Magnetic field effects on compact stars

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CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

FIAS Frankfurt Institute for Advanced Studies

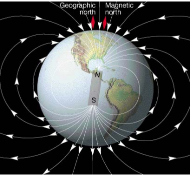
HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

DAAD

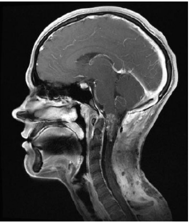
Plan of the talk

- Motivation
- Effects of magnetic field on the Equation of State
- Magnetized Neutron Stars: fully-general relativistic approach
  Langage Objet pour la RElativité NaumériquE (LORENE)
- Results
- Summary

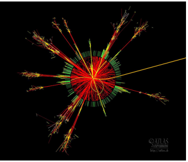
Motivation: magnetic fields



Earth: B$\sim$ 0.5 G

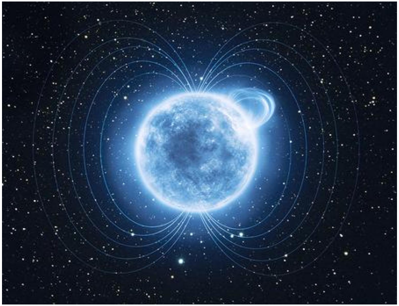


MR: B$\sim 10^3$ G



Atlas: B$\sim 10^{20}$ G

Neutron stars with a strong magnetic field:
Duncan and Thompson (1992), Thompson and Duncan (1996).

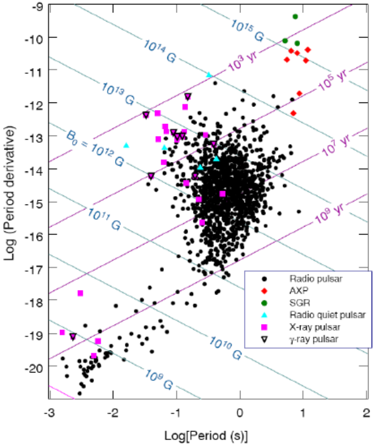


Typical NS: $\mathbf{B_s \sim 10^{12}}$ **G**
Magnetars: $\mathbf{B_s > 10^{14}}$ **G**

Motivation: magnetic fields

Surface magnetic field and at the pole:

$B_d = 3.2 \times 10^{19} \sqrt{P\dot{P}}\,\mathrm{G}$



Virial theorem: $\mathbf{B_c \sim 10^{18}}$ G

Origin?

The Ultimate Convection Oven

THIS PICTURE LEAVES a basic question unanswered: Where did the magnetic field come from in the first place? The traditional assumption was: it is as it is, because it was as it was. That

Duncan, Thompson, Kouveliotou

How to model highly magnetized stars

Einstein Equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Geometry

1. Spherical: TOV
2. Perturbation
3. Fully-GR

Energy Content

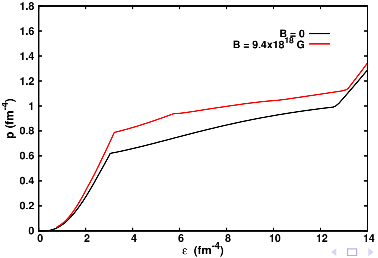
1. Matter: particles
2. Fields: magnetic field

Magnetized EoS

I. An extended hadronic and quark SU(3) non-linear realization of the sigma model that describes magnetized hybrid stars containing nucleons, hyperons and quarks. See, e.g. **Hempel M. at all (2013); Dexheimer V., Schramm S. (2008, 2010)**.
II. The anomalous magnetic moment of the hadrons.
III. Landau levels ν:

$$E^*_{i_{\nu s}} = \sqrt{k_{z_i}^2 + \left(\sqrt{M_i^{*2} + 2\nu|q_i|B} - s_i\kappa_i B\right)^2}$$

IV. Effect of B on the EoS:



Fully-General Relativistic Approach

• Stationary neutron stars with no magnetic-field-dependent EoS were studied by Bonazzola (1993), Bocquet (1995).
• magnetic fields effects in the EoS was presented in Chatterjee (2014), for a **quark EoS**.
• Our case: **nucleons, hyperons, mixed phase with quarks, AMM** of all hadrons (even the uncharged ones):

I. much more complex EoS
II. much higher magnetization

Mathematical setup

- The energy-momentum tensor: Chatterjee at all. 2014

$$
\begin{aligned}
T^{\mu\nu} = &(e+p)u^\mu u^\nu + pg^{\mu\nu} \\
&+ \frac{m}{B}\left(b^\mu b^\nu - (b\cdot b)(u^\mu u^\nu + g^{\mu\nu})\right) \\
&+ \frac{1}{\mu_0}\left(-b^\mu b^\nu + (b\cdot b)u^\mu u^\nu + \frac{1}{2}g^{\mu\nu}(b\cdot b)\right)
\end{aligned}
$$

where m and B are the lengths of the magnetization and magnetic field 4-vectors.

- In the rest frame of the fluid:

$$T^{\mu\nu} = \mathit{fluid} + \mathit{magnetization} + \mathit{field}\ (z\ \mathit{direction})$$

$$
T^{\mu\nu} = \begin{pmatrix}
e+\frac{B^2}{2\mu_0} & 0 & 0 & 0 \\
0 & p-mB+\frac{B^2}{2\mu_0} & 0 & 0 \\
0 & 0 & p-mB+\frac{B^2}{2\mu_0} & 0 \\
0 & 0 & 0 & p-\frac{B^2}{2\mu_0}
\end{pmatrix}
$$

Mathematical setup

- Stationary and axisymmetric space-time, the metric is written as:

$$ds^2 = -N^2 dt^2 + \Psi^2 r^2 \sin^2\theta (d\phi - N^\phi dt)^2 + \lambda^2 (dr^2 + r^2 d\theta^2)$$

where N^ϕ, N, Ψ and λ are functions of (r, θ).

- A poloidal magnetic field satisfies the circularity condition:

$$A_\mu = (A_t, 0, 0, A_\phi)$$

- The magnetic field components as measured by the observer $(\mathcal{O}_0)$ with n^μ velocity can be written as:

$$B_\alpha = -\frac{1}{2}\epsilon_{\alpha\beta\gamma\sigma} F^{\gamma\sigma} n^\beta = \left(0, \frac{1}{\Psi r^2 \sin\theta}\frac{\partial A_\phi}{\partial\theta}, -\frac{1}{\Psi \sin\theta}\frac{\partial A_\phi}{\partial r}, 0\right)$$

$$A_t, A_\phi \rightarrow Maxwell\ \ Equations.\ Static\, case:\ no\ \ electric\ \ field$$

3+1 decomposition of $T_{\mu\nu}$

- Total energy density (*fluid* + *field*): Chatterjee at all. 2014

$$E = \Gamma^2(e+p) - p + \frac{1}{2\mu_0}(B^i B_i)$$

- and the momentum density flux can be written as:

$$J_\phi = \Gamma^2(e+p)U + \frac{1}{\mu_0}\left(\frac{m}{B}B^i B_i U\right).$$

- 3-tensor stress components are given by:

$$S^r{}_r = p + \frac{1}{2\mu_0}(B^\theta B_\theta - B^r B_r) + \frac{2m}{B}\frac{B^\theta B_\theta}{\Gamma^2}$$
$$S^\theta{}_\theta = p + \frac{1}{2\mu_0}(B^r B_r - B^\theta B_\theta) + \frac{2m}{B}\frac{B^r B_r}{\Gamma^2}$$

$$S^\phi{}_\phi = p + \Gamma^2(e+p)U^2 + \frac{1}{2\mu_0}\left[B^i B_i + \frac{2m}{B}(1+\Gamma^2 U^2)\frac{B^i B_i}{\Gamma^2}\right]$$

with $\Gamma = (1-U^2)^{-\frac{1}{2}}$ the Lorenz factor and U the fluid velocity defined as:

$$U = \frac{\Psi r \sin\theta}{N}(\Omega - N^\phi)$$

- Remember: $p = p(h, B)$, with $h(r,\theta) := \ln\left(\frac{e+p}{m_b n_b c^2}\right)$

Field equations: our 4 unknowns N, N^ϕ, Ψ, λ

- **Einstein equations:** $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta_3 \nu = 4\pi G \lambda^2 \left(E + S_i^i\right) + \frac{\Psi^2 r^2 \sin^2\theta}{2N^2} (\partial N^\phi)^2 - \partial\nu \partial(\nu + \beta)$$

$$\tilde{\Delta}(N^\phi r \sin\theta) = -16\pi G \frac{N\lambda^2}{\Psi} \frac{J_\phi}{r\sin\theta} - r\sin\theta \partial N^\phi \partial(3\beta - \nu)$$

$$\Delta_2[(N\Psi - 1) r \sin\theta] = 8\pi G N \lambda^2 \Psi r \sin\theta (S_r^r + S_\theta^\theta)$$

$$\Delta_2(\nu + \alpha) = 4\pi G \lambda^2 (E + S_\phi^\phi) + \frac{\Psi^2 r^2 \sin^2\theta}{2N^2} (\partial N^\phi)^2 - \partial\nu \partial(\nu + \beta)$$

- **Definitions:**

$\nu = \ln N$, $\alpha = \ln\lambda$, $\beta = \ln\Psi$

$$\Delta_2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial^2\theta}\right)$$

$$\Delta_3 = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial^2\theta} + \frac{1}{r^2\tan\theta}\frac{\partial}{\partial\theta}\right)$$

$$\tilde{\Delta_3} = \Delta_3 - \frac{1}{r^2\sin^2\theta}$$

$E = E^{(PF)} + E^{(EM)}$

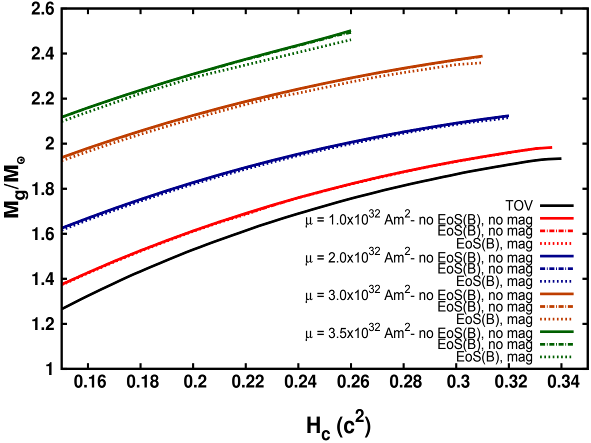
$S_i^i = S_i^{(PF)\,i} + S_i^{(EM)\,i}$ **(i = r, θ and ϕ)**

Structure of the star

- **Mass**
  $M = \int \lambda^2 \Psi r^2 \times \left[N(E+S) + 2N^{\phi}\Psi(E+p)Ur\sin\theta \right] \sin\theta dr d\theta d\phi$

- **Circumferential Radius**
  $R_{circ} = \Psi(r_{eq}, \frac{\pi}{2}) r_{eq}$

Increasing of the mass due to the magnetic field and effect of EoS(B) and magnetization *m*



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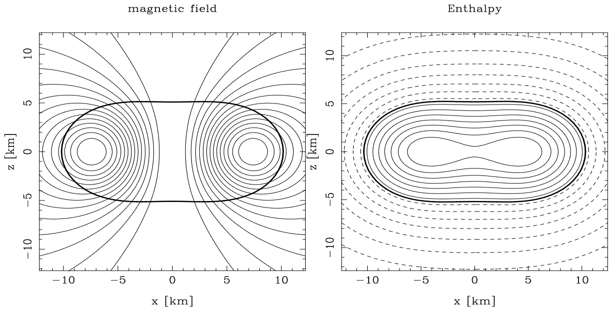
→ **Very small reduction** of stellar masses due to magnetization (negative sign in $T^{\mu\nu}$).

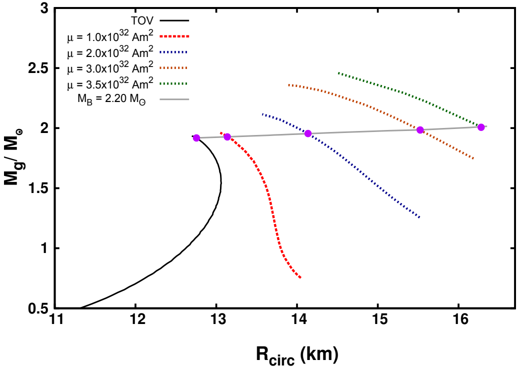
→ Effect on the maximum mass through the effect on the equation of state is **negligible**.

Deformation due to the magnetic field



→ The maximum mass for the value $\mu = \mathbf{3.5} \times \mathbf{10^{32}}\ \mathrm{Am}^2$.
→ It corresponds to a central enthalpy of $H_c = 0.26\, c^2$ ($\mathbf{n} = \mathbf{0.463}\ \mathrm{fm}^{-3}$).
→ The gravitational mass obtained for the star is **2.46** $\mathrm{M}_\odot$ for a central magnetic field of **1.62** $\times$**10**$^{\mathbf{18}}$ **G**.
→ The ratio between the magnetic pressure and the matter pressure in the center for this star is **0.793**.

Mass-Radius Diagram for different fixed magnetic moments μ



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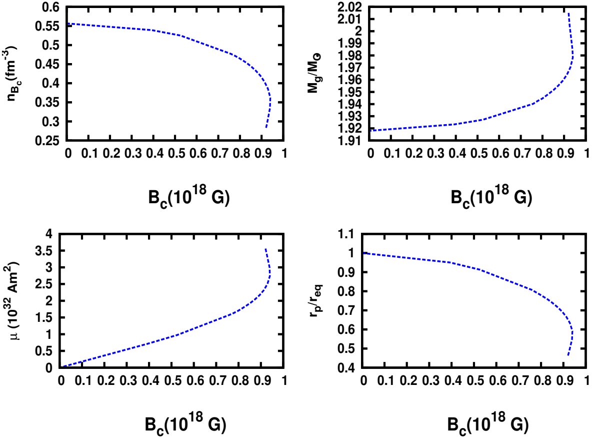
→ Effects of the magnetic field into the equation of state and the magnetization are also **included**.

→ The gray line shows an equilibrium sequence for a fixed baryon mass of $2.2\,\mathrm{M}_\odot$.

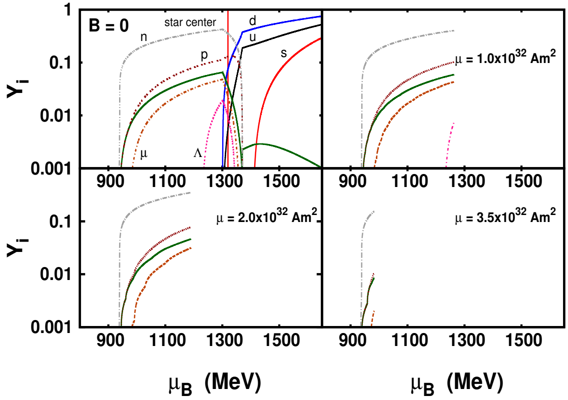
→The full **purple circles** represent a possible **evolution** from a highly magnetized neutron star to a non-magnetized and spherical star.

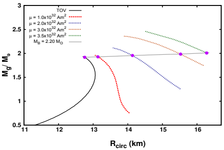
Global Quantities for a star with fixed $\mathrm{M_B} = 2.20\,\mathrm{M_\odot}$



→ Change in behaviour for $\mathbf{B_c \sim 0.9 - 1.0 \times 10^{18}}$ **G**. At this point, the magnetic force has pushed the matter off-center and a **topological change to a toroidal configuration** can take place Cardall (2001).
→ The ratio between the polar and the equatorial radii can reach **50%** for a magnetic field strength of $\sim \mathbf{1 \times 10^{18}}$ **G** at the center.

Population change for a star with $M_B = 2.20\,M_\odot$





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→ As one increases the magnetic field, the **particle population** changes inside the star.

→ These stars are represented in MR diagram by the full purple circles.

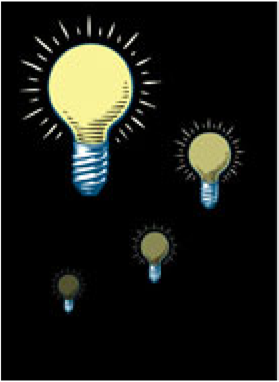
→ Younger stars that possess strong magnetic fields might go through a **phase transition** later along their evolution, when their central densities increase enough for the **hyperons and quarks** to appear.

Properties of White Dwarfs

→ The **sizes** are the size of the planet Earth
→ **Densities** $10^{5-9} \mathrm{g/cm^3}$
→ Typical **composition** : C and/or O
→ Gravity is balanced by the **electron degeneracy pressure**
→ The masses are up to **1.4 Msun, the Chandrasekhar limit**

Progenitors of **Type Ia supernovae**: Chandrasekhar White Dwarfs

Standard candles



EXPANSION OF THE UNIVERSE 2011

Saul Perlmutter
Brian P. Schmidt
Adam G. Riess

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

Properties of White Dwarfs

→ But, motivated by **observations** of supernova that appears to be **more luminous** than expected (e.g. SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc), it has been argued that the **progenitor** of such super-novae should be a white dwarf with mass above the well-known Chandrasekhar limit: **2.0 - 2.8 Msun** .

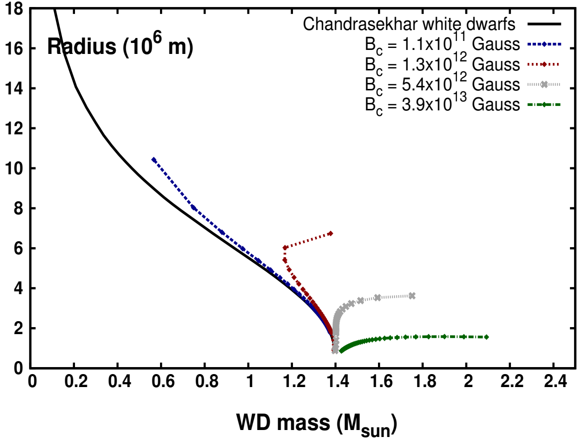
→ Several magnetized WDs discovered with **surface fields of** $10^5 - 10^9$ **G**

→ For a typical white dwarf: **Bmax** $\sim 10^{13}$ **G**

→ It has been suggested that strongly magnetized white dwarfs can **violate the Chandrasekhar mass limit** significantly (Kundu, Mukhopadhyay 2012)

- The new mass limit could explain **super-luminous Type Ia supernovae** from exploding white dwarfs

Mass-radius diagram for magnetized white dwarfs



Franzon, B. ; Schramm, S. 2015, Physical Review D, 92, 083006

→ **Magnetic field** effects can considerably increase the star masses and, therefore, might be the **source of superluminous SNIa.**

Summary

- Self-consistent stellar model including a poloidal magnetic field
- Effects of the magnetic field on the equation of state, including the magnetization.
- Leading contribution to the macroscopic properties of stars, like mass and radius, comes from the pure field contribution of the energy-momentum tensor.
- Assuming that the magnetic field decays over time, stars would not only become less massive and smaller over time, but also go through phase transitions to more exotic phases.
- Observables: distinct change in the **cooling** and stellar **braking index: in preparation**.
- Magnetic field effects can considerably increase WD masses

Thank you!