bama.ua.edu/~mkaminski3/UA_Workshop_2015/Overview.htm

APPENDIX: chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD U(1))

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral
magnetic
effect

Axial current (e.g. QCD axial U(1))

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral
vortical
effect

chiral
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effect

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Axial current (e.g. QCD axial U(1))

$$J_A^\mu = \cdots + \underbrace{\xi \omega^\mu}_{\text{chiral vortical effect}} + \underbrace{\xi_B B^\mu}_{\text{chiral separation effect}} + \xi_{AA} B_A^\mu$$