Viscosity in General Relativity

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AstroCoffee,
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Mathematically, the theory is very rich from both its analytic and geometric points of view. Over the past few decades, the subject of mathematical general relativity has matured into an active and exciting field of research among mathematicians.

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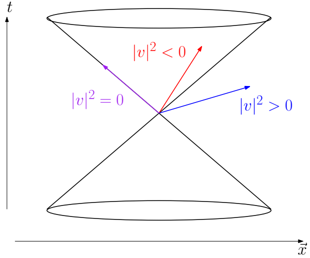

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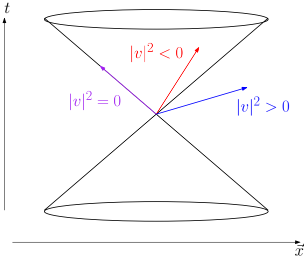

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Causality: The four-velocity of any physical entity satisfies $|v|^2 = \eta_{\alpha\beta} v^\alpha v^\beta \leq 0$. "Nothing propagates faster than the speed of light."

From special to general relativity

In general relativity the metric η is no longer fixed but changes due to the presence of matter/energy: $\eta_{\alpha\beta} \to g_{\alpha\beta}(x)$, where $x = (x^0, x^1, x^2, x^3)$ are space-time coordinates.

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The values of $g_{\alpha\beta}(x)$ depend on the matter/energy near x. Thus distances and lengths vary according to the distribution of matter and energy on space-time. This distribution, in turn, depends on the geometry of the space-time, i.e., it depends on $g_{\alpha\beta}(x)$.

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The Ricci and scalar curvature

The Ricci curvature of g is

$$R_{\alpha\beta} = g^{\mu\nu}\left(\frac{\partial^2 g_{\alpha\beta}}{\partial x^\mu \partial x^\nu} + \frac{\partial^2 g_{\mu\nu}}{\partial x^\alpha \partial x^\beta} - \frac{\partial^2 g_{\alpha\nu}}{\partial x^\mu \partial x^\beta} - \frac{\partial^2 g_{\mu\beta}}{\partial x^\alpha \partial x^\nu}\right) + F_{\alpha\beta}(g, \partial g).$$

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Thus, Einstein's equations are a system of second order partial differential equations for $g_{\alpha\beta}$ (and whatever other fields come from $T_{\alpha\beta}$).

Coupling gravity and matter

Consider Einstein's equations

$$(\star)\begin{cases} R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} = T_{\alpha\beta}, \\ \nabla^{\alpha} T_{\alpha\beta} = 0, \end{cases}$$

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Matter fields = everything that is not gravity.

To couple Einstein's equations to any matter field, all we need is $T_{\alpha\beta}$.

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Here, u is a (time-like) unit (i.e., $|u|^2 = g_{\alpha\beta} u^\alpha u^\beta = -1$) vector field representing the four-velocity of the fluid particles; p and ϱ are real valued functions describing the pressure and energy density of the fluid.

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The system is closed by an equation of state: $p = p(\varrho)$.

Causality in general relativity

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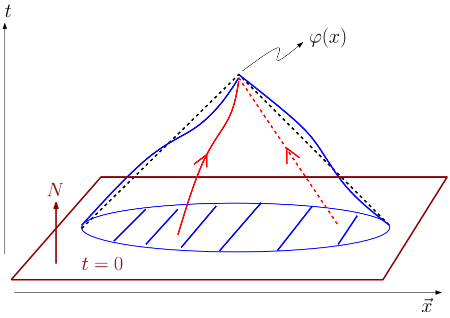
Note that that the causal structure is far more complicated than in Minkowski space since $g_{\alpha\beta} = g_{\alpha\beta}(x)$. One can better formulate causality in terms of the domain of dependence of solutions to Einstein's equations:



Causality in GR.

A theory is causal if for any field φ its value at x depends only on the "past domain of dependence of x."

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The introduction of fluids with viscosity in general relativity is well-motivated from a physical perspective:

- Real fluids have viscosity.
- Cosmology. Perfect fluids exhibit no dissipation. Maartens ('95): "The conventional theory of the evolution of the universe includes a number of dissipative processes, as it must if the current large value of the entropy per baryon is to be accounted for. (...) important to develop a robust model of dissipative cosmological processes in general, so that one can analyze the overall dynamics of dissipation without getting lost in the details of particular complex processes."

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- The treatment of viscous fluids in the context of special relativity is also of interest in heavy-ion collisions (Rezzolla and Zanotti, '13).

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All we need then is $T^{NS}_{\alpha\beta}$ ($T_{\alpha\beta}$ for Navier-Stokes).

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The action $\mathcal{S}$ also determines $T_{\alpha\beta}$.

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The stress-energy tensor is given by

$$T_{\alpha\beta} = \frac{1}{\sqrt{-\det(g)}} \frac{\delta \mathcal{L}}{\delta g^{\alpha\beta}}.$$

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Remark: stress-energy for the Navier-Stokes equations in non-relativistic physics is constructed "by hand."

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$$T^{E}_{\alpha\beta} = (p+\varrho)u_\alpha u_\beta + p g_{\alpha\beta} - (\zeta - \frac{2}{3}\vartheta)\pi_{\alpha\beta}\nabla_\mu u^\mu \\ - \vartheta \pi^\mu_\alpha \pi^\nu_\beta (\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \kappa(q_\alpha u_\beta + q_\beta u_\alpha),$$

where $\pi_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$, ζ and ϑ are the coefficients of bulk and shear viscosity, respectively, κ is the coefficient of heat conduction, and q_α is the heat flux.

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where $\pi_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$, ζ and ϑ are the coefficients of bulk and shear viscosity, respectively, κ is the coefficient of heat conduction, and q_α is the heat flux.

$T^{E}_{\alpha\beta}$ reduces to the stress-energy tensor for a perfect fluid when $\zeta = \vartheta = \kappa = 0$, it is a covariant generalization of the non-relativistic stress-energy tensor for Navier-Stokes, and satisfies basic thermodynamic properties.

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2. Find a stress-energy tensor that avoids the assumptions of Hiscock and Lindblom.

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Two possible choices to circumvent this problem are:

1. Extend the space of variables of the theory, introducing new variables and equations based on some physical principle. Second order theories.

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Despite the results of Hiscock and Lidblom, $T^{E}_{\alpha\beta}$ is still used in applications (particularly in cosmology) for the construction of phenomenological models.

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In the MIS theory, the quantities Π, $\Pi_{\alpha\beta}$, and Q_α are treated as new variables on the same footing as ϱ, u^α, etc.

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For the MIS and other second order theories:

- Causality for certain values of the variables.

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- Causal under all physically relevant scenarios?
- Coupling to Einstein's equations? (Existence of solutions?)

Back to first order theories

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Lichnerowicz

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$$T_{\alpha\beta} = (p+\varrho)u_\alpha u_\beta + pg_{\alpha\beta} - (\zeta - \frac{2}{3}\vartheta)\pi_{\alpha\beta}\nabla_\mu C^\mu \\ - \vartheta\pi^\mu_\alpha\pi^\nu_\beta(\nabla_\mu C_\nu + \nabla_\nu C_\mu) - \kappa(q_\alpha C_\beta + q_\beta C_\alpha) + 2\vartheta\pi_{\alpha\beta}u^\mu\nabla_\mu h,$$

where $h = \frac{p+\varrho}{n}$ $(n > 0)$ is the specific enthalpy of the fluid and

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Lichnerowicz's stress-energy tensor had been mostly ignored for many years, but recently it has been showed as potentially viable candidate for relativistic viscosity.

Some results

Using Lichnerowicz's stress-energy tensor, it is possible to show (D–, '14; D– and Czubak '16; D–, Kephart, and Scherrer, '15):

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- Applications to cosmology lead to different models, in particular big-rip scenarios.
- None of these results consider all dissipative variables (e.g. shear viscosity but no bulk viscosity, etc).

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Geroch and Lindblom ('90) developed a general framework for theories of relativistic viscosity that leads to causal dynamics under many circumstances. One then has to has to show that a particular theory (e.g. MIS) fits in the formalism under the conditions that give rise to causality.

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– Thank you for your attention –