Three Waves for Quantum Gravity

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Outline

- Introduction
- Effective field theory methods applied to quantum gravity
- Applications to black holes and gravitational waves
- Conclusions

Effective field theory

- Loosely speaking the idea is to integrate out degrees of freedom that are irrelevant at the energy at which we do physics.

- Typical example: Quantum Chromodynamics (QCD): at energies above the confinement scale or some 250 MeV we deal with quarks, while below we have bound states of quarks: e.g. mesons.

- We can build an effective field theory by considering a theory of mesons (Chiral Perturbation Theory) which is an effective field theory for QCD.

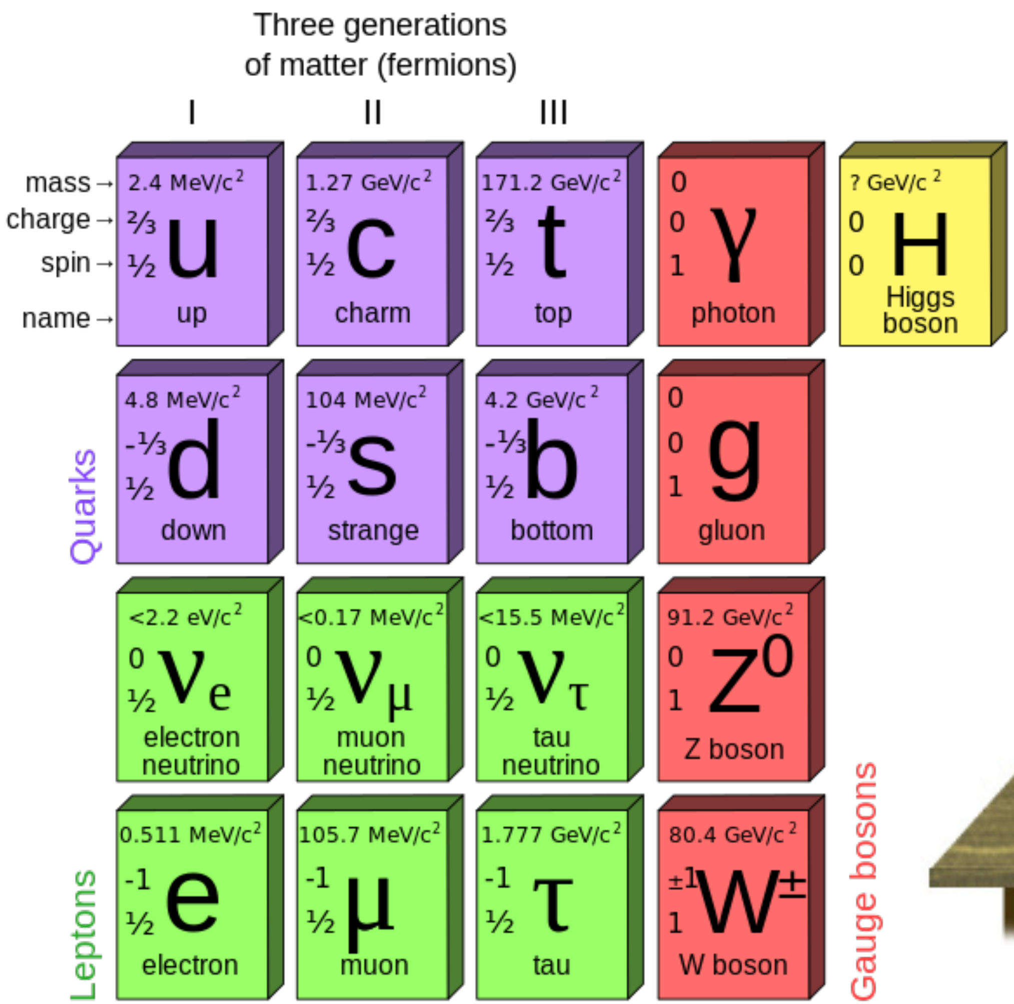
- The same mathematical techniques can be applied to quantum gravity.

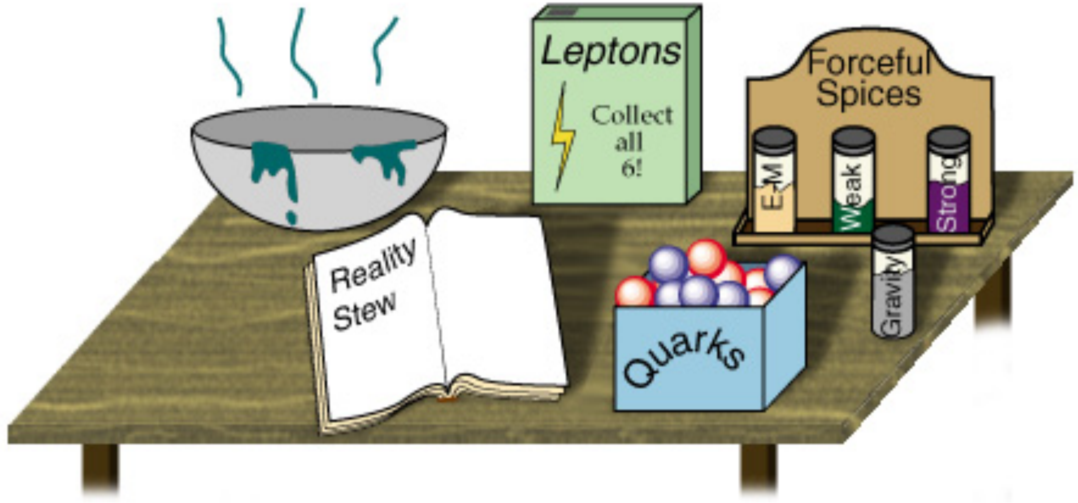
- It is not necessary to know the fundamental theory to write down the corresponding effective field theory: we need to know
  - the symmetries of the problem
  - the field content at low energy

Why quantize gravity? Because we have to as other forces of nature are quantized!

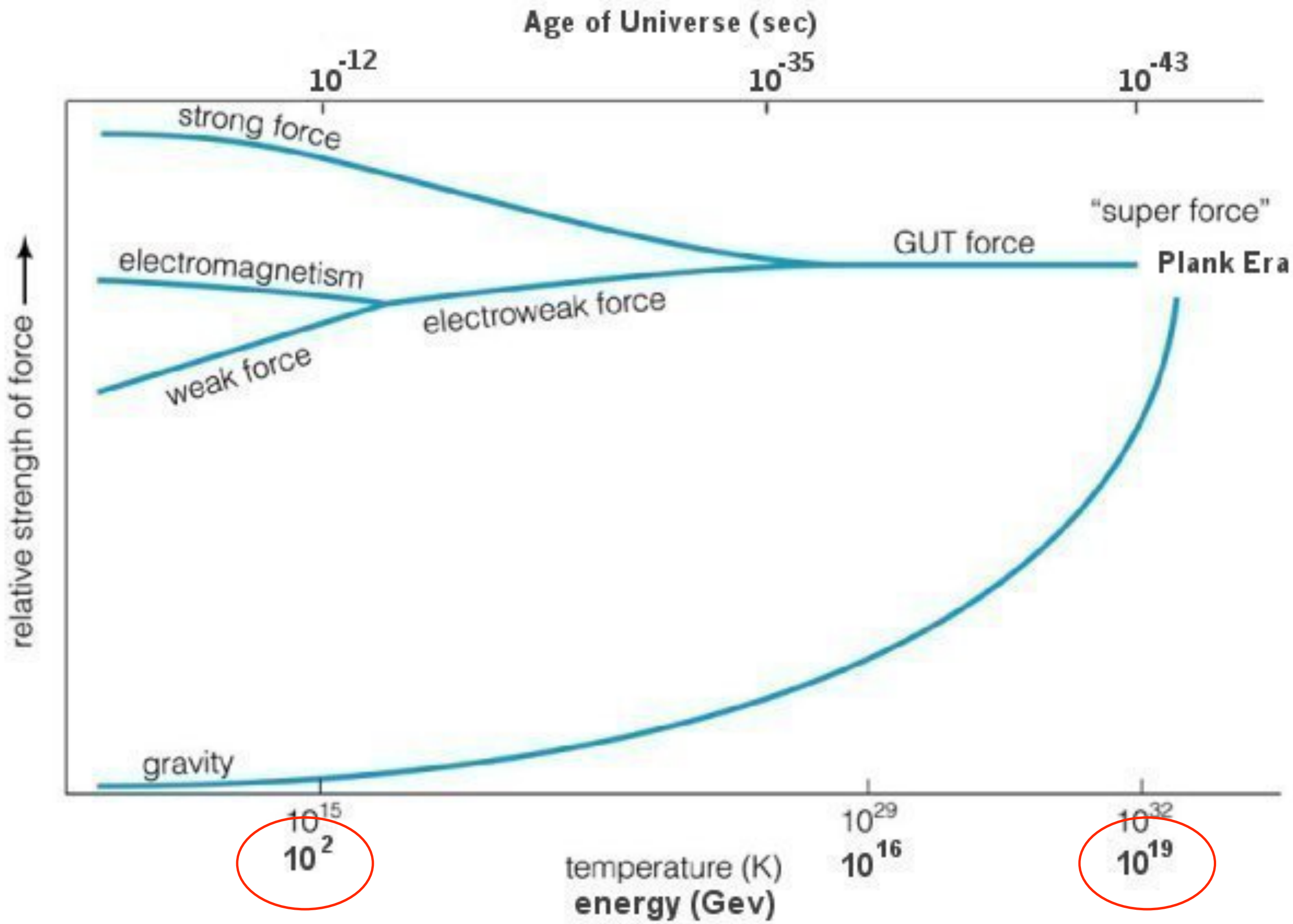
standard model

$m_H \approx 125$ GeV





When does gravity become comparable in strength to other forces?



N.B.: This is the standard picture. I will show you that the Planck mass i.e. 10^{19} GeV could be much smaller!

The Planck mass is the energy scale at which quantum gravitational effects become important.

A grand unification? Is there actually only one fundamental interaction?

$$M_P = \sqrt{\frac{\hbar c}{G_N}}$$

Effective action for GR

- How can we describe general relativity quantum mechanically?
- Well known issues with linearized GR: it is not renormalizable.
- This is the reason d'être of string theory, loop quantum gravity etc…
- How much can we understand using QFT techniques?
- We have good reasons to think that length scales smaller than the Planck scale are not observables due to the formation of small black holes.
- Effective field theories might be all we need to discuss physics at least up to the Planck scale.

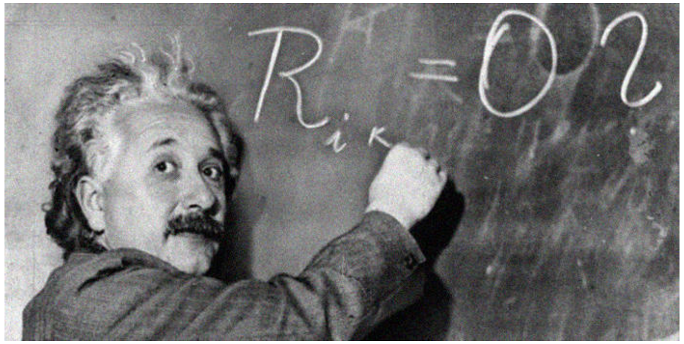
- The goal is to try to make the link with observables.
- Or at least with with thought experiments.
- It is very conservative.
- What can we learn using techniques we actually understand well, and which are compatible with nature as we know it: standard model and GR.

- I am going to assume general covariance (diffeomorphism invariance)
- Quantum gravity has only 2 dofs namely the massless graviton (which has 2 helicity states).
- We know the particle content of the “matter theory” (SM, GUT, inflation etc).
- We can write down an effective action for quantum gravity.

- This program was started by Feynman in the 60’s using linearized GR.

- Try to find/calculate observables

- Try to find consistency conditions which could guide us on our path towards a quantization of GR.



Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The Ricci scalar R and tensor $R_{\mu\nu}$ contain two derivatives of the metric.

They thus have mass dimension 2, this is important to organize the effective field theory.

G is Newton's constant, it is related to the Planck mass.

$T_{\mu\nu}$ is the energy-momentum tensor: this is your particle physics model.

It can be derived from the Hilbert-Einstein action:

$$S = \frac{1}{2\kappa} \int R \sqrt{-g}\, \mathrm{d}^4 x$$

$$g = \det(g_{\mu\nu})$$

$$\kappa = 8\pi G c^{-4}$$

Perturbative linearized general relativity

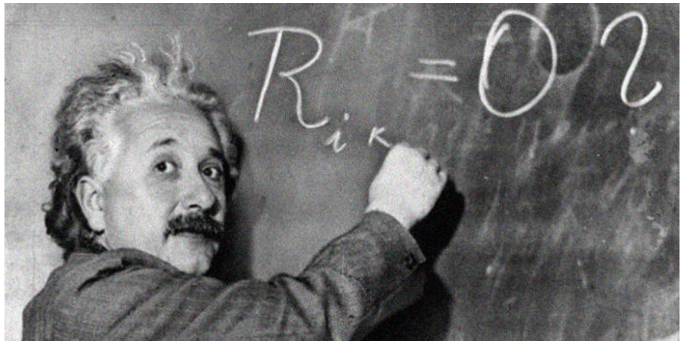
Matter coupling to gravity is described by general relativity:

$$S[g,\phi,\psi,A_\mu] \;\; = -\int d^4x\sqrt{-\det(g)} \;\; \left(\frac{1}{16\pi G_N}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \xi R\phi^2 + \right.$$
$$\left. + e\bar{\psi}i\gamma^\mu D_\mu\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Linearized general relativity can be regarded as an effective field theory valid up to the reduced Planck mass

$$L = -\frac{1}{4}h^{\mu\nu}\Box h_{\mu\nu} + \frac{1}{4}h\Box h - \frac{1}{2}h^{\mu\nu}\partial_\mu\partial_\nu h + \frac{1}{2}h^{\mu\nu}\partial_\mu\partial_\alpha h^\alpha_\nu - \frac{\sqrt{2}}{\bar{M}_P}h^{\mu\nu}T_{\mu\nu} + \mathcal{O}(\bar{M}_P^{-2})$$

The theory is non-renormalizable, but as we shall see some predictions are still possible.



Effective action for quantum gravity

The Hilbert-Einstein action

$$S = \frac{1}{2\kappa}\int R\sqrt{-g}\,\mathrm{d}^4x$$

receives corrections from quantum gravity, integrating out fluctuations of the graviton (and other matter fields depending on the energy under consideration),
one obtains:

$$\begin{aligned} S \;=\; \int d^4x\,\sqrt{-g}\,\Big[\Big(\frac{1}{2}M^2 + \xi H^\dagger H\Big)\mathcal{R} - \Lambda_C^4 + c_1\mathcal{R}^2 + c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_4\Box\mathcal{R} \\ + b_1\mathcal{R}\log\frac{\Box}{\mu_1^2}\mathcal{R} + b_2\mathcal{R}_{\mu\nu}\log\frac{\Box}{\mu_2^2}\mathcal{R}^{\mu\nu} + b_3\mathcal{R}_{\mu\nu\rho\sigma}\log\frac{\Box}{\mu_3^2}\mathcal{R}^{\mu\nu\rho\sigma} + \mathcal{O}(M_\star^{-2}) + \mathcal{L}_{SM}\Big] \end{aligned}$$

The non-local part of the EFT

- The Wilson coefficients of the non-local operators are universal predictions of quantum gravity:

$$S_{QL} = \int d^4x\sqrt{g}\left(\alpha R \log\left(\frac{\Box}{\mu_\alpha^2}\right) R + \beta R_{\mu\nu} \log\left(\frac{\Box}{\mu_\beta^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} \log\left(\frac{\Box}{\mu_\gamma^2}\right) R^{\mu\nu\alpha\beta}\right)$$

	α	β	γ
Scalar	$5(6\xi - 1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	430	-1444	424

All numbers should be divided by $11520\pi^2$.

NB: they are calculated using dim-reg.

(see e.g. Birrell and Davies, Quantum Fields in Curved Space-Time, more recently Donoghue et al.)

- The Wilson coefficients of the local operators on the other hand are not calculable: this the price to pay.

Green's function

- Varying with respect to the metric one obtains the 2-point function

$$G^{-1}(p^2) = 2\,p^2\left[1 - 16\,\pi\,c_2\,\frac{p^2}{m_{\rm P}^2} - \frac{N\,p^2}{120\,\pi\,m_{\rm P}^2}\ln\left(-\frac{p^2}{\mu^2}\right)\right] \qquad N = 1/3N_s + N_\psi + 4N_V$$

- It is clear that it has more than one pole. Setting c_2=0 for a moment:

$$\begin{aligned} q_1^2 &= 0, \\ q_2^2 &= \frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}, \\ q_3^2 &= (q_2^2)^*, \end{aligned}$$

- Complex poles: EQG breaks down and potentially well below the Planck scale.
- Sign of strong dynamics kicking in.
- Plays an important role in unitarizing perturbative amplitudes in the large N (self-healing)

Poles and Quantum Black Holes?

- It is tempting to interpret these poles as black hole precursors.
- In the SM $N_s = 4,\ N_f = 45$, and $N_V = 12$
- We thus find

$$(7-3i)\times 10^{18}\ \text{GeV}\ \text{and}\ (7+3i)\times 10^{18}\ \text{GeV}$$

- using

$$p_0^2 = (m - i\Gamma/2)^2$$

- The first one corresponds to a state with mass 7×10^{18} GeV

  and width 6×10^{18} GeV

- Note that the 2nd pole has the wrong sign for particle between the mass and the width.

Acausal versus nonlocal effects

- Remember that the 2nd pole has the wrong sign between the mass and width terms for a particle: it is a ghost.

- Acausal effects: connection to black hole information paradox? Could be canceled by e.g. Lee and Wick's mechanism.

- Acausal effects can be replaced by non local effects

$$S = \int d^4x \sqrt{g} \left[R \log\left(\frac{\Box}{\mu^2}\right) R \right] \qquad L(x,y) = \langle x | \log\left(\frac{\Box}{\mu^2}\right) | y \rangle$$

by reinterpreting the log term (the non-local function can be calculated using the Green's function of the box operator).

- Can these effects soften singularities?

Study of the EFT

Let me linearize the EFT

$$S = \int d^4x \left\{ -\frac{1}{4} h^{\mu\nu} \left[-\left(c_2 + (b_2 + 4b_3) \log\left(\frac{\Box}{\mu^2}\right)\right) \kappa^2 \Box + 2 \right] \Box P^{(2)}_{\mu\nu\rho\sigma} h^{\rho\sigma} \right.$$
$$\left. + \frac{1}{2} h^{\mu\nu} \left[2 \left(3c_1 + c_2 + (3b_1 + b_2 + b_3) \log\left(\frac{\Box}{\mu^2}\right)\right) \kappa^2 \Box + 2 \right] \Box P^{(0)}_{\mu\nu\rho\sigma} h^{\rho\sigma} + \kappa h_{\mu\nu} T^{\mu\nu} \right\}$$

with the projectors:

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(L_{\mu\rho}L_{\nu\sigma} + L_{\mu\sigma}L_{\nu\rho}) - \frac{1}{3}L_{\mu\nu}L_{\rho\sigma},$$

$$P^{(0)}_{\mu\nu\rho\sigma} = \frac{1}{3}L_{\mu\nu}L_{\rho\sigma} \qquad \text{where } L_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu\partial_\nu/\Box$$

- To identify the field content of the EFT, let's calculate the Green's function expectation value between two conserved sources

$$\frac{\kappa^2}{4}\left[\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{2}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2}-\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{3}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{2}{\kappa^2\left(c_2+(b_2+4b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}+\frac{T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{1}{\kappa^2\left(3c_1+c_2+(3b_1+b_2+b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}\right]$$

- where $\kappa^2 = 32\pi G$

- We see that:
  - the 1st term corresponds to a massless spin-2 field: the "classical" graviton
  - the 2nd term corresponds to a massive spin-2 field with an overall minus sign (ghost)

$$m_2^2=\frac{2}{(b_2+4b_3)\kappa^2 W\left(-\frac{2\exp\frac{c_2}{(b_2+4b_3)}}{(b_2+4b_3)\kappa^2\mu^2}\right)}$$

  - the 3rd term corresponds to a massive spin-0 field

$$m_0^2=\frac{1}{(3b_1+b_2+b_3)\kappa^2 W\left(-\frac{\exp\frac{3c_1+c_2}{(3b_1+b_2+b_3)}}{(3b_1+b_2+b_3)\kappa^2\mu^2}\right)}$$

- Note that this extends the classical result of Stelle.
- It is crucial to realize that these are classical fields.

Universal features of quantum gravity

Using EFT techniques, we have identified universal (model independent) features of quantum gravity:

- The scale of quantum gravity is dynamical,

$$\sqrt{\frac{120\pi}{G_N N}}$$

  it depends on the number of fields in the theory.

- Strong interactions kick in at this energy scale.

- Space-time becomes non-local.

- There are three classical fields in the low energy regime of quantum gravity.

Lagrangian for low energy physics

- It is easy to work out the coupling of the classical degrees of freedom

$$S = \int d^4x \Bigg[\left(-\frac{1}{2} h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu \Box h_\nu{}^\nu - h^{\mu\nu} \partial_\mu \partial_\nu h_\alpha{}^\alpha + h^{\mu\nu} \partial_\rho \partial_\nu h^\rho{}_\mu \right)$$
$$+ \left(-\frac{1}{2} k_{\mu\nu} \Box k^{\mu\nu} + \frac{1}{2} k_\mu{}^\mu \Box k_\nu{}^\nu - k^{\mu\nu} \partial_\mu \partial_\nu k_\alpha{}^\alpha + k^{\mu\nu} \partial_\rho \partial_\nu k^\rho{}_\mu \right.$$
$$\left. - \frac{m_2^2}{2} \left(k_{\mu\nu} k^{\mu\nu} - k_\alpha{}^\alpha k_\beta{}^\beta \right) \right)$$
$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{m_0^2}{2} \sigma^2 - \sqrt{8\pi G_N} (h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}} \sigma \eta_{\mu\nu}) T^{\mu\nu} \Bigg] .$$

- This can also be done without linearizing the theory by going to the Einstein frame:

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}\Big[\bar{R} - \tfrac{3}{2}\left(A^{-1}(\phi_{\sigma\tau})\right)_{\mu}{}^{\nu}\,\bar{\nabla}^{\mu}\chi\bar{\nabla}_{\nu}\chi - \tfrac{3}{2}\left(\det A(\phi_{\sigma\tau})\right)^{-1/2}\left(1-e^{-\chi}\right)^2 \\
&\quad -\bar{g}^{\mu\nu}\left(C^{\lambda}{}_{\mu\rho}(\phi_{\sigma\tau})C^{\rho}{}_{\nu\lambda}(\phi_{\sigma\tau}) - C^{\lambda}{}_{\mu\nu}(\phi_{\sigma\tau})C^{\rho}{}_{\rho\lambda}(\phi_{\sigma\tau})\right) \\
&\quad +\tfrac{1}{4}{m_2}^2\left(\det A(\phi_{\sigma\tau})\right)^{-1/2}\left(\phi_{\mu\nu}\phi^{\mu\nu}-\phi^2\right) + \bar{\mathcal{L}}_M(e^{-\chi}\tilde{g}_{\mu\nu}(\phi_{\sigma\tau}),\phi_\alpha)\Big].
\end{aligned}
$$

- We see that the "classical" graviton plays the role of the metric and determines the geometry.

- It couples in a universal manner to matter as usual (this is just GR).

- The massive classical fields are not gravitational fields in the sense that they do not affect the invariant length or geometry.

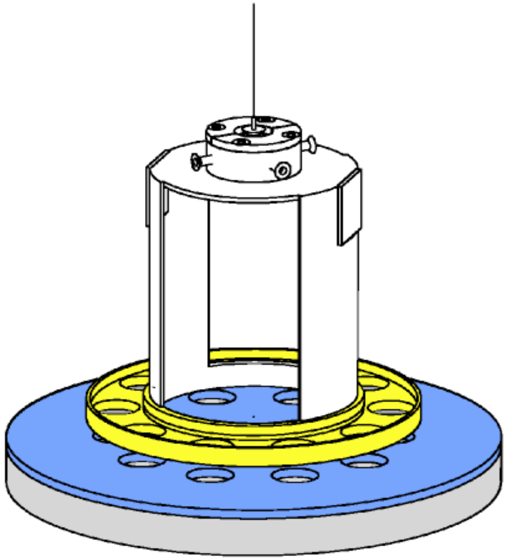
- The coupling of the massive spin-2 object to matter is universal while that of the massive spin-0 is not: it does not couple to massless vector fields as it couples to the trace of the energy-momentum tensor.

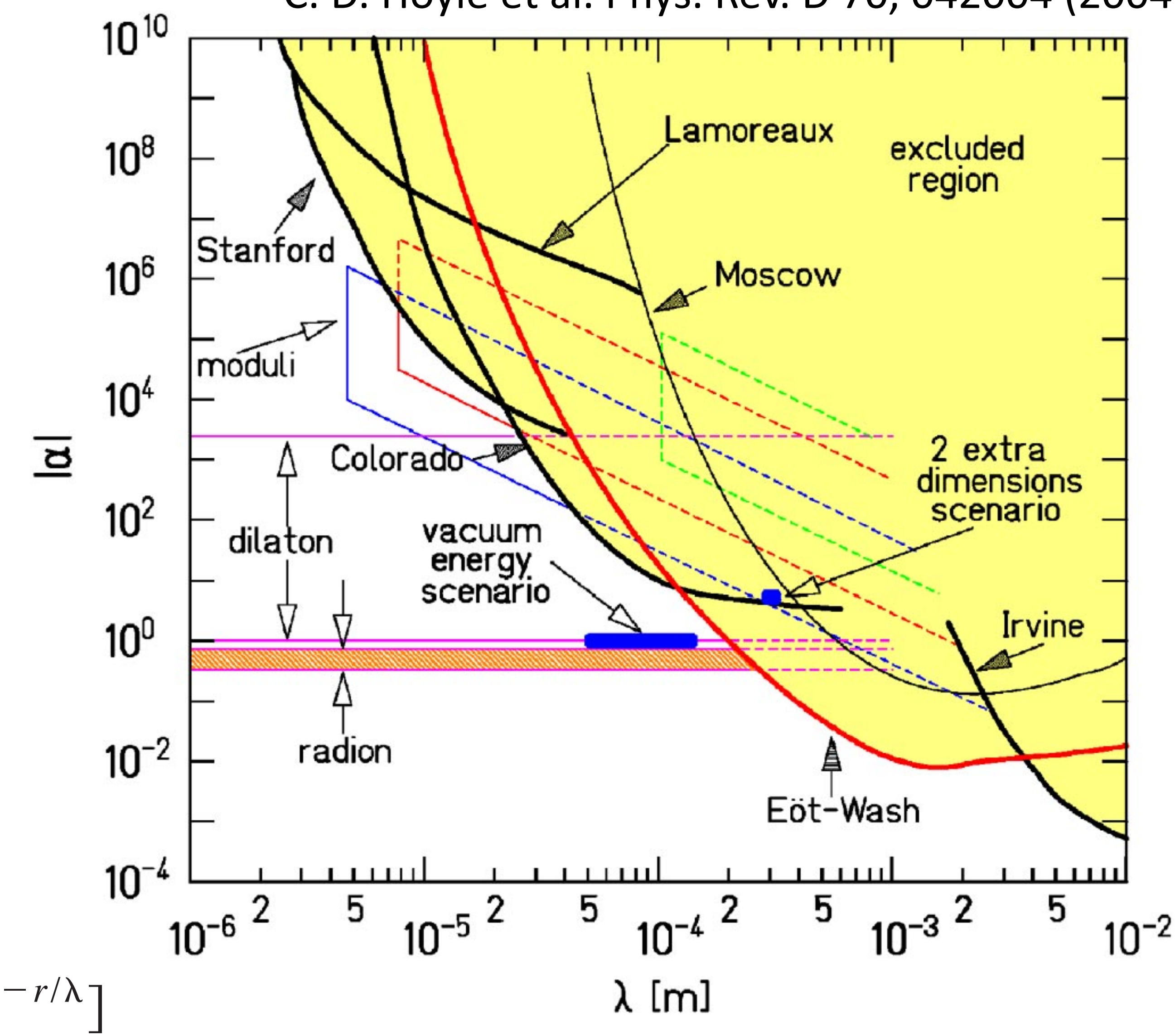
Classical fields

- There are some interesting consequences if one tries to interpret these fields as dark matter or inflaton: they are classical fields
  - Is dark matter an emergent phenomenon?
  - R^2 inflation: as the scalar field is purely classical it could lead to the expansion of the universe but not to the density fluctuations, another field (maybe the Higgs field) would be needed
- As we are dealing with a classical field, the fact that the massive spin-2 is a ghost is not an obvious problem, it simply means that it couples to matter with a negative Planck mass.
- We do not find any sign of instability.
- As we shall see shortly, the massive spin 2 object simply leads to a repulsive force.

Eöt-Wash pendulum experiment

C. D. Hoyle et al. Phys. Rev. D 70, 042004 (2004)





$$V(r) = -G\frac{m_1 m_2}{r}[1+\alpha e^{-r/\lambda}]$$

Quantum gravity correction to Newton's Law

- We can easily deduce the quantum gravitational corrections to the Newtonian potential of a point mass

$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-Re(m_0)r} - \frac{4}{3}e^{-Re(m_2)r}\right)$$

- Note that the imaginary parts of the masses cancel out.

- There is no contradiction with Donoghue's result.

- In the absence of accidental fine cancellations between both Yukawa terms, the current bounds imply m_0 , $m_2 > (0.03\ \text{cm})^{-1} = 6.6 \times 10^{-13}$ GeV.

- Note that the experiment performed by Hoyle et al. is probing separations between 10.77 mm and 137 μm, a cancelation between the two Yukawa terms on this range of scales seems impossible without modifying general relativity with new physics to implement a screening mechanism.

Gravitational waves

- The new classical fields could be produced in high energetic astrophysical or cosmological events.

- In binary system, only the massive spin-2 can be produced, it has 5 polarizations. As the trace of the energy-momentum tensor is conserved the spin-0 cannot be produced.

- In phase transitions, both the massive spin-0 and spin-2 modes could be produced.

- There are thus three kinds of waves in quantum gravity: the massless gravitational waves that have just been observed and massive waves.

- We may have seen a superposition of these modes if the mass of the massive spin-2 mode is low enough.

- Clearly there is enough energy in a typical merger:

$$36\, M_\odot + 29\, M_\odot \rightarrow 62 M_\odot + 3 M_\odot \text{(gravitational wave)}$$

- $3M_\odot$ corresponds to 3×10^{57} GeV, detailed calculation is in progress.

- The bound on the quantum gravitational corrections to Newton's potential imply that quantum gravity could only impact the final moments of the inspiraling of binary of two neutron stars or of two black holes.

- Their effect will only become relevant at distances shorter than 0.03 cm.

- The quantum gravitational correction to the orbital frequency of a inspiraling binary system is given by

$$\omega^2 = \frac{Gm}{r^3}\left(1 + \frac{1}{3}e^{-Re(m_0)r} - \frac{4}{3}e^{-Re(m_2)r}\right)$$

- Using the standard relation between energy and power, we can obtain the quantum corrected frequency and quantum corrected amplitude:

$$\begin{aligned} f_{GW}(t) &= \frac{\omega(t)}{\pi} \\ A_{GW}(t) &= \frac{1}{d_L}\frac{2G_N}{c^4}2\mu\omega(t)r^2(t) \end{aligned}$$

- While it is easy to calculate f_{GW} and A_{GW} explicitly, it is clear that the quantum gravitational corrections to the emission of gravitational waves can only become relevant when the two objects are closer than 0.03 cm

- This distance is well within the Schwarzschild radius of any astrophysical black hole and clearly tools from numerical relativity need to be employed to obtain a reliable computation.

- Maybe the situation is not so bad for black holes as the mass is centered around the “singularity”.

- Besides the usual massless gravitational waves, there are two new kind of radiations, namely the massive spin-0 and spin-2.

- They could be produced in energetic astrophysical or cosmological events.
-
- However, in the case of a binary system, because the center of mass of the system is conserved, the spin-0 wave cannot be produced.

- On the other hand, the massive spin-2 could be emitted in the last moment of a merger when the two inspiraling objects are closer than the inverse of the mass of the massive spin-2 field.

- A lengthy calculation leads to a remarkable result: the energy E carried away by the massive spin-2 mode from a binary system per frequency is identical to that of massless spin-2 mode:

$$\frac{dE_{massive}}{d\omega} = \frac{G_N}{45}\omega^6 \langle Q_{ij}Q^{ij}\rangle \theta(\omega - m_2)$$

- The total wave emission by a binary system is thus given by

$$\frac{dE}{d\omega} = \frac{dE_{massless}}{d\omega} + \frac{dE_{massive}}{d\omega}$$

- The massive spin-2 wave will only be produced when the two black holes are close enough from another.

- If we denote the distance between the black holes of masses m_A and m_B by d, we obtain the frequency of the inspiral ω:

$$\omega^2 = \frac{G_N(m_A + m_B)}{d^3}$$

- To estimate how close the two black holes have to be to generate enough energy to produce a massive wave compatible the pendulum bound, we set $\omega = (0.03\ \mathrm{cm})^{-1}$ and use the masses of the first merger observed by the LIGO collaboration $m_A = 36\ M_\odot$ $m_B = 29 M_\odot$.

- We find that for a wave of mass $(0.03\ \mathrm{cm})^{-1}$ to be produced the two black holes would have to be at 16 cm from another.

- Clearly this is again well within the horizon of any astrophysical black holes and a reliable simulation will require a technically challenging simulation using numerical methods.

- However, our results demonstrate that massive spin-2 waves can be produced in the merger of astrophysical objects such as black holes or neutron stars and this effect must be taken into account in future numerical studies.

- Clearly the massive modes will only be produced in the final stage of the inspiral process at the time of the merger and ringdown.

- This represents a unique opportunity to probe quantum gravity with astrophysical events in a fully non-speculative manner.

- It is also possible to envisage the production of these new quantum gravitational massive classical modes during first order phase transitions if such phases took place early on in the cosmological evolution of our universe.

- Clearly, the occurrence of a first order phase transition in the early universe is a speculative topic as there is no such phase transition within the electroweak standard model.

- Our work represents an additional complication for the study of early universe phase transitions as beyond the massless gravitational waves, the new massive modes could be produced.

- Indeed, the collision of bubbles and damping of plasma inhomogeneities could have generated a stochastic background of massive gravitational waves beyond the massless ones that are expected.

- This implies that some of the energy of these processes could be lost in massive modes. This fact has been overlooked so far when doing simulations for LISA

Summary of EQG and bounds on its parameters

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2 \qquad M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 9.7 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

Singularities

- Within this framework, what can we say about singularities in cosmology and black holes?

- Recently Donoghue and El-Menoufi have argued that the late time singularity in FRLW cosmology could be avoided due to the non-local operators we have discussed

From 1402.3252

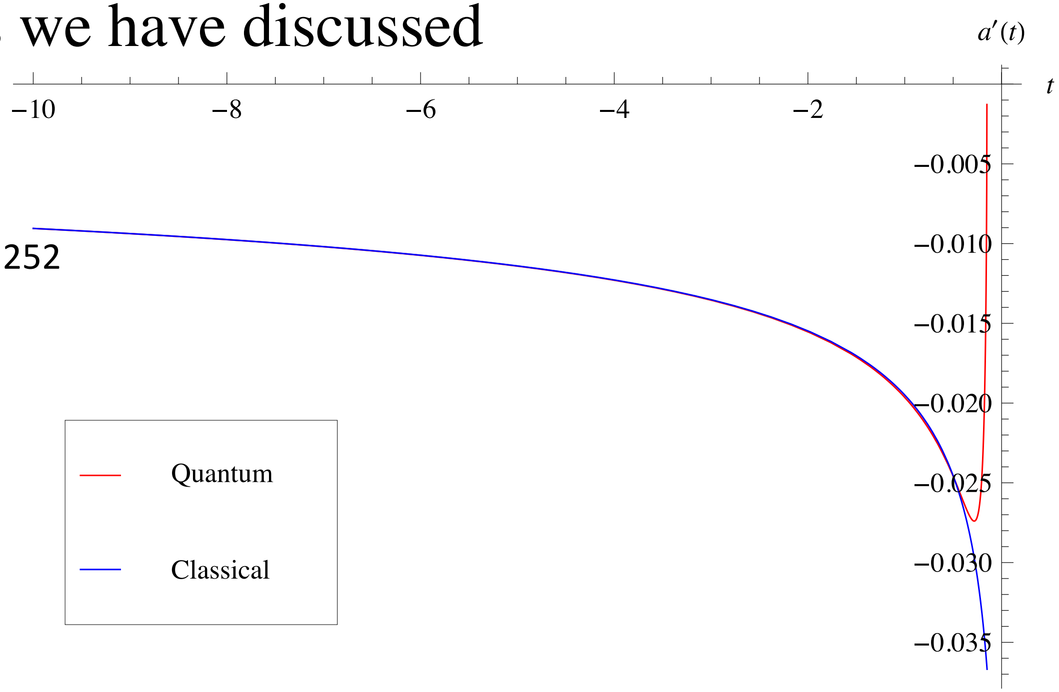


FIG. 7: Collapsing dust-filled universe with $\mu_R = 1$ and a single scalar field. The time derivative of the scale factor quickly stops diverging when the quantum correction becomes active.

- Donoghue and El-Menoufi find that the big-crunch singularity can be avoided for certain number of fields.

- Singularity avoidance should be universal, would one avoid them in black holes as well?

Quantum Corrections to Black Holes

- Black holes are amongst the simplest and yet most mysterious objects in our universe.

- No-hair theorem implies that they are described by only a few parameters: their masses, angular momenta and charges.

- Despite this apparent simplicity, they are incredibly challenging as understanding their physics requires merging quantum mechanics and general relativity.

- Investigating the effects of this non-locality in black hole physics is our main motivation to consider quantum corrections to spherically symmetric solutions in general relativity.

- In particular, we revisit the issue of quantum corrections to the Schwarzschild black hole solution which have been studied in the past by Duff and Donoghue et al. .

- We identify a complication which has not been realized previously, namely that of how to define a black hole.

- A mathematically consistent way to define a black hole is to define it as a static vacuum solution, i.e., an eternal black hole.

- If this definition is adopted, we obtain a result that differs from previous investigations.

- In particular, we will see that the classical black hole solution remains a solution in quantum gravity up to quartic order in the non-local curvature expansion.

- While eternal black holes are mathematically well defined, they may not capture the full physical picture.

- A real, astrophysical, black hole is the final state of the evolution of a matter distribution, for example of a heavy star, after it has undergone gravitational collapse.

- This process is certainly not happening in vacuum.

- This raises the question of how to define a real astrophysical black hole and of how to calculate quantum corrections to its metric.

- A non-vanishing energy-momentum tensor could be used to model a collapsing star.

- At a time when the star has not yet collapsed into a black hole, the star can be described as a static source at a specific time in its evolution.

- Another complication appears due to the non-locality: we are forced to integrate the modified equations of motion over regions of Planck size curvature.

- One may thus worry about the sensitivity of the EFT to regions of space-time with high curvature and, in particular, to the singularity at $r = 0$.

- Clearly the effective field theory breaks down in regions of large curvature, which in turn raises the question if the latter could offer a reliable picture in our case.

- However, the ultimate ultra-violet physics that dominates regions of large curvature should not affect observables at long distances, i.e. the exterior region of a black hole.

- Indeed, in an EFT, one expects short distance physics to decouple at low energies.

- We are making this conservative assumption.

Quantum gravity corrections to the equations of motion

We can now derive the equations of motion

$$G_{\mu\nu} + H_{\mu\nu} + H^{\mathrm{q}}_{\mu\nu} = 0$$

where $G_{\mu\nu}$ is the Einstein tensor, $H_{\mu\nu}$ and $H_{\mu\nu}{}^{q}$ represent respectively the local and non-local parts of the quantum correction to the field equations.

$$H_{\mu\nu} = \bar{c}_1(\mu)\left(2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2g_{\mu\nu}\Box R + 2\nabla_\mu\nabla_\nu R\right)$$
$$+ \bar{c}_2(\mu)\left(\nabla_\alpha\nabla_\mu R^\alpha_\nu + \nabla_\alpha\nabla_\nu R^\alpha_\mu - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 2R^\alpha_\mu R_{\nu\alpha}\right)$$

$$H^{\mathrm{q}}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\left[\alpha R\ln\left(\frac{\Box}{\mu^2}\right)R + \beta R_{\alpha\beta}\ln\left(\frac{\Box}{\mu^2}\right)R^{\alpha\beta} + \gamma R_{\alpha\beta\sigma\tau}\ln\left(\frac{\Box}{\mu^2}\right)R^{\alpha\beta\sigma\tau}\right]$$
$$- \left[2\alpha\frac{\delta R}{\delta g^{\mu\nu}}\ln\left(\frac{\Box}{\mu^2}\right)R + 2\beta\frac{\delta R^\beta_\alpha}{\delta g^{\mu\nu}}\ln\left(\frac{\Box}{\mu^2}\right)R^\alpha_\beta + \gamma\frac{\delta R_{\beta\alpha\sigma\tau}}{\delta g^{\mu\nu}}\ln\left(\frac{\Box}{\mu^2}\right)R^{\beta\alpha\sigma\tau}\right.$$
$$\left. + \gamma\frac{\delta R^{\beta\alpha\sigma\tau}}{\delta g^{\mu\nu}}\ln\left(\frac{\Box}{\mu^2}\right)R_{\beta\alpha\sigma\tau}\right] \quad .$$

Absence of perturbative correction to Schwarzschild black hole

- We write the metric as follows

$$g_{\mu\nu} = g^{\mathrm{Sch.}}_{\mu\nu} + g^{\mathrm{q}}_{\mu\nu}$$

where g^q represents the quantum correction to Schwarzschild solution.

- Linearizing the field equations around g^{Sch}, one finds

$$G^{\mathrm{L}}_{\mu\nu}\left[g^{\mathrm{q}}\right] + H_{\mu\nu}\left[g^{\mathrm{Sch.}}\right] + H^{\mathrm{q}}_{\mu\nu}\left[g^{\mathrm{Sch.}}\right] = 0$$

where the linearized Einstein tensor reads.

- It is well known that $H_{\mu\nu}[g^{Sch}] = 0$

- A lengthy calculation shows that $H_{\mu\nu}{}^{q}[g^{Sch}] = 0$.

- There are no correction to Schwarzschild's metric at 2[nd] order in curvature

- While that there are no corrections at quartic order in curvature, which is in sharp contrast with previous results, there will be corrections at higher order for example higher dimensional operators such as

$$c_6 R^{\mu\nu}{}_{\alpha\sigma} R^{\alpha\sigma}{}_{\delta\gamma} R^{\delta\gamma}{}_{\mu\nu}$$

- will lead to quantum corrections of the Schwarzschild solution.

- We are doing perturbation around the standard Schwarzschild solution

$$A(r) = 1 - \frac{2MG}{r} + h(r)$$

- Far away from the hole, we find

$$h(r) = c_6 \frac{576\pi G_N^3 M^2}{r^6}$$

- This simply demonstrates that the Schwarzschild solution is not a solution of the field equations when higher dimensional operators of dimensions d≥6 are included.

Singularity avoidance?

- An immediate consequence of our result is that the singularity avoidance observed by Donoghue and El-Menoufi is non-universal as the very same operators do not cure the curvature singularity of an eternal Schwarzschild black hole.

- However, it is important to keep in mind that these results are obtained in perturbation theory.

- We have seen that there are indications that perturbation theory will break down below the reduced Planck mass.

- Strong dynamics is expected to resolve singularities.

Corrections to the gravitational field of a static source

- It is interesting to understand the effect of the quantum induced non-locality on the field of a static spherically symmetric object such as a star.
- We aim for a simplified treatment and thus we only consider $\alpha R \ln\left(\frac{\Box}{\mu^2}\right) R$
- We use a perturbative approach again compatible with our EFT approach.

$$g = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

- The solution to Einstein equation for a constant density star is known in closed form.

$$A(r) = \left[1 - \frac{2G\mathcal{M}(r)}{r}\right]^{-1}, \quad \mathcal{M}(r) = \int^r \rho\,\mathrm{d}V = \int_0^r 4\pi r'^2 \rho(r')\,\mathrm{d}r.$$

$$B(r) = \exp\left\{-\int_r^\infty \frac{2G}{r'^2}[\mathcal{M}(r') + 4\pi r'^3 P(r')]A(r')\,\mathrm{d}r'\right\}.$$

In these equations ρ is the density of the star and P its pressure.

- Outside the star, the non-locality introduces a non-trivial contribution

$$G^{\mathrm{L}}_{\mu\nu} = \alpha(16\pi G_N)^2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\int_{\mathrm{S}} d^4x'\sqrt{g}\,L(x,x';\mu)\,T$$

where the integral extends only over the source region,
$T = \rho_0 - 3\,P$ is the trace of the energy-momentum tensor, ρ_0 is the mass density and P is the pressure.

- Both the pressure and metric functions are known in the interior of the star

$$P(r) = \rho_0\frac{(1-2GMr^2/R_S^3)^{1/2}-(1-2GM/R_S)^{1/2}}{(1-2GM/R_S)^{1/2}-3(1-2GMr^2/R_S^3)^{1/2}}$$

where R_S is the radius of the star.

- Far away from the source, we find the leading correction

$$g_{\rm tt}^{\rm q} = \frac{18\alpha l_{\rm P}^2}{R_S^2}\frac{2G_N M}{r}, \quad g_{\rm rr}^{\rm q} = \frac{12 G_N M \alpha l_{\rm P}^2}{r^3}$$

- Note that it is not possible to recover our previous result for an eternal Schwarzschild black hole by taking the limit $R_S = 0$ as this limit is ill-defined.

- This is not really a surprise!

- In 1987 Geroch and Trashen have shown that in general relativity, the only sensible delta-function sources in generally covariant theories are sources of spatial codimension one.

- So one cannot consider a point source as in the linearized theory.

- Instead, the simplest source that one can consider in the full nonlinear theory is a thin spherical shell of radius.

- This is also a source of discrepancy with previous works that have considered delta-functions as sources.

- We have shown that the Schwarzschild black hole solution remains a solution including non-local quantum corrections up to quadratic order in curvatures.

- Our findings emphasize the need to be very careful when discussing quantum corrections to black holes which need to be defined carefully.

- While, from a mathematical point of view, an eternal black hole is a static vacuum solution, astrophysical black holes are not.

- They are surrounded by matter and are themselves the result of the gravitational collapse of matter.

- Calculating quantum gravitational corrections to real astrophysical black holes is thus a fantastically difficult task which cannot be done easily analytically.

- This investigation requires us to study a dynamical process where a matter distribution, e.g., a star, collapses to form a black hole and to follow quantum effects throughout the process.

- Our work represents a first step in that direction.

- We have found that an observer far away from a star experiences a correction to Newton's law that depends on the size of the star.

- Long after the star has collapsed, the far field behavior of the remaining object should approach that of an eternal black hole.

- At this stage of the evolution, the observer would find only cubic order in curvature corrections to Newton's law.

Conclusions

- We have discussed a conservative effective action for quantum gravity (EQG) within usual QFTs such as the standard model or GUT.
- EQG can make predictions which can be confronted to data.
- We have seen some universal features of quantum gravity: the Planck scale is dynamical, space-time becomes non-local at that scale & strong dynamics at the Planck scale.
- We have discussed a novel application to quantum corrections to black holes and found new unexpected results.

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Thanks for your attention!