Entanglement in the sky

Eugenio Bianchi

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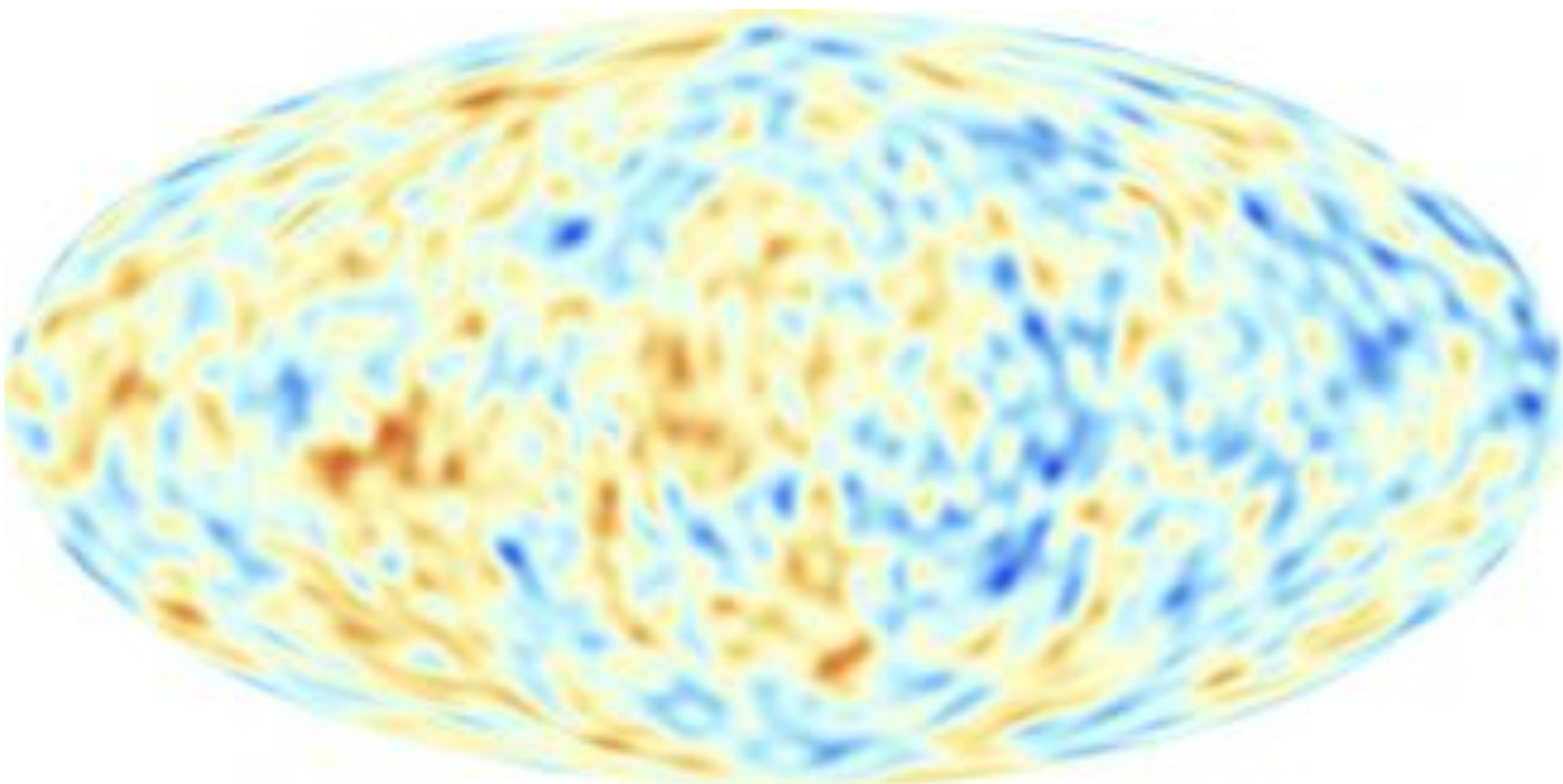


Frankfurt Institute for Advanced Studies - Astrocoffee, March 2016

Entanglement in the sky

Plan:

* The vacuum

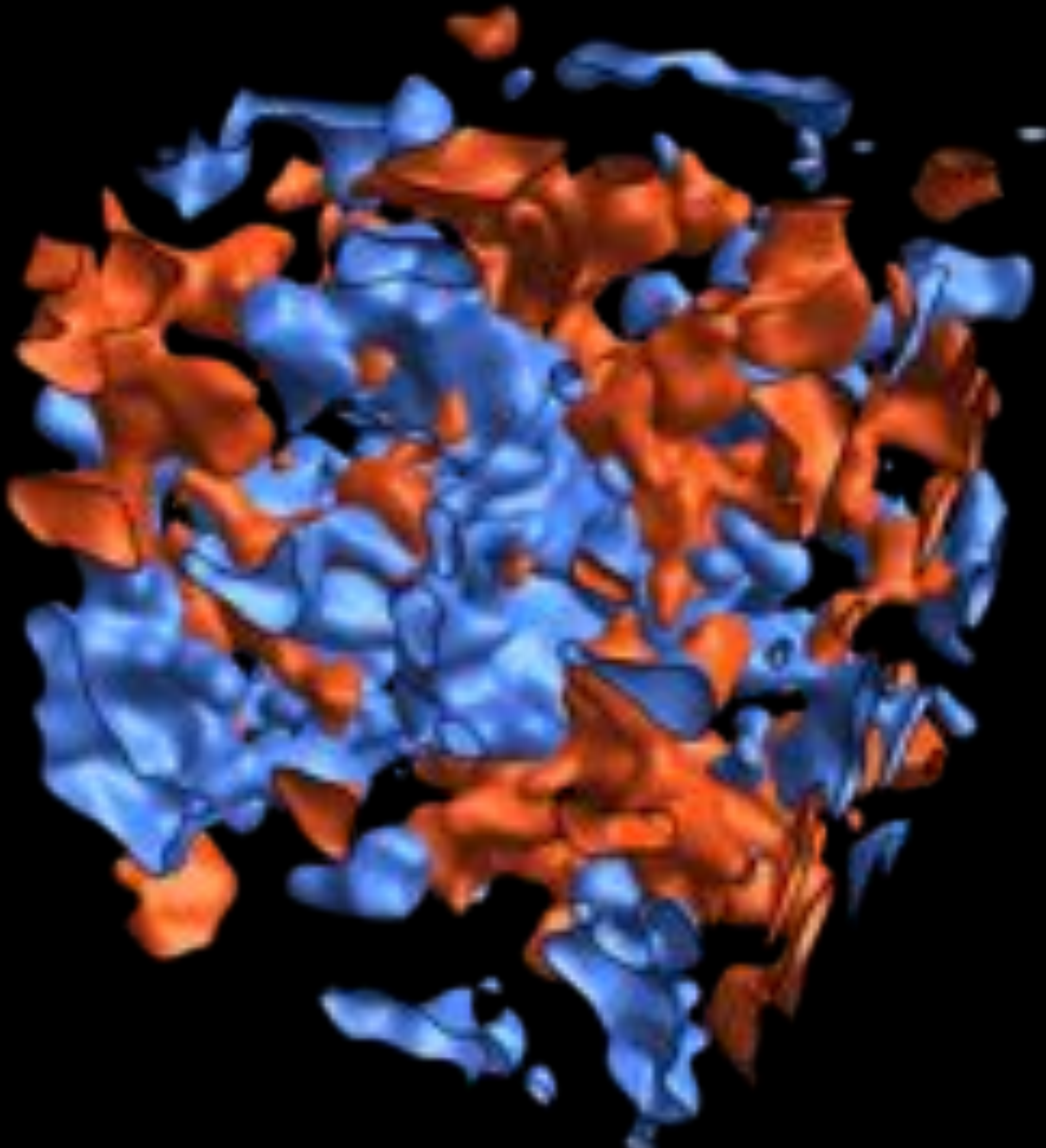
  - entanglement, squeezing and the CMB
  - origin of primordial entanglement

* Inflation and pre-inflationary initial conditions

  - PLANCK constraints on models
  - gravity-driven inflation, initial conditions and entanglement

The Vacuum

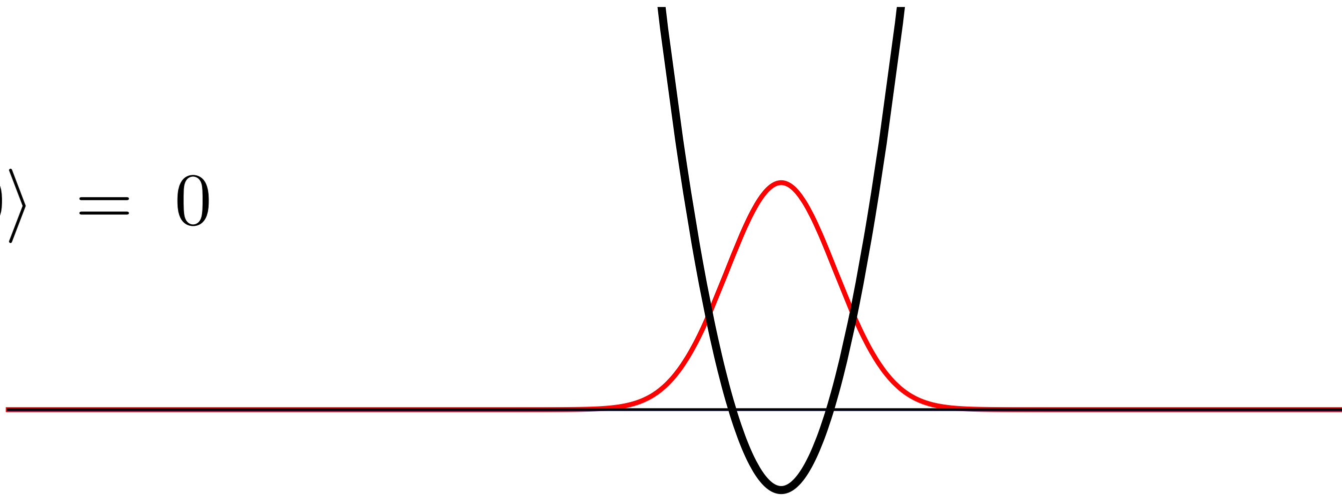
The Vacuum State of a Quantum Field



The Vacuum State of a Quantum Field

No particles $a(\vec{k})\,|0\rangle \;=\; 0$

Vanishing expectation value $\langle 0|\,\varphi(\vec{x})\,|0\rangle \;=\; 0$



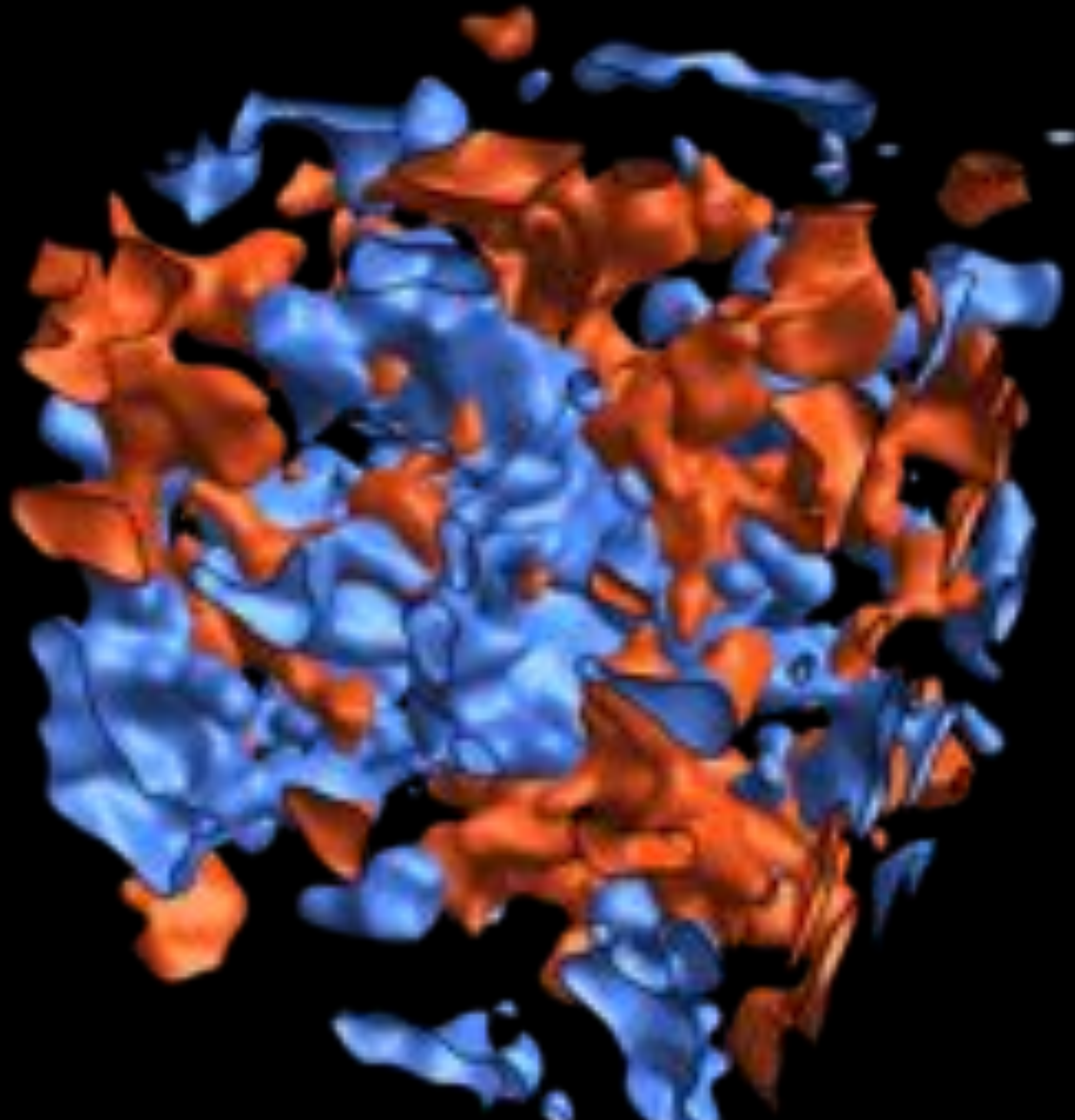
Non-vanishing correlations

$$\langle 0|\,\varphi(\vec{x})\;\varphi(\vec{y})\,|0\rangle \;=\; \int \frac{d^3\vec{k}}{(2\pi)^3}\,\frac{1}{|\vec{k}|}\,\cos\left(\vec{k}\cdot(\vec{x}-\vec{y})\right) \;=\; \frac{1}{2\pi^2}\,\frac{1}{|\vec{x}-\vec{y}\,|^2}$$

Fluctuations of the field averaged over a region of size R

$$(\Delta\varphi_R)^2 \equiv \;\langle 0|\varphi_R\,\varphi_R|0\rangle \;-\left(\langle 0|\varphi_R|0\rangle\right)^2 \;\sim\; \frac{1}{R^2}$$

The Vacuum State of a Quantum Field



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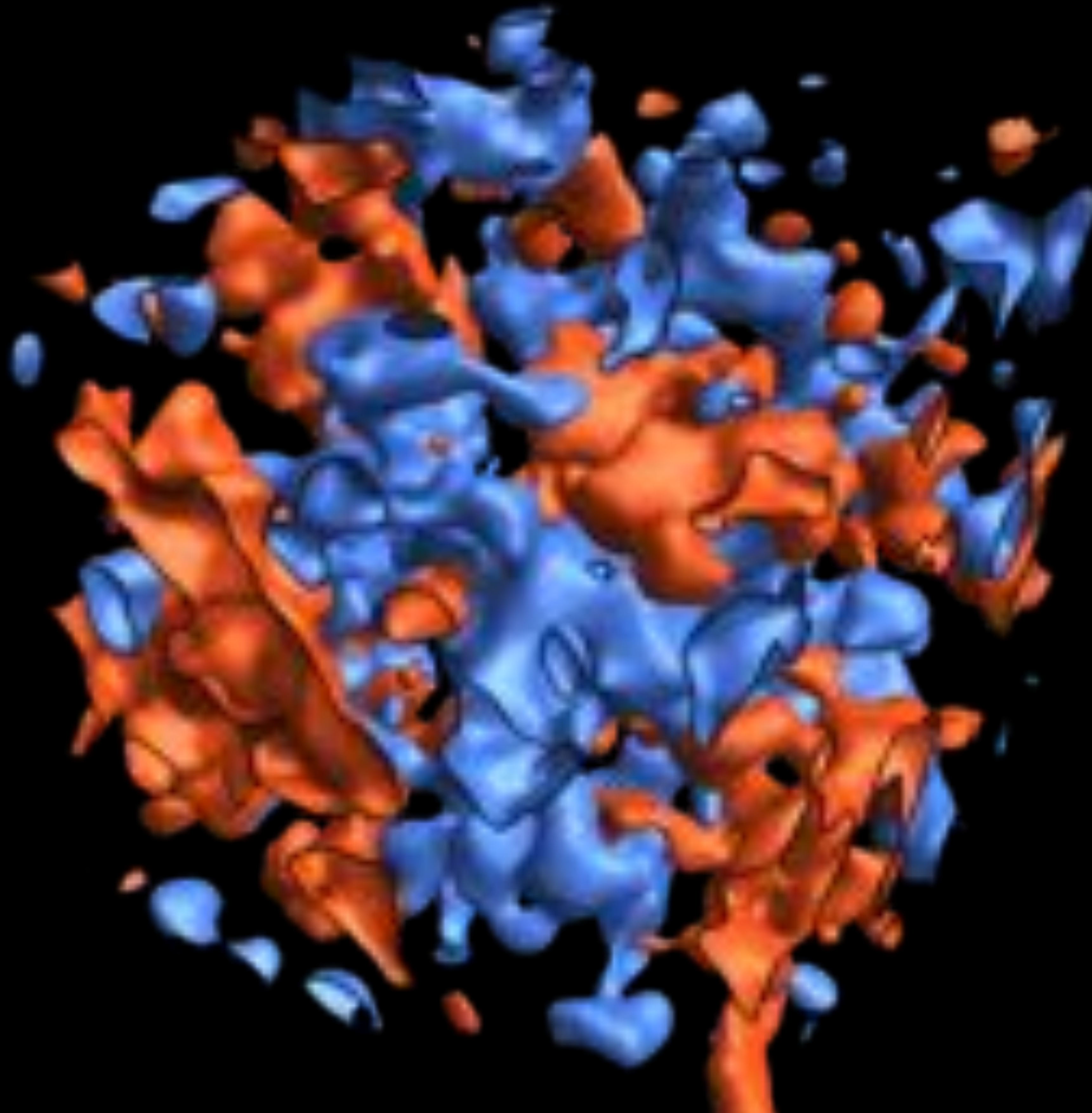

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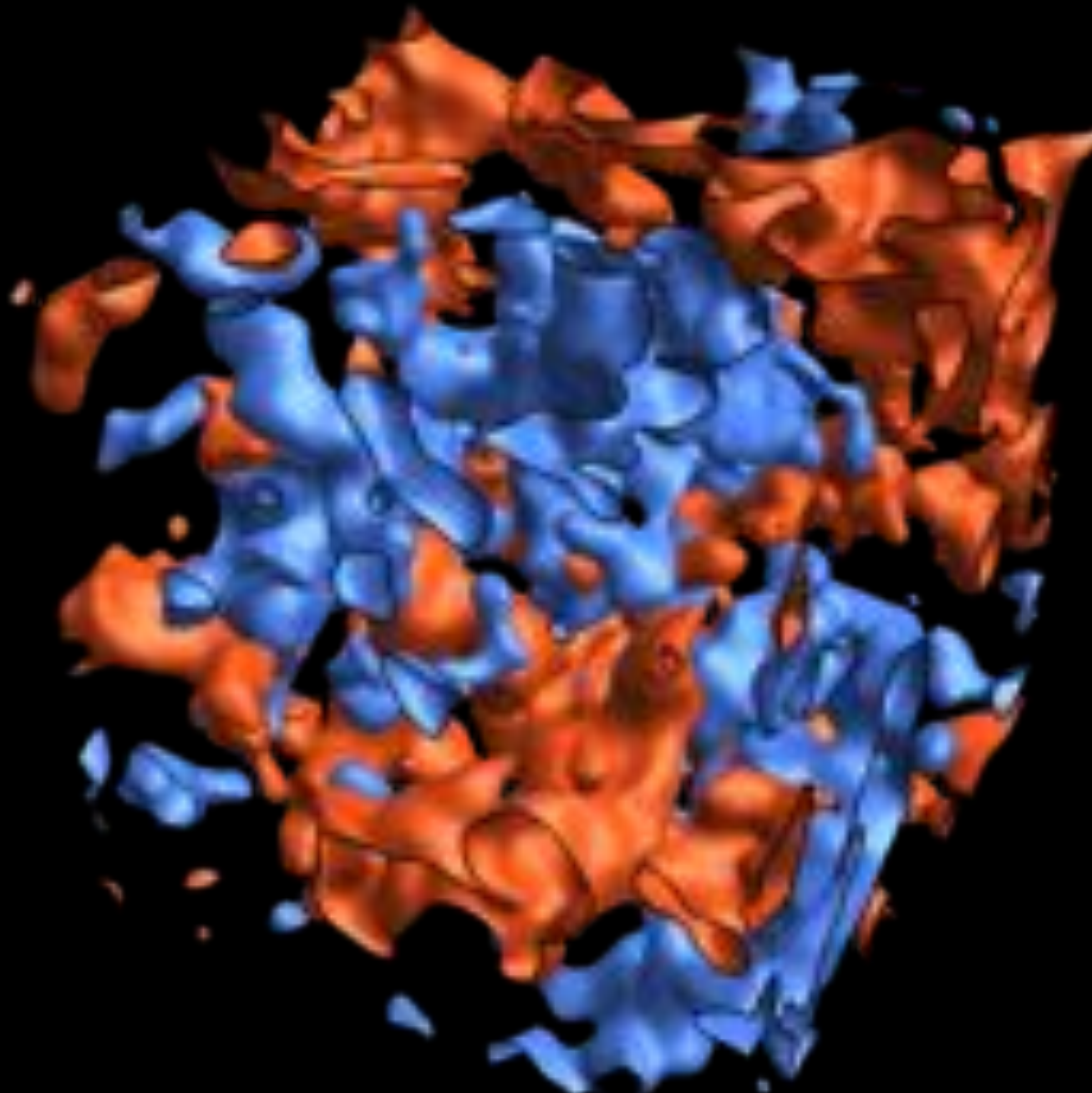
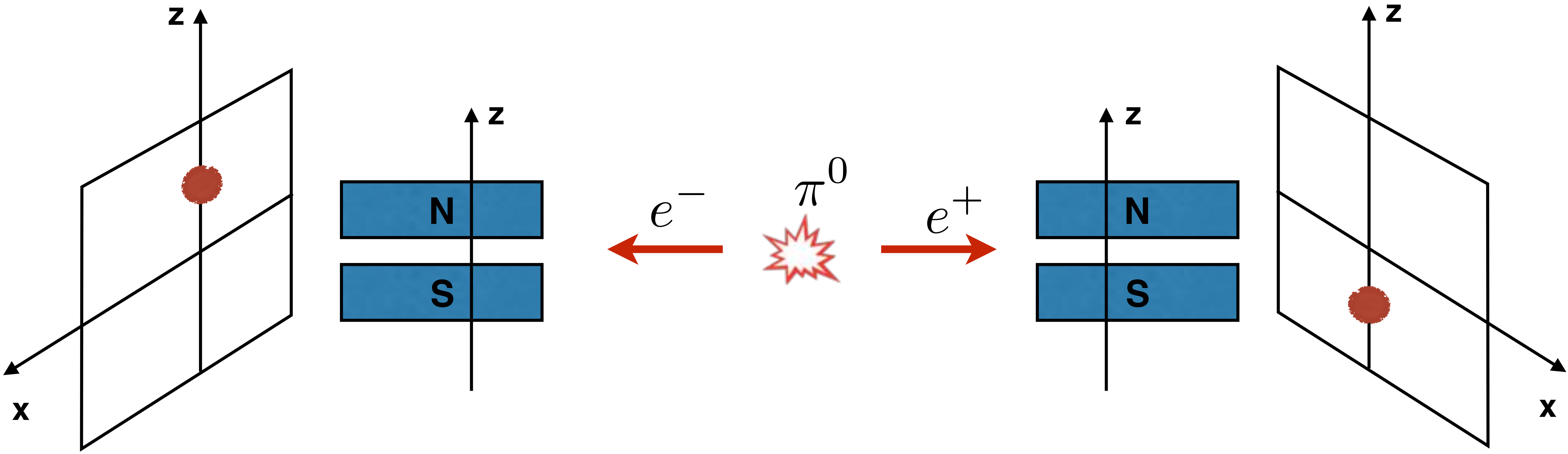


EPR pair: $\pi^0 \to e^- + e^+$

Entanglement

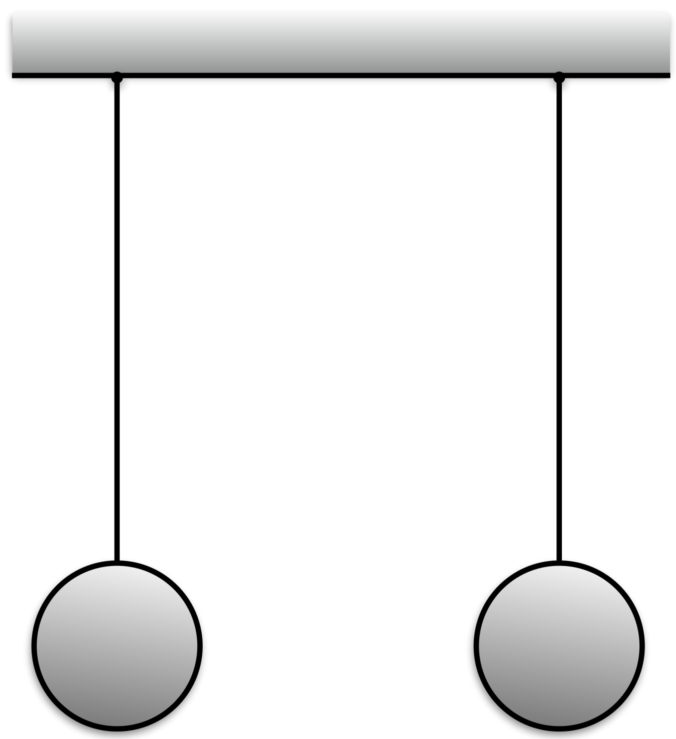
Einstein-Podolsky-Rosen (1935)



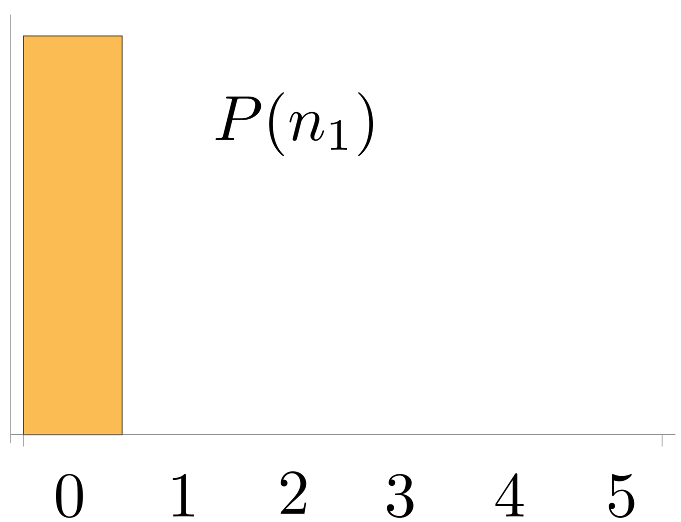
$$|s\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\Big)$$

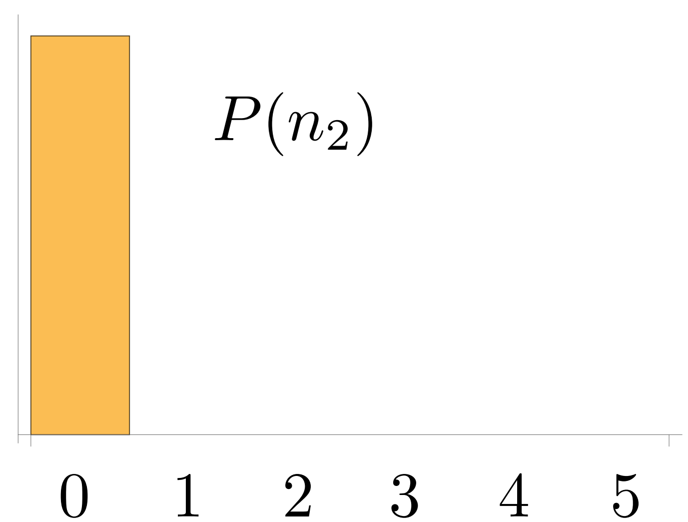
Entangled oscillators

Ground state

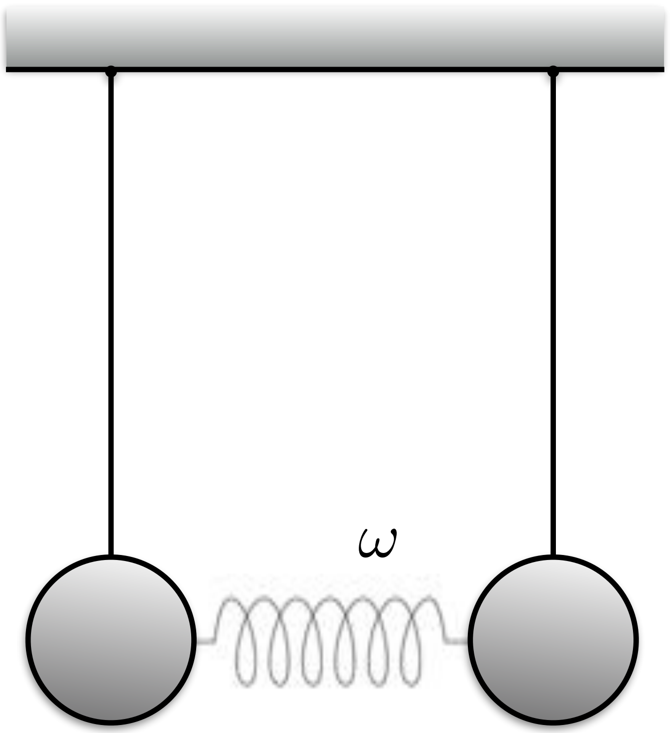


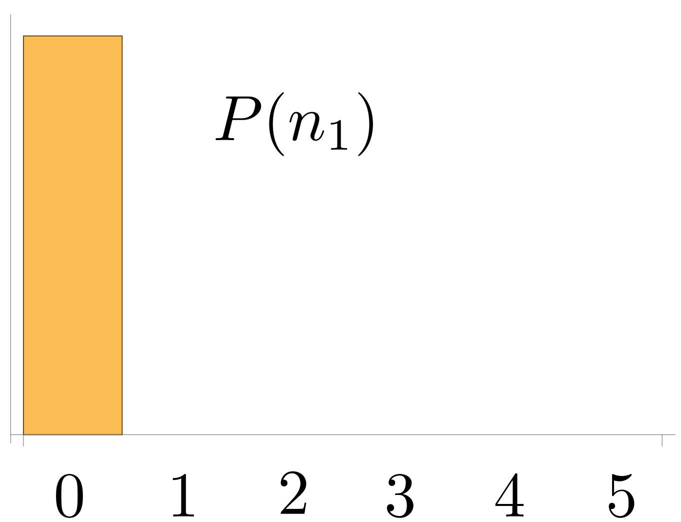
$|0\rangle|0\rangle$

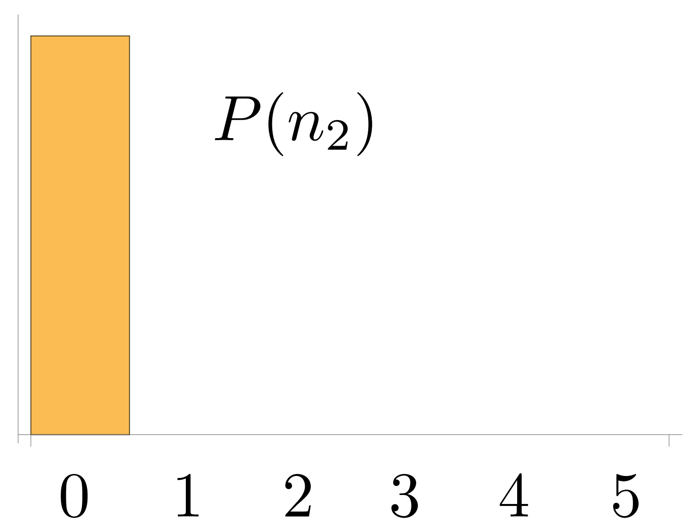




Entangled oscillators







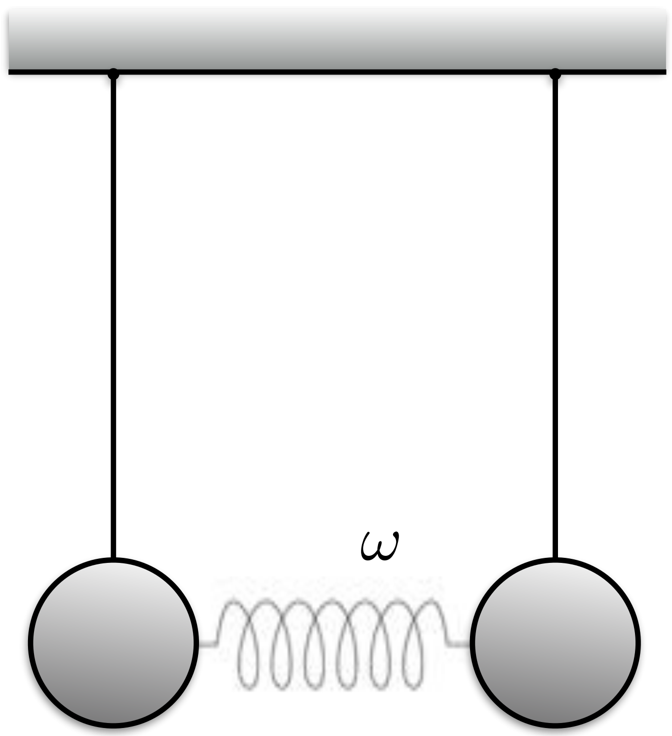
Ground state

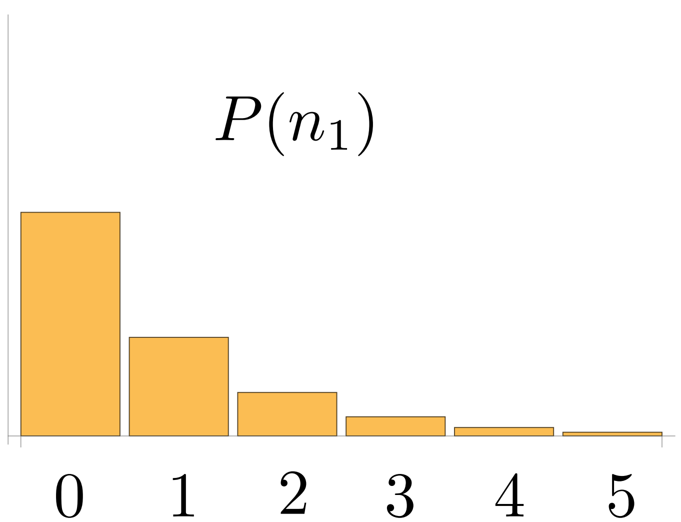
without the spring

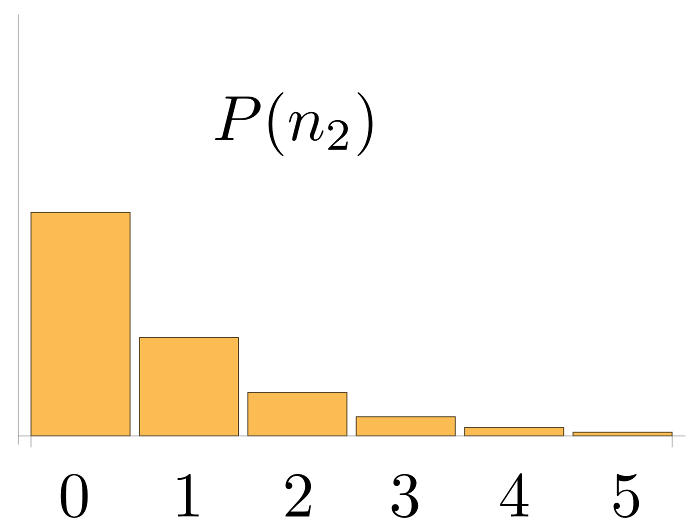
$|0\rangle|0\rangle$

with the spring

Entangled oscillators







Ground state

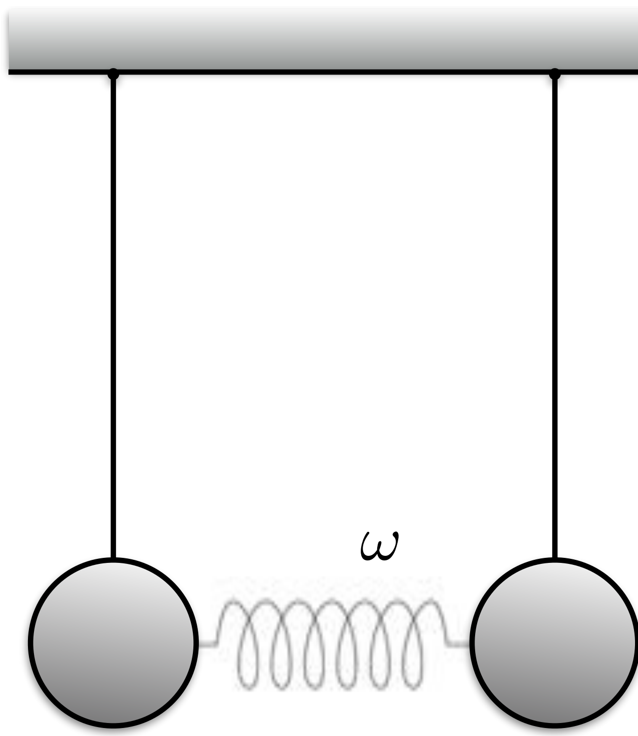
without the spring

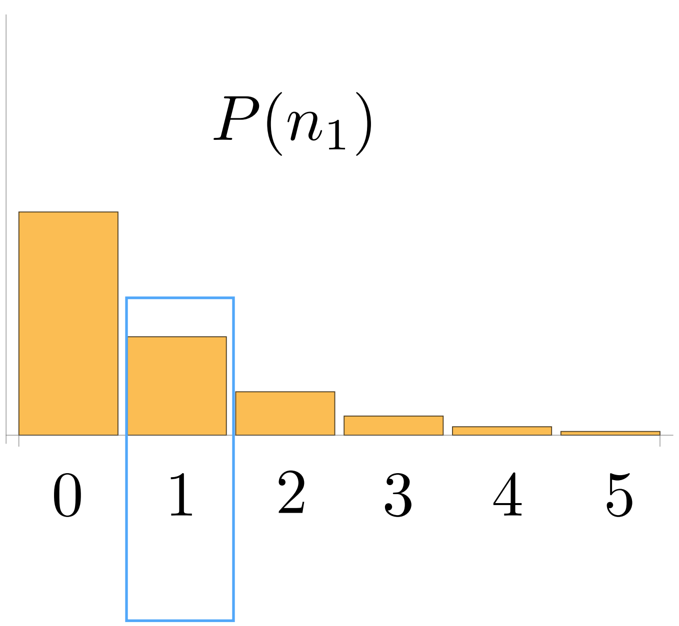
$$|0\rangle|0\rangle$$

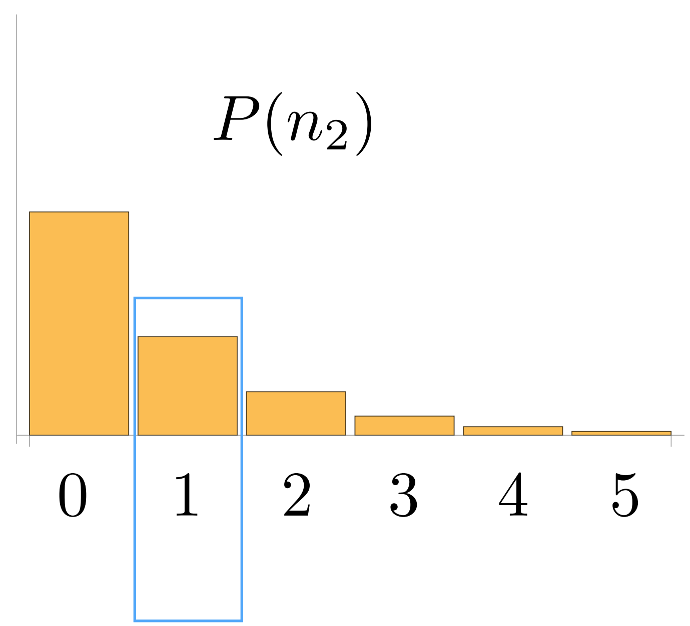
with the spring

$$|\Omega\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle|n\rangle$$

Entangled oscillators







Ground state

without the spring

$$|0\rangle|0\rangle$$

with the spring

$$|\Omega\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle|n\rangle$$

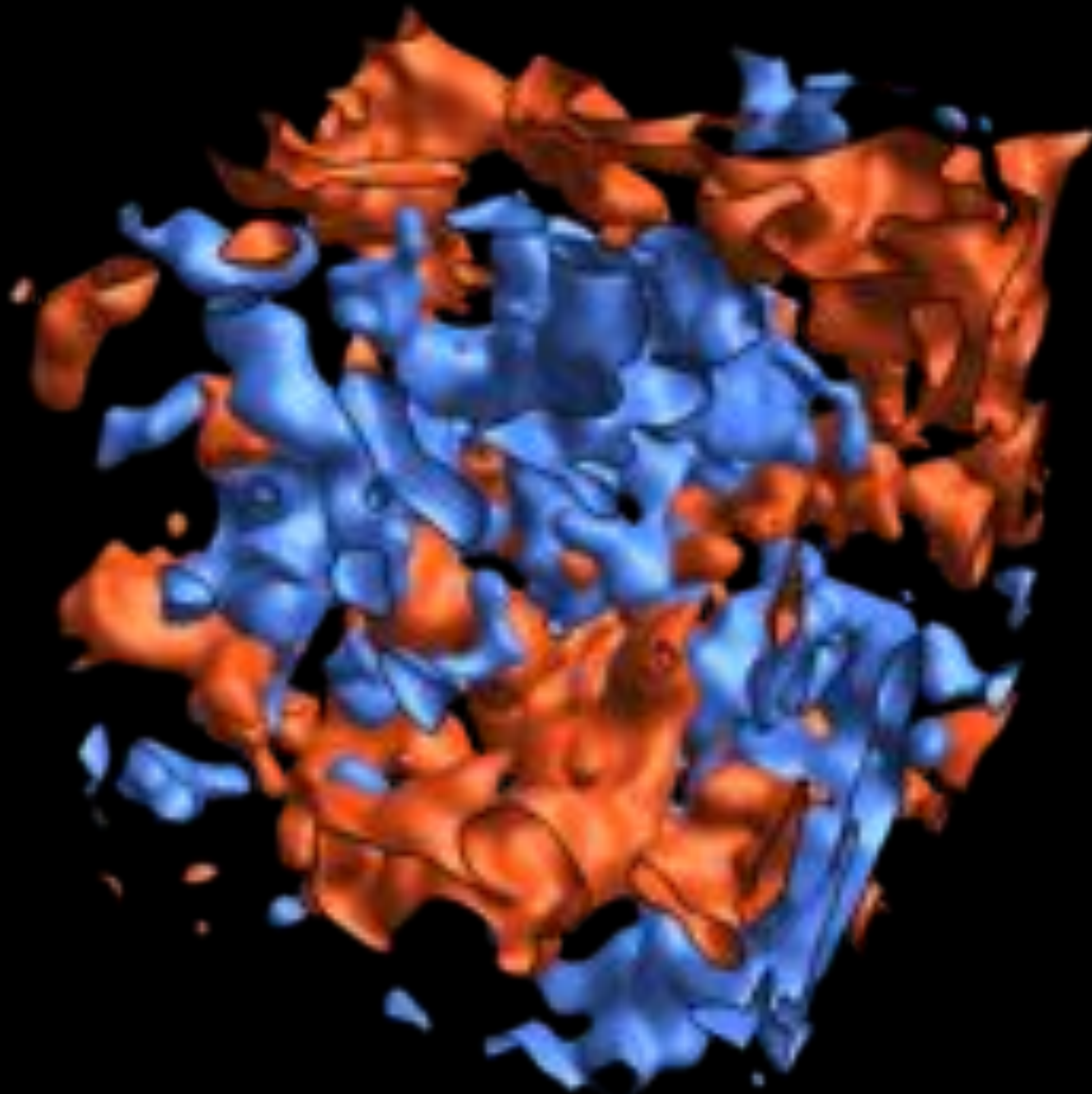
Thermal ensemble from entanglement

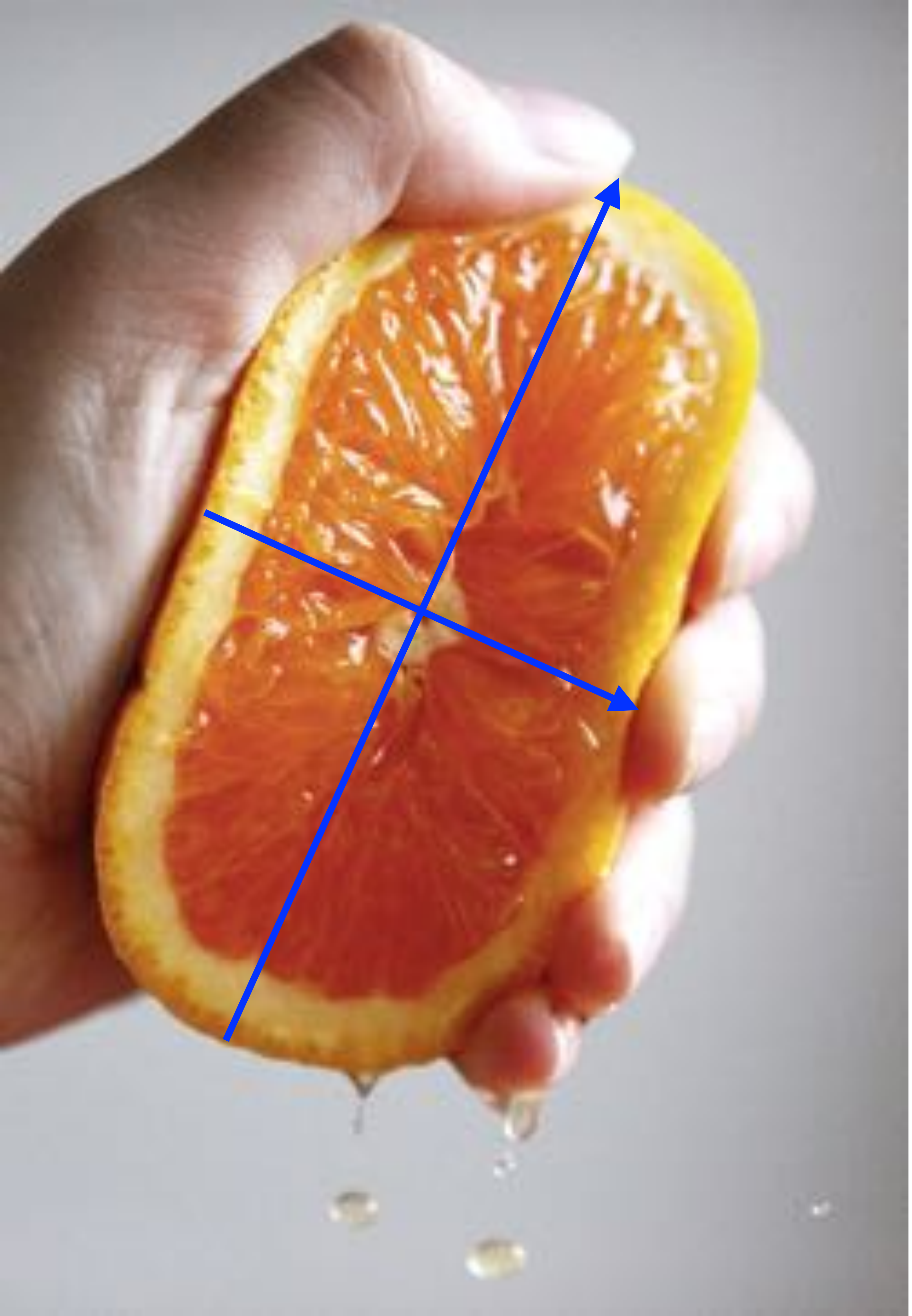
$$\rho_2 = \mathrm{Tr}_1|\Omega\rangle\langle\Omega| = \frac{1}{Z}\sum_{n=0}^{\infty}e^{-\beta n}|n\rangle\langle n|$$

Entanglement entropy

$$S_2 = -\mathrm{Tr}(\rho_2\log\rho_2)$$

The vacuum state of a quantum field is highly entangled

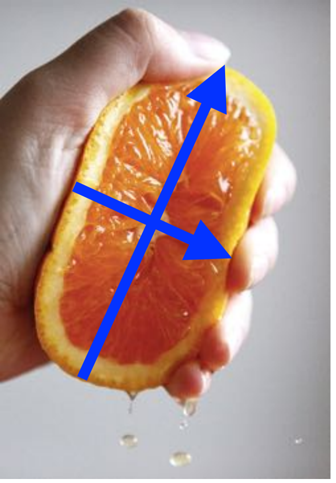




Squeezing the vacuum

examples:

- Schwinger effect
- Hawking radiation
- Primordial density fluctuations



Squeezing the vacuum and entanglement entropy growth

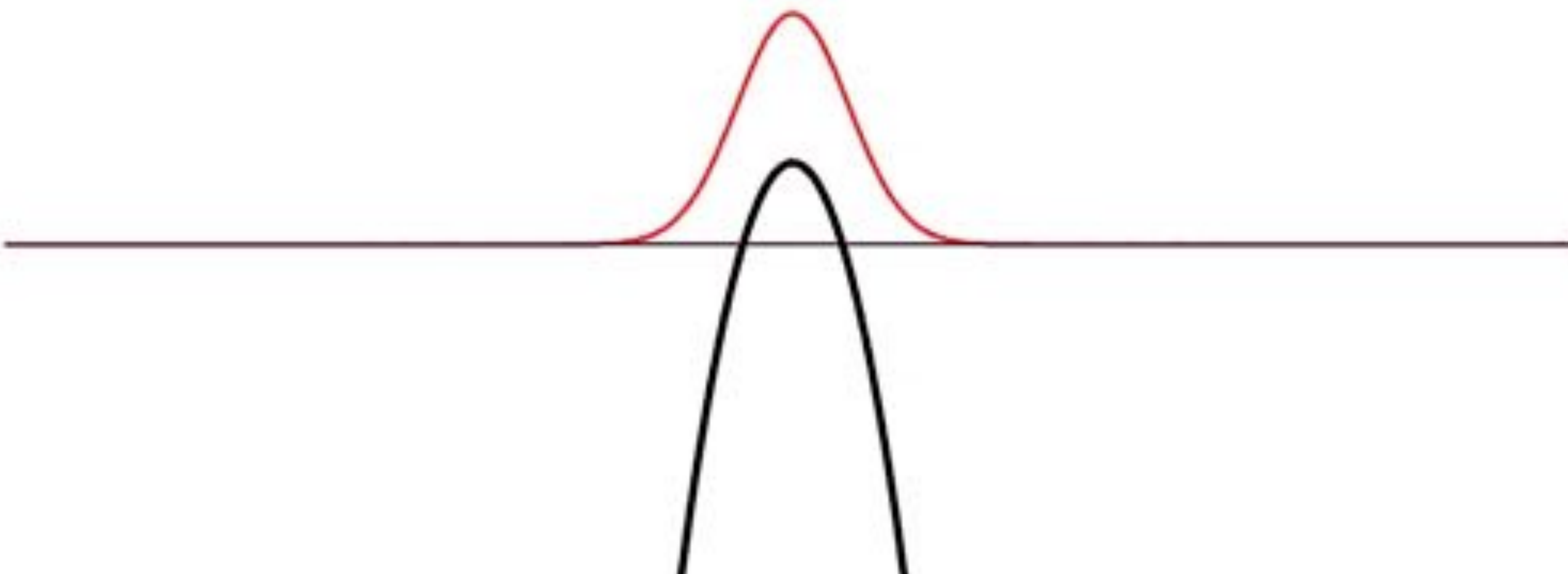
Two new results:

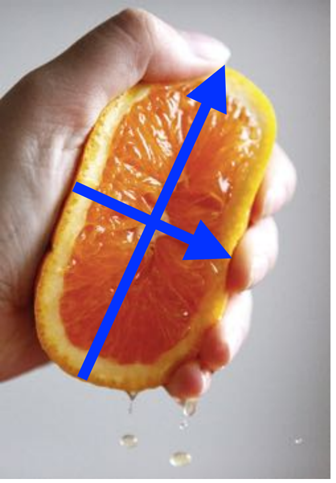
(1) Entanglement growth at instabilities

$$S_A(t) \equiv -\mathrm{Tr}_A\big(\rho(t)\log\rho(t)\big) \;\sim\; \Big(\sum_i \lambda_i\Big)\, t$$

linear growth: rate given by the sum of the positive Lyapunov exponents

Bianchi-Hackl-Yokomizo Phys.Rev D (2015)





Squeezing the vacuum and entanglement entropy growth

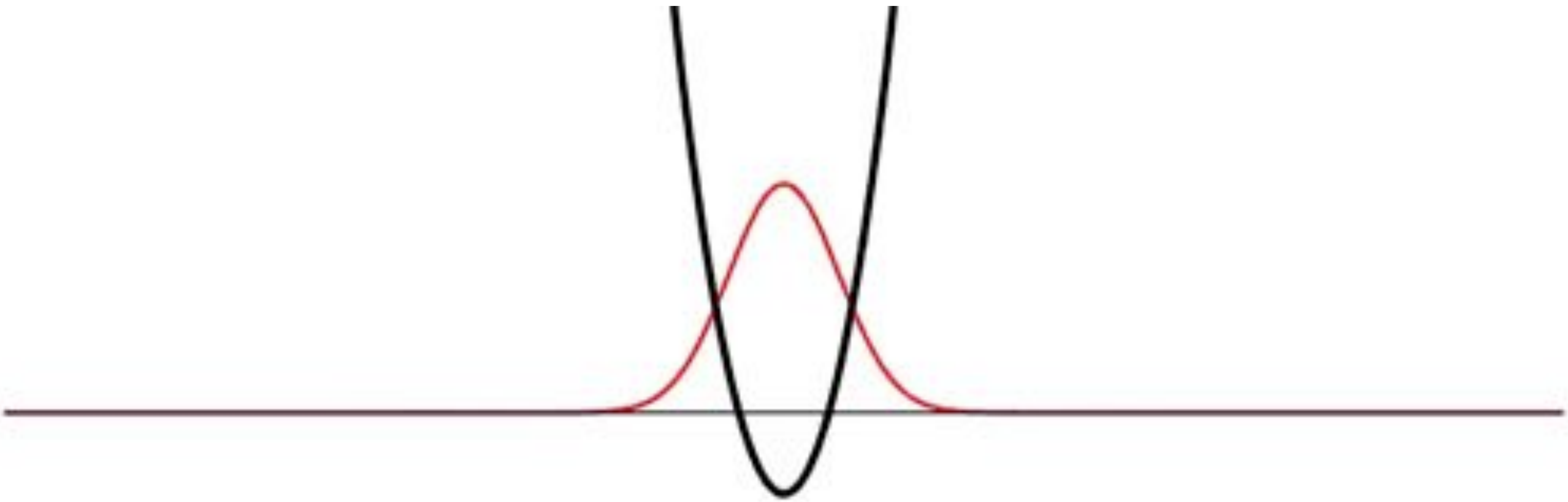
Two new results:

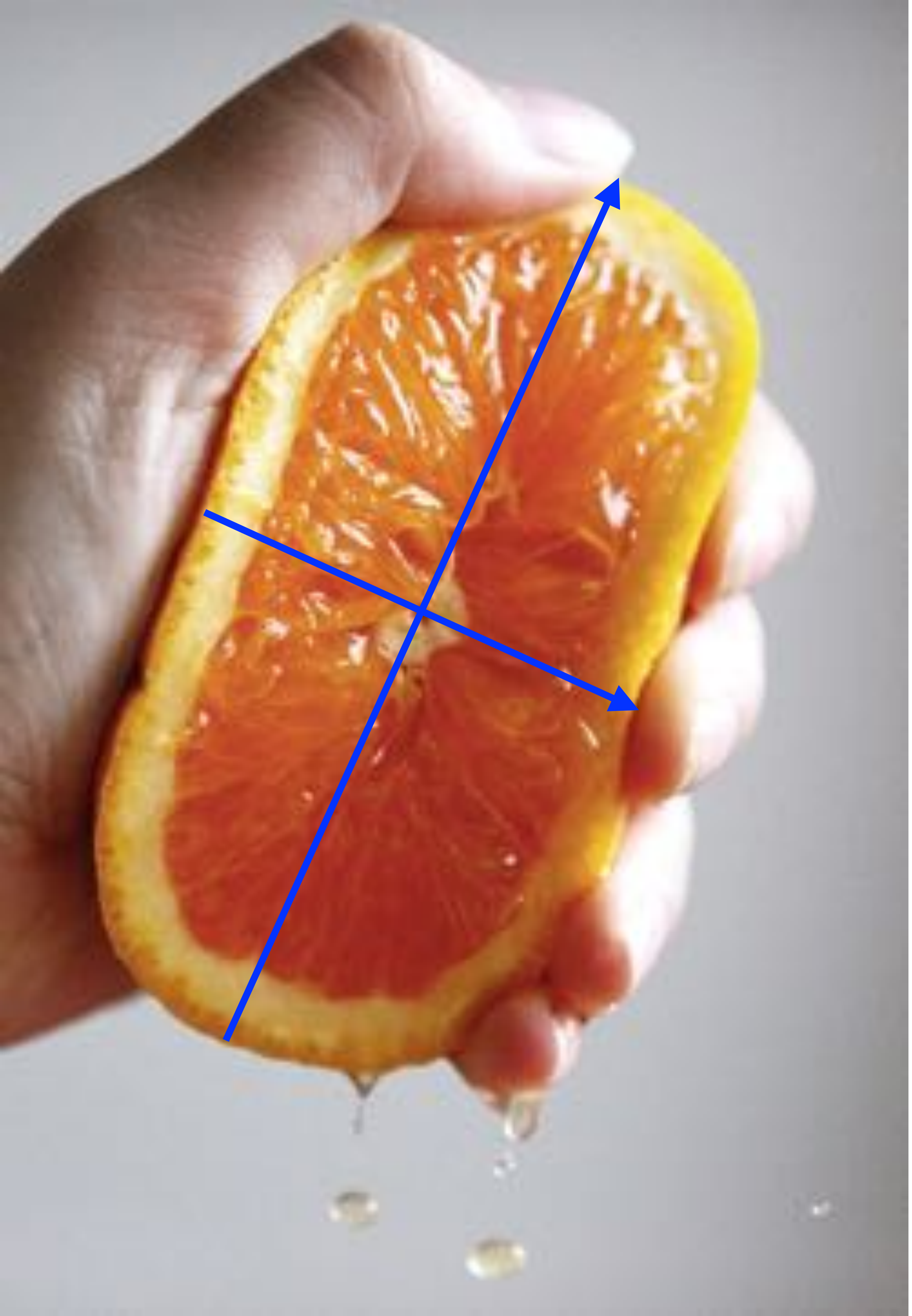
(2) Entanglement growth at parametric resonances

$$S_A(t) \equiv -\mathrm{Tr}_A\big(\rho(t)\log\rho(t)\big) \;\sim\; \Big(\sum_i \mu_i\Big)\, t$$

linear growth: rate given by the sum of the positive Floquet exponents

Bianchi-Hackl-Yokomizo *to appear* (2016)





Squeezing the vacuum

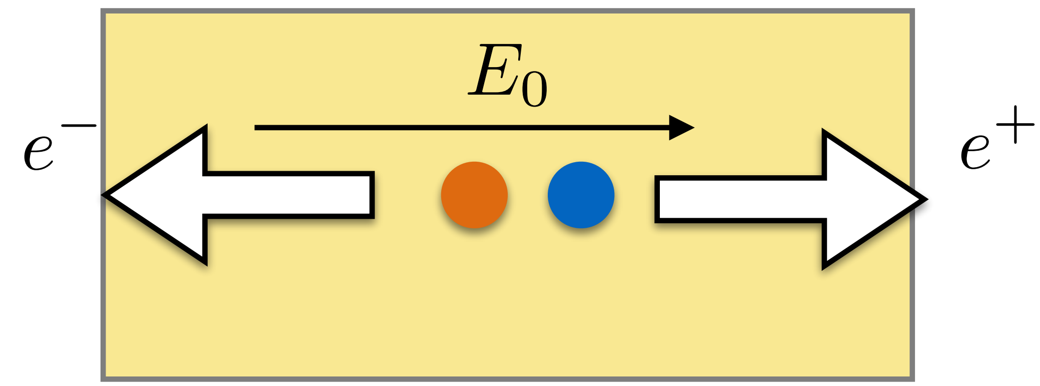
examples:

- Schwinger effect
- Hawking radiation
- Primordial density fluctuations

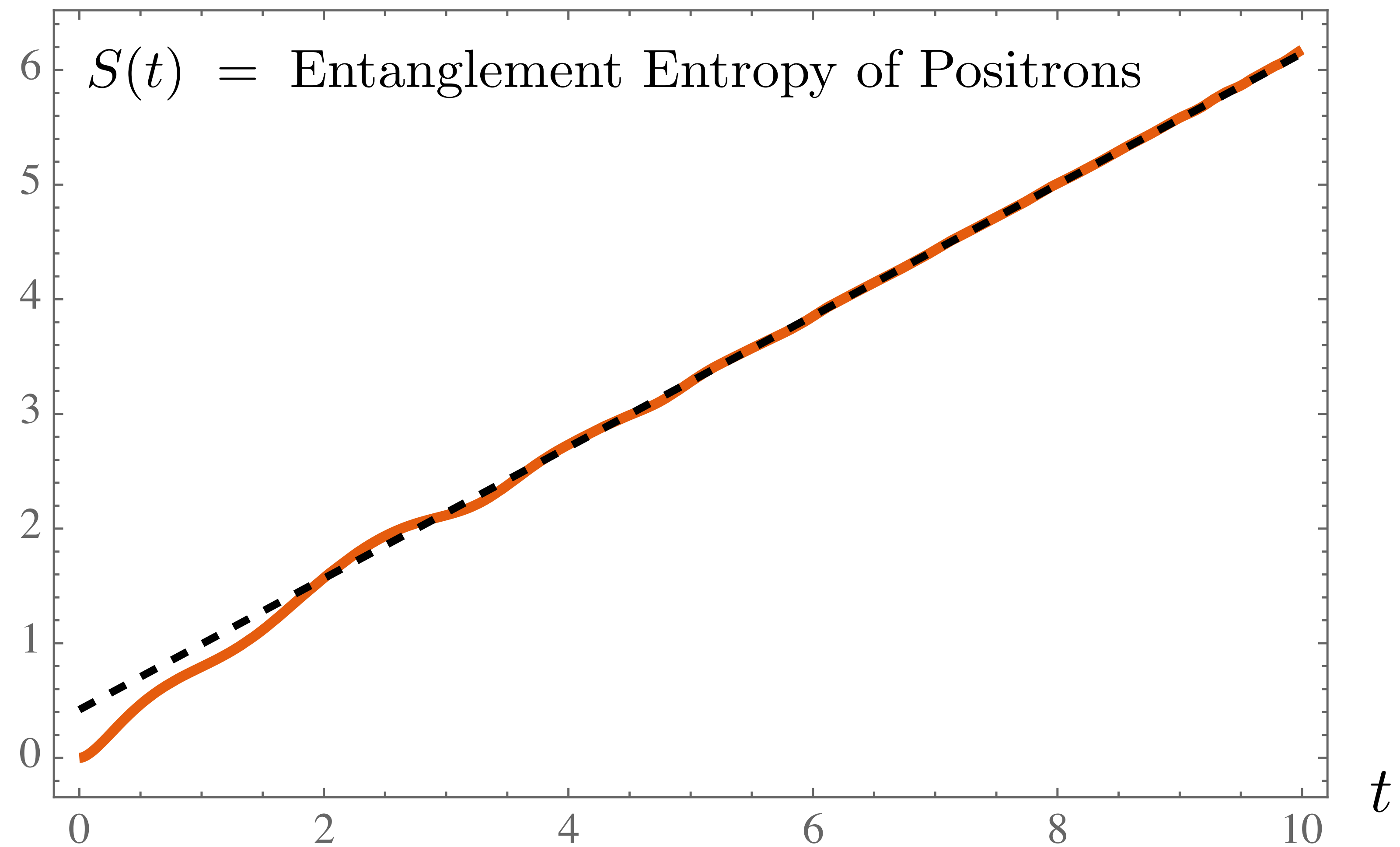
Schwinger effect (1951)

Vacuum fluctuations are unstable in the presence of a strong electric field

$$\hbar c \frac{e\,E_0}{(mc^2)^2} > 1$$



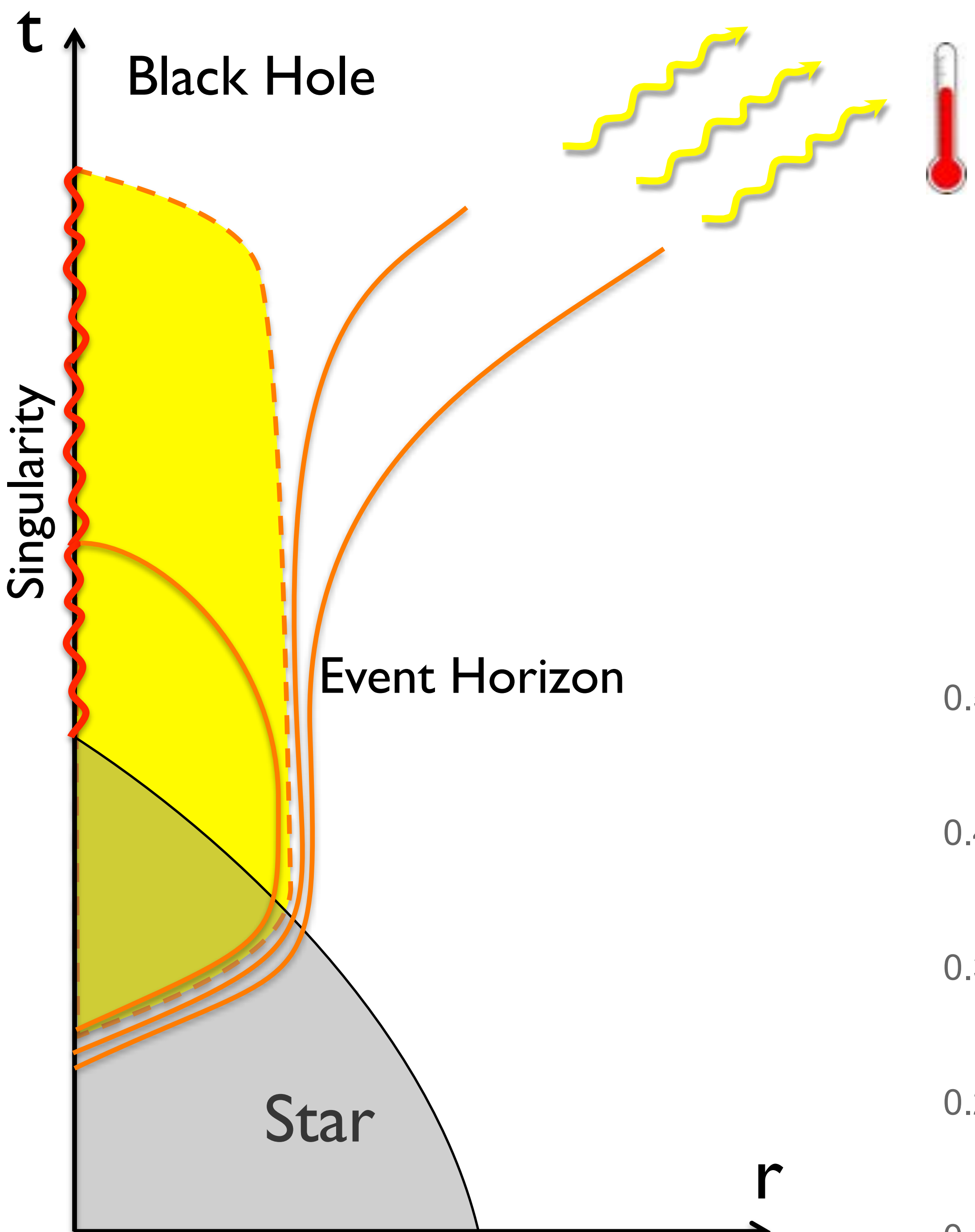
Extraction of *entangled* pairs of electrons and positrons from vacuum fluctuations



EB, Hackl and Yokomizo (2016)

Hawking radiation (1974)

Vacuum fluctuations are unstable in the presence of a strong gravitational field



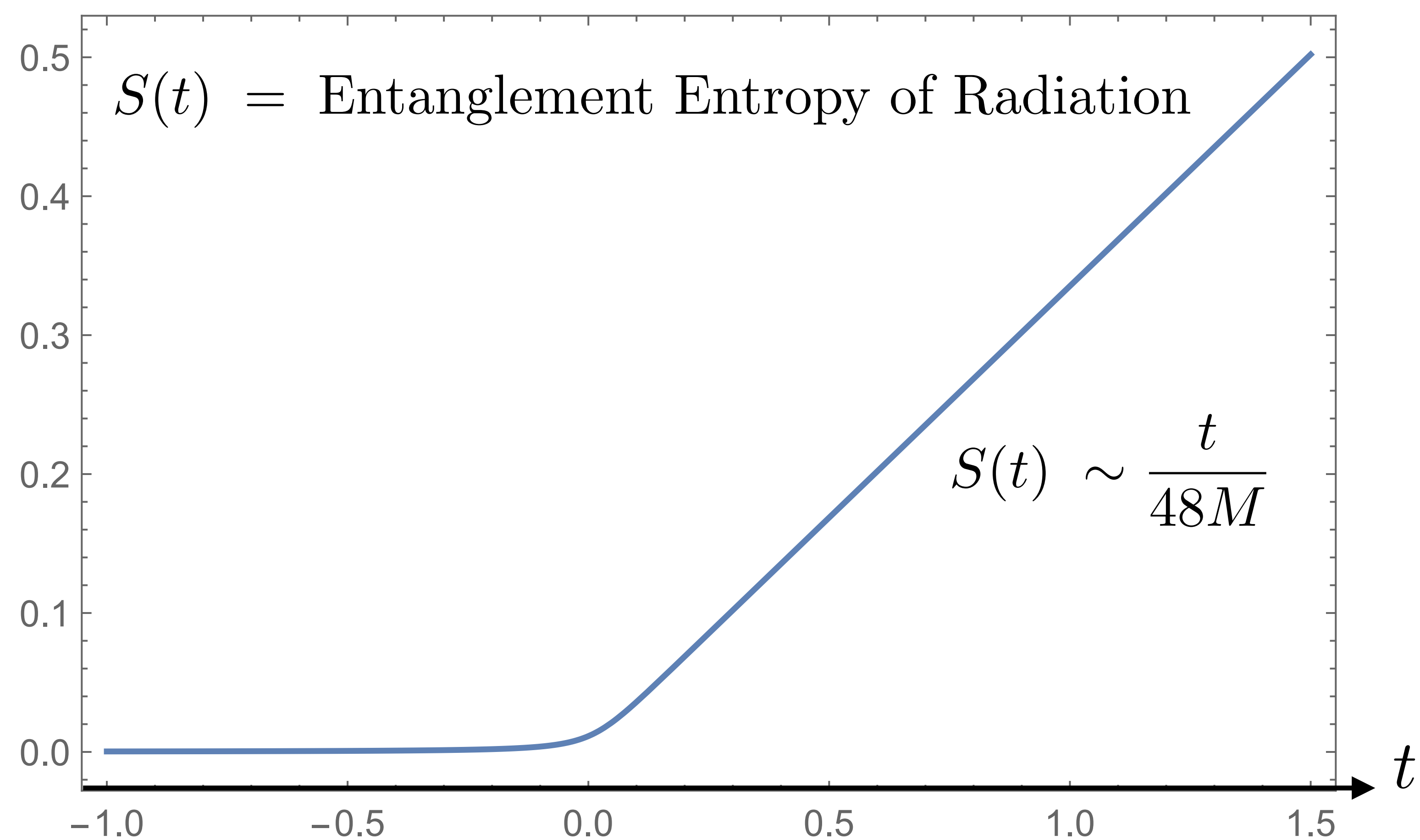
* emission of radiation

Temperature $T = \dfrac{\hbar}{8\pi M}$

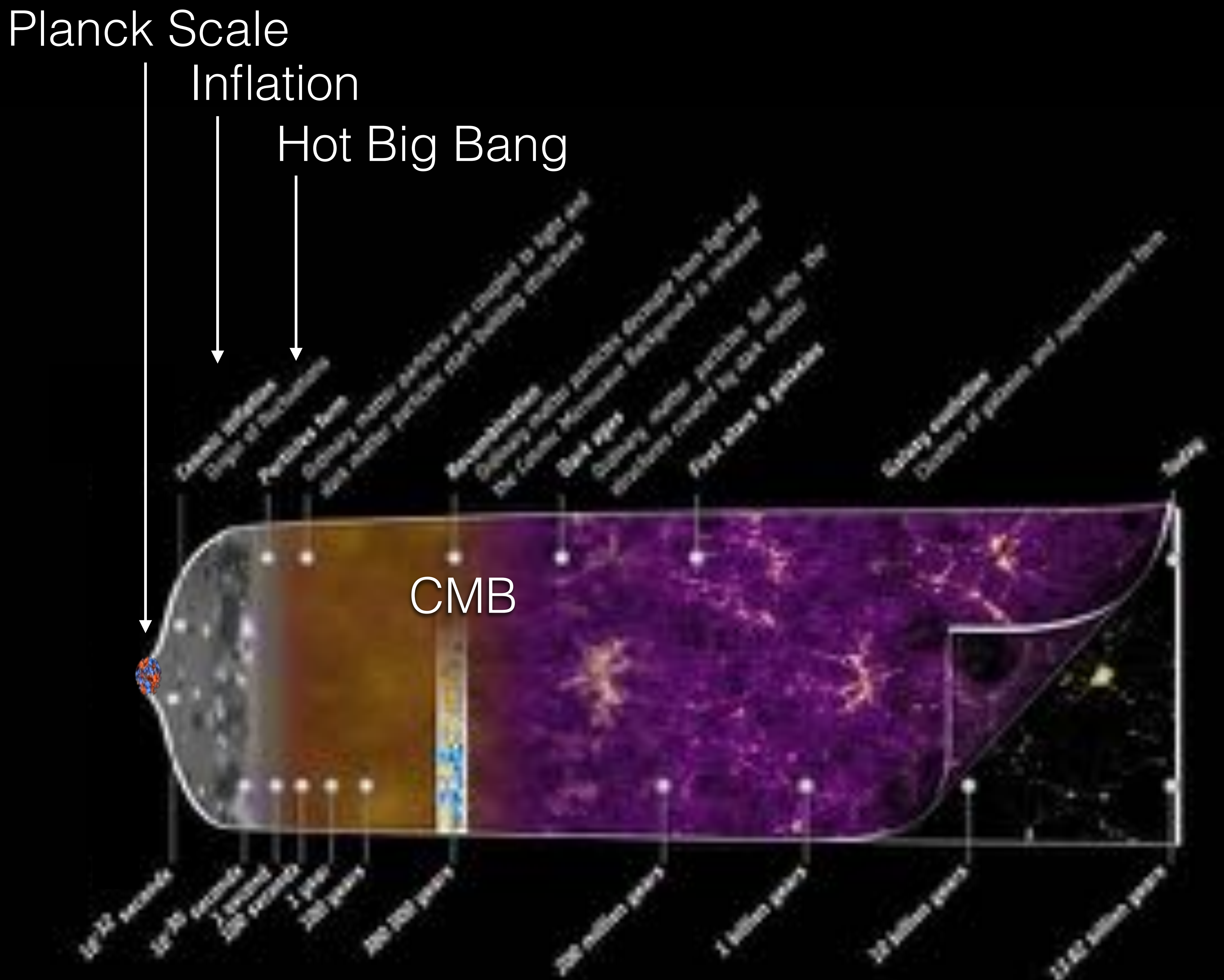
Energy flux $F = \sigma\, A\, T^4 \;\sim\; \dfrac{\hbar}{M^2}$

* energy conservation

Mass loss $\dot{M}(t) = -F(t) \;\sim\; \dfrac{\hbar}{M^2}$

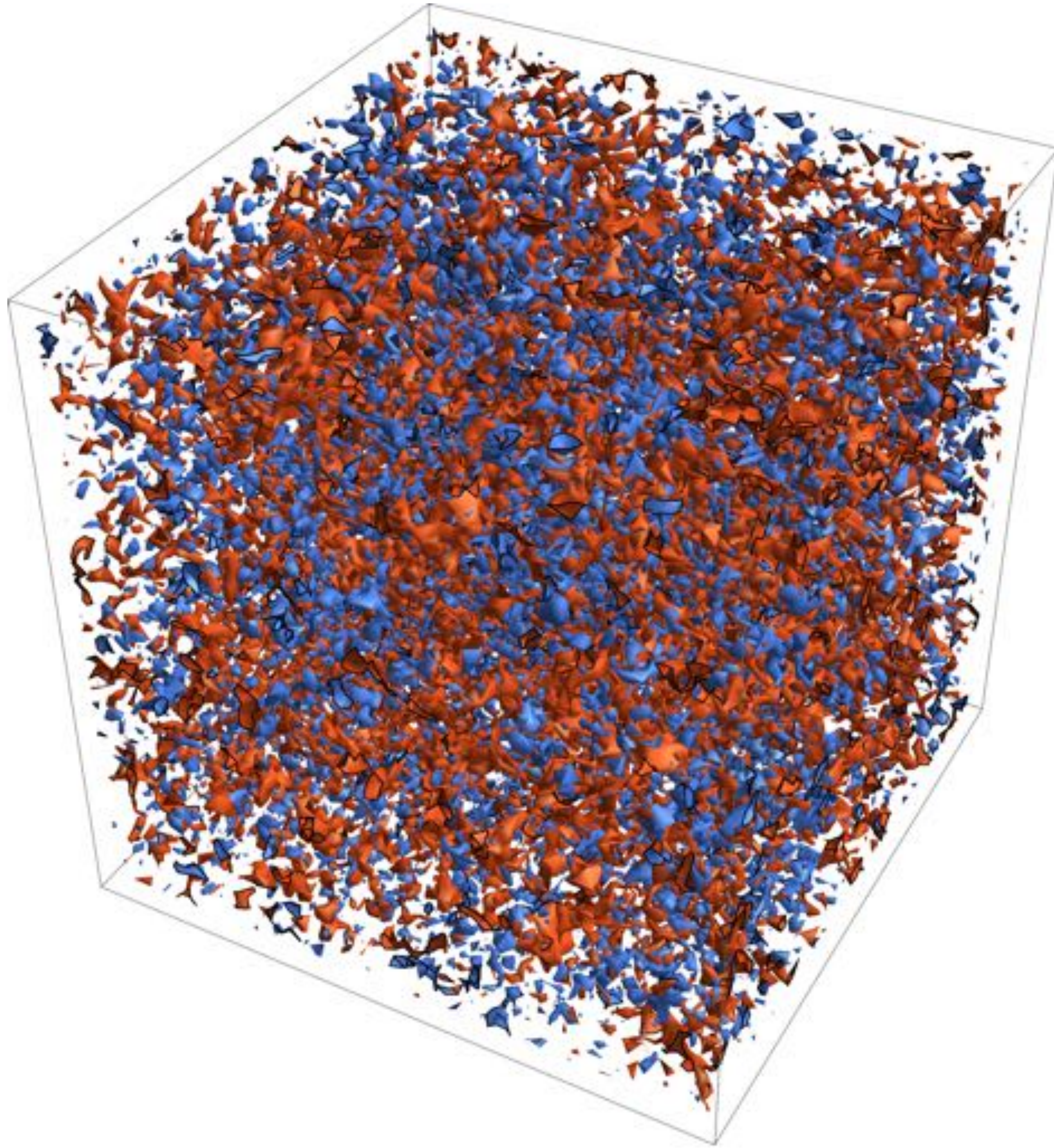


EB, Smerlak, *Phys.Rev.D* (2014)
EB, De Lorenzo and Smerlak, *JHEP* (2015)


Planck Scale
Inflation
Hot Big Bang
CMB

The vacuum of a quantum field before inflation

$$P(k) = \frac{1}{k}$$

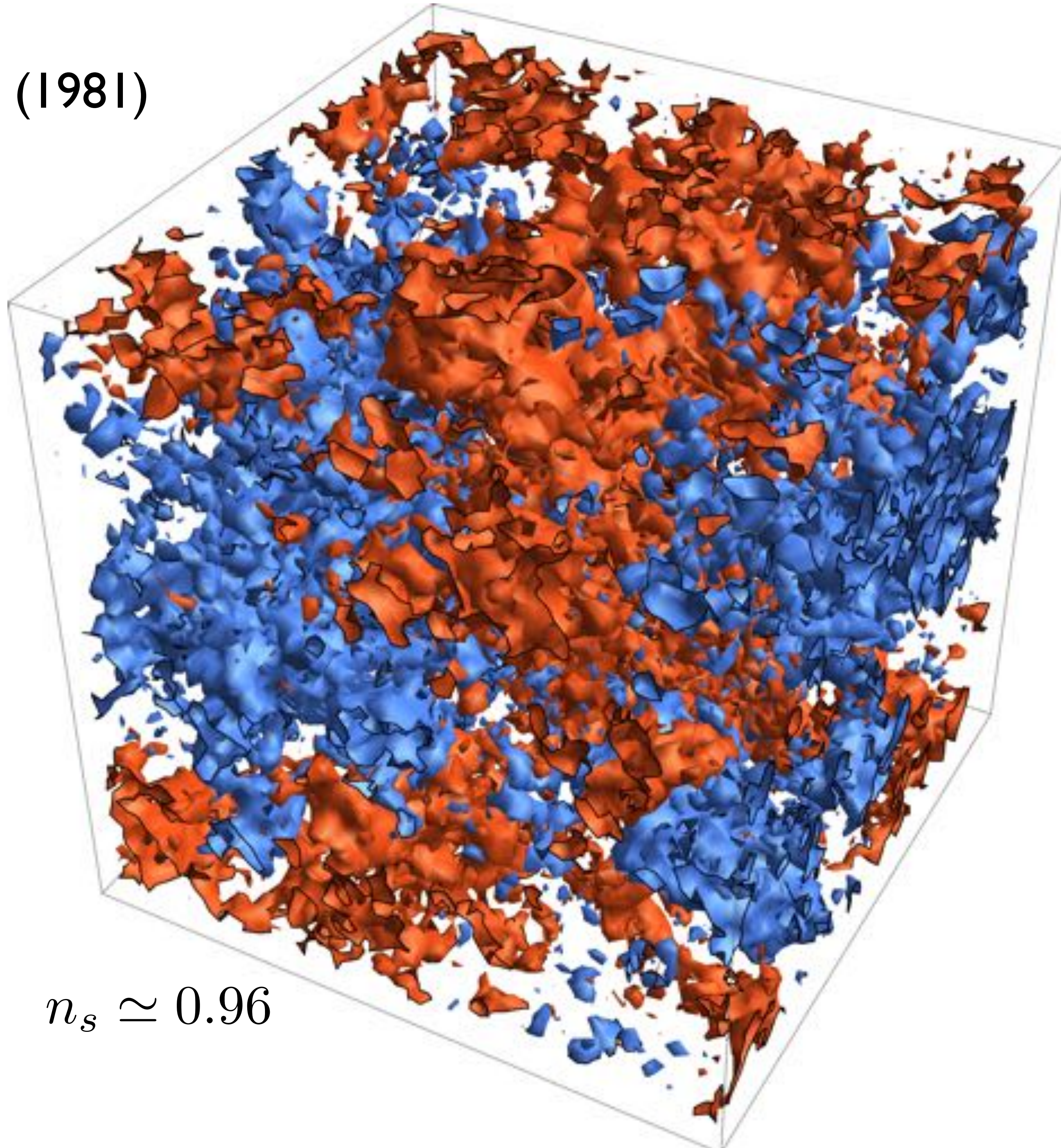


The vacuum of a quantum field after inflation

Mukhanov-Chibisov (1981)

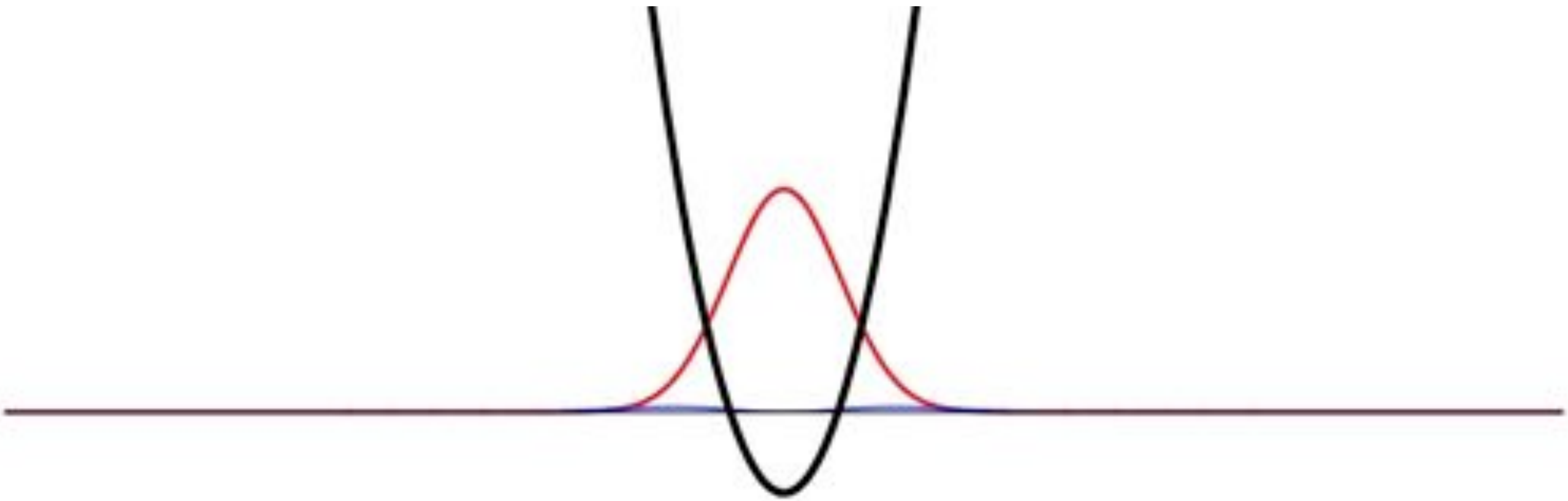
$$P(k) = \frac{(aH)^2}{k^{3n_s}}$$

$$n_s \lesssim 1$$



$$n_s \simeq 0.96$$

Mechanism: amplification of vacuum fluctuations by instabilities

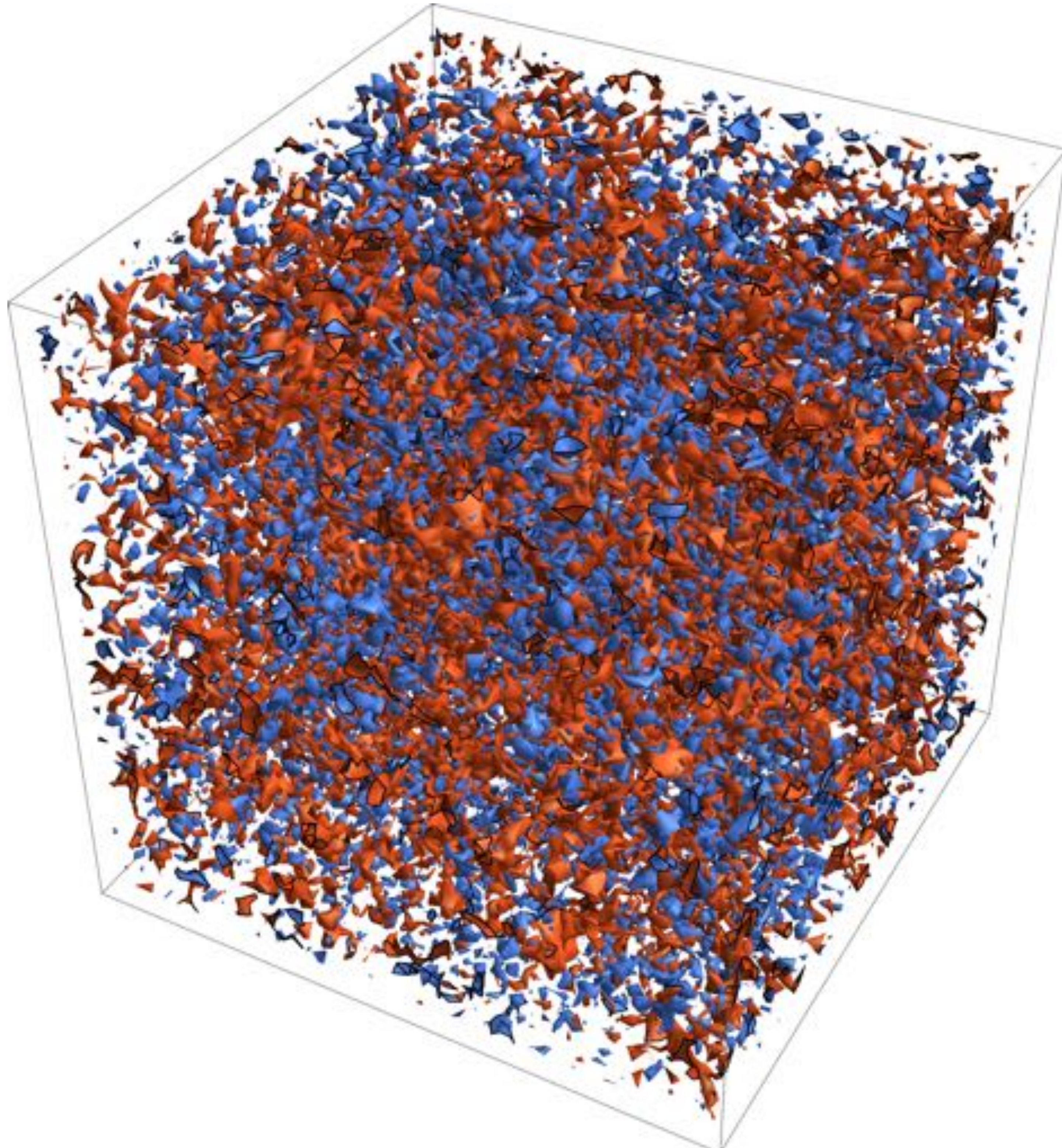


Harmonic oscillator with
time-dependent frequency

$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\left(k^2 - f(t)\right) q^2$$

The vacuum of a quantum field before inflation

$$P(k) = \frac{1}{k}$$



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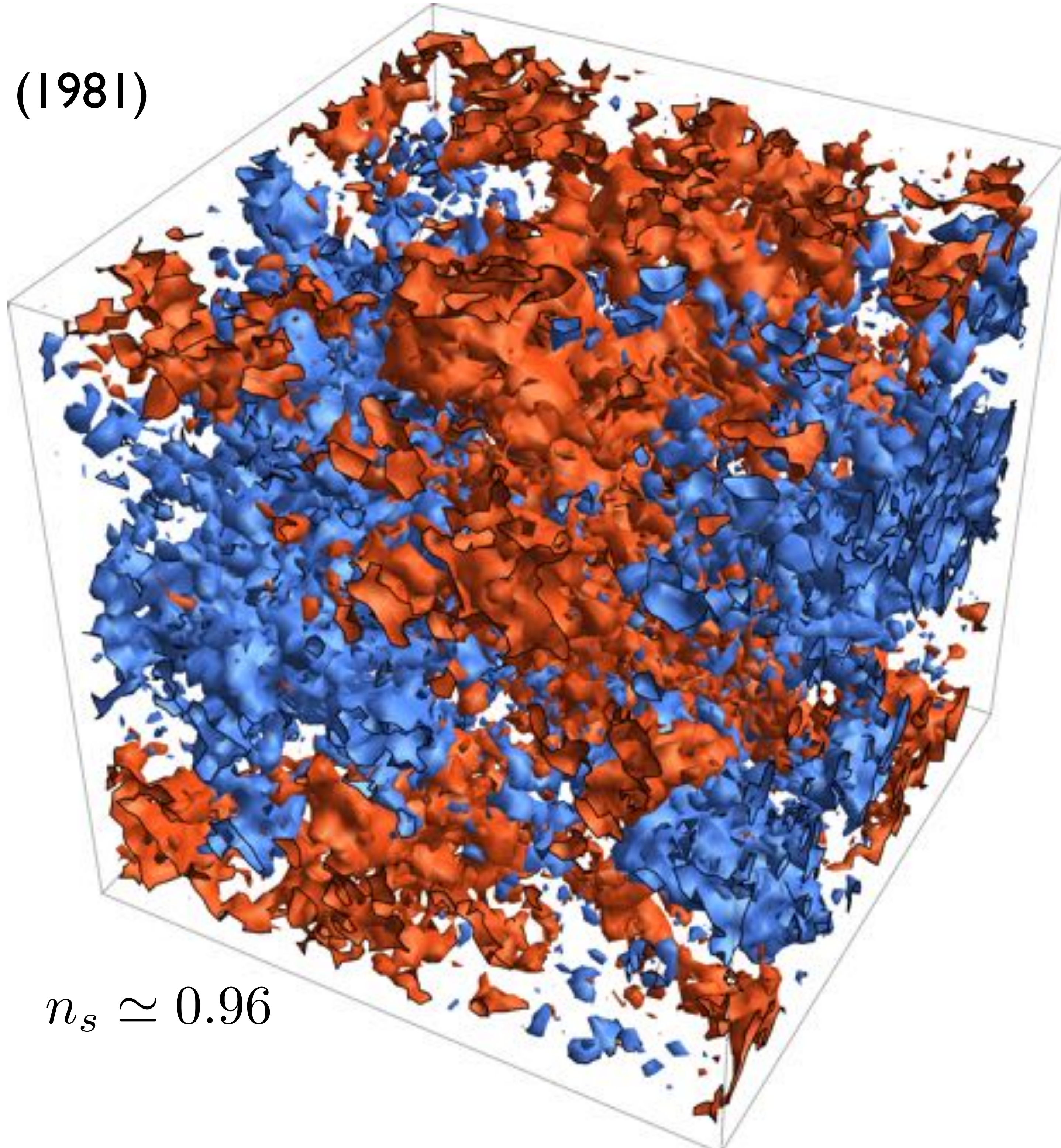
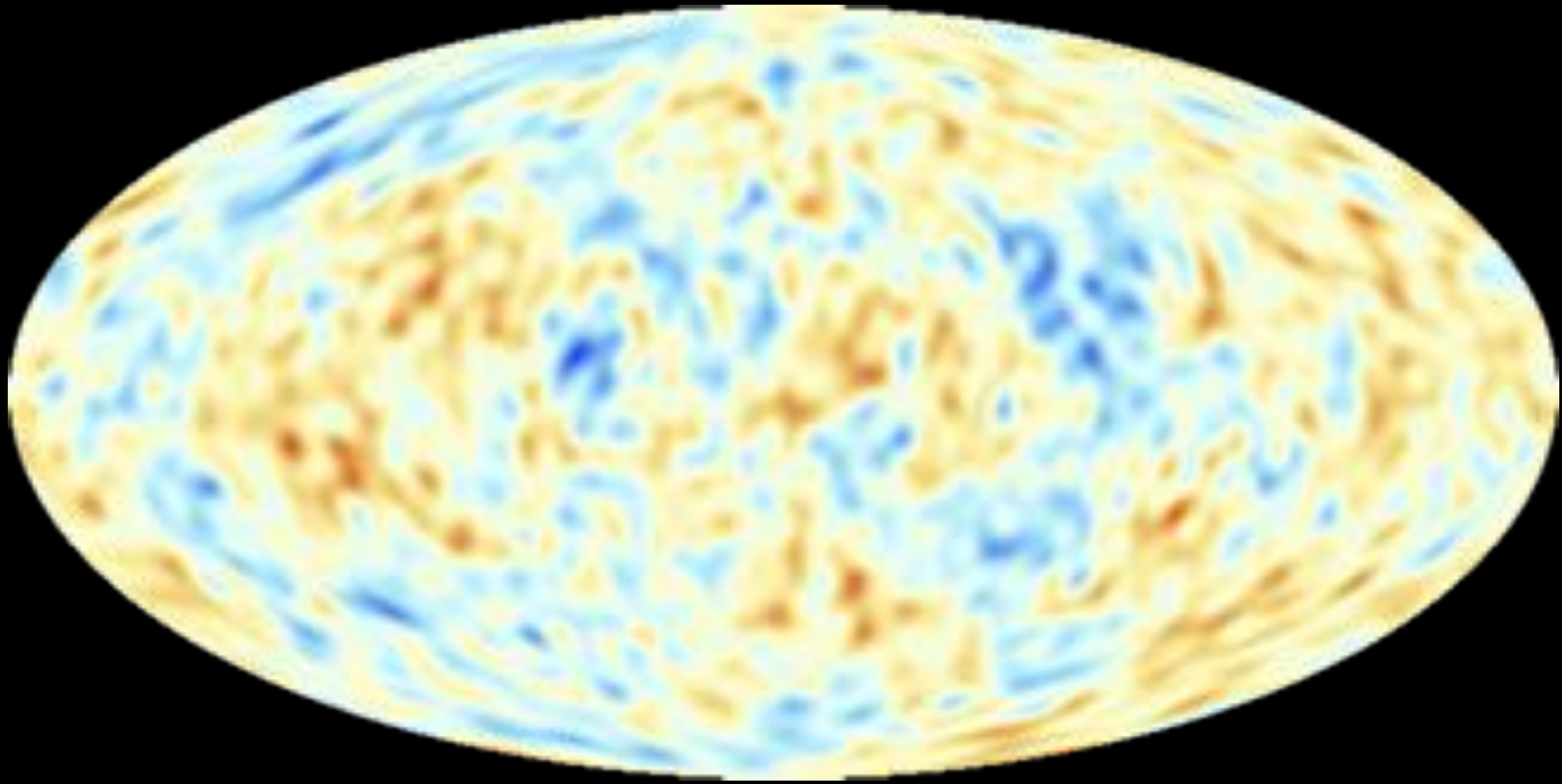
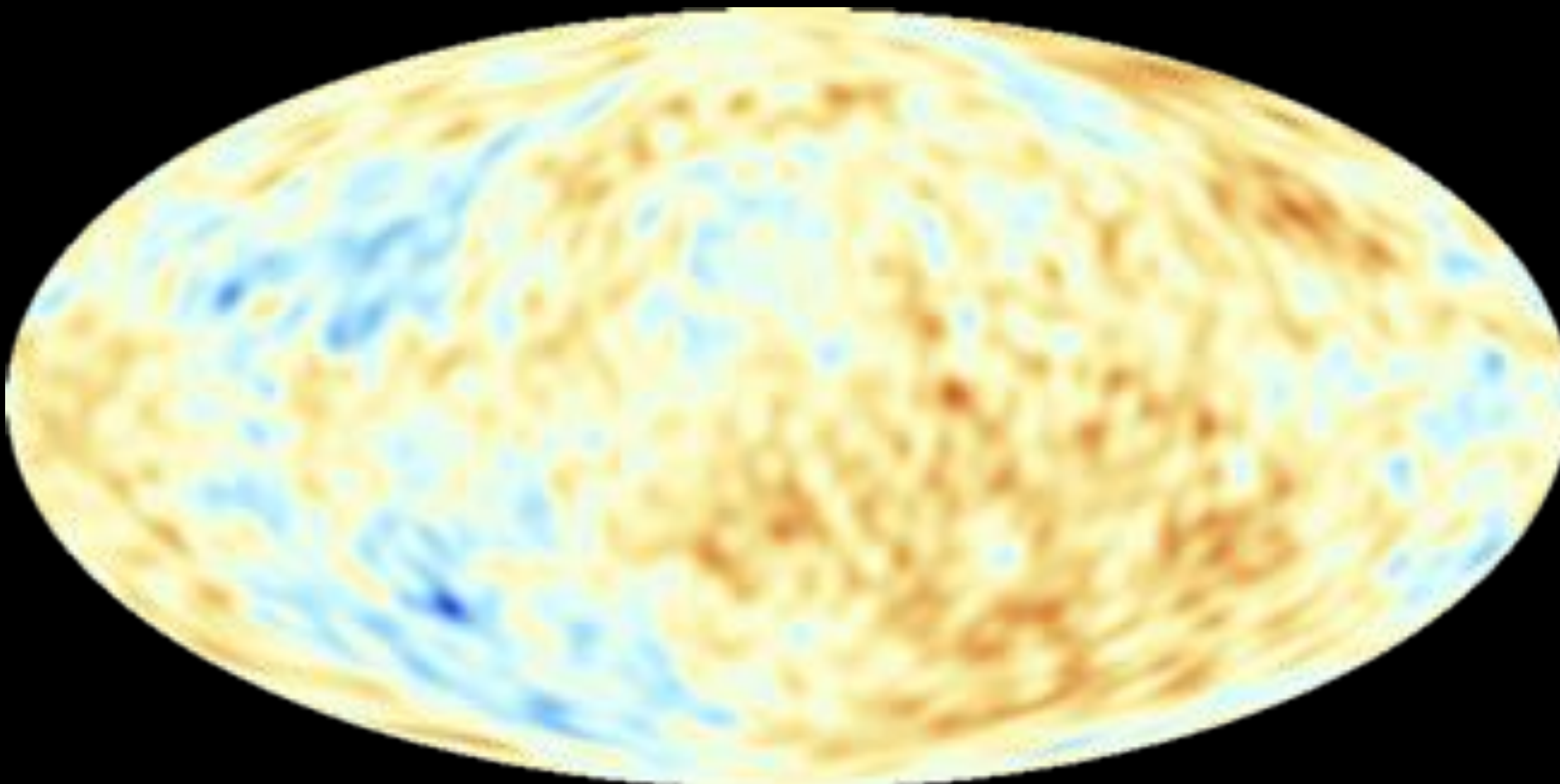
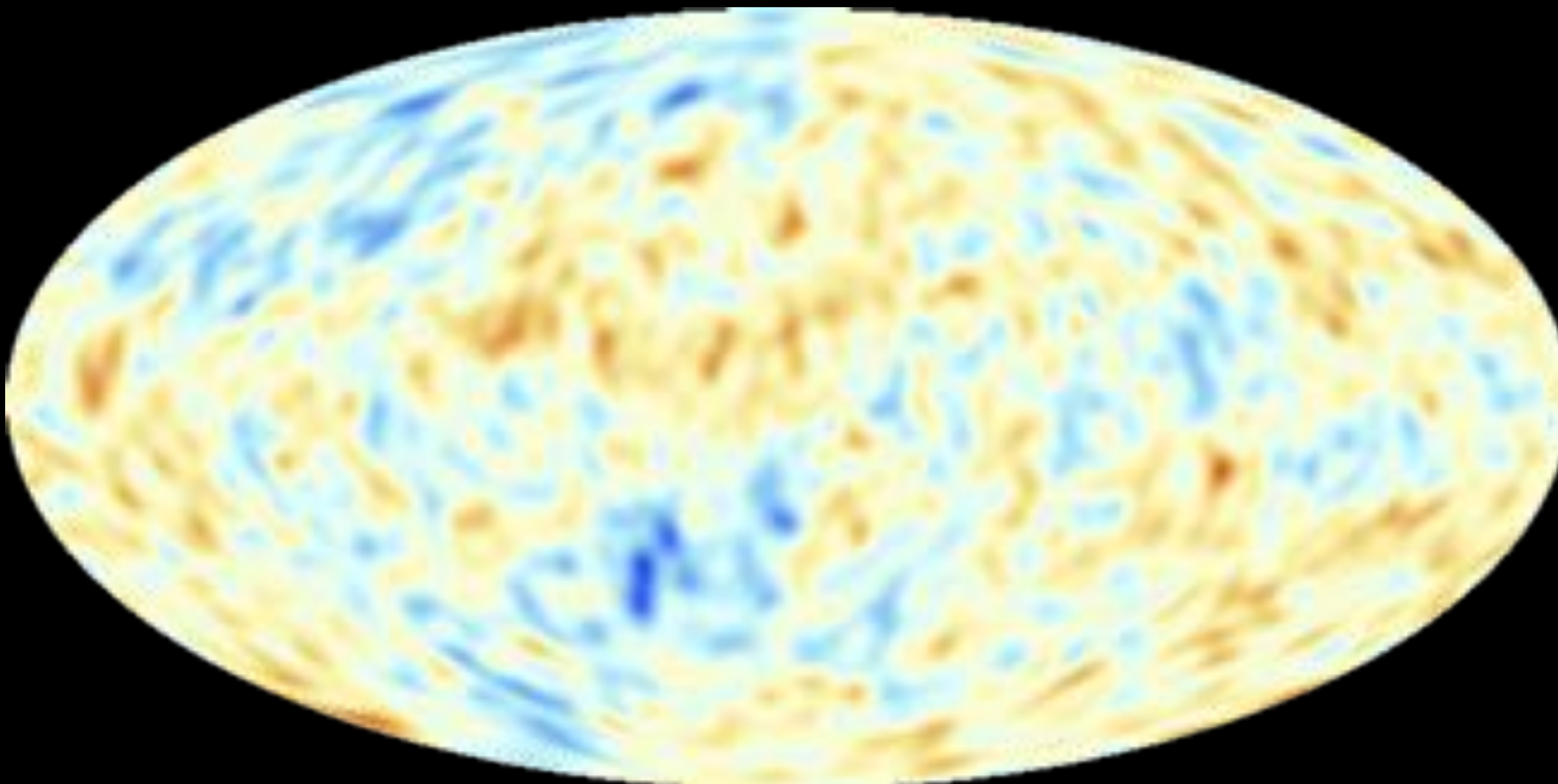
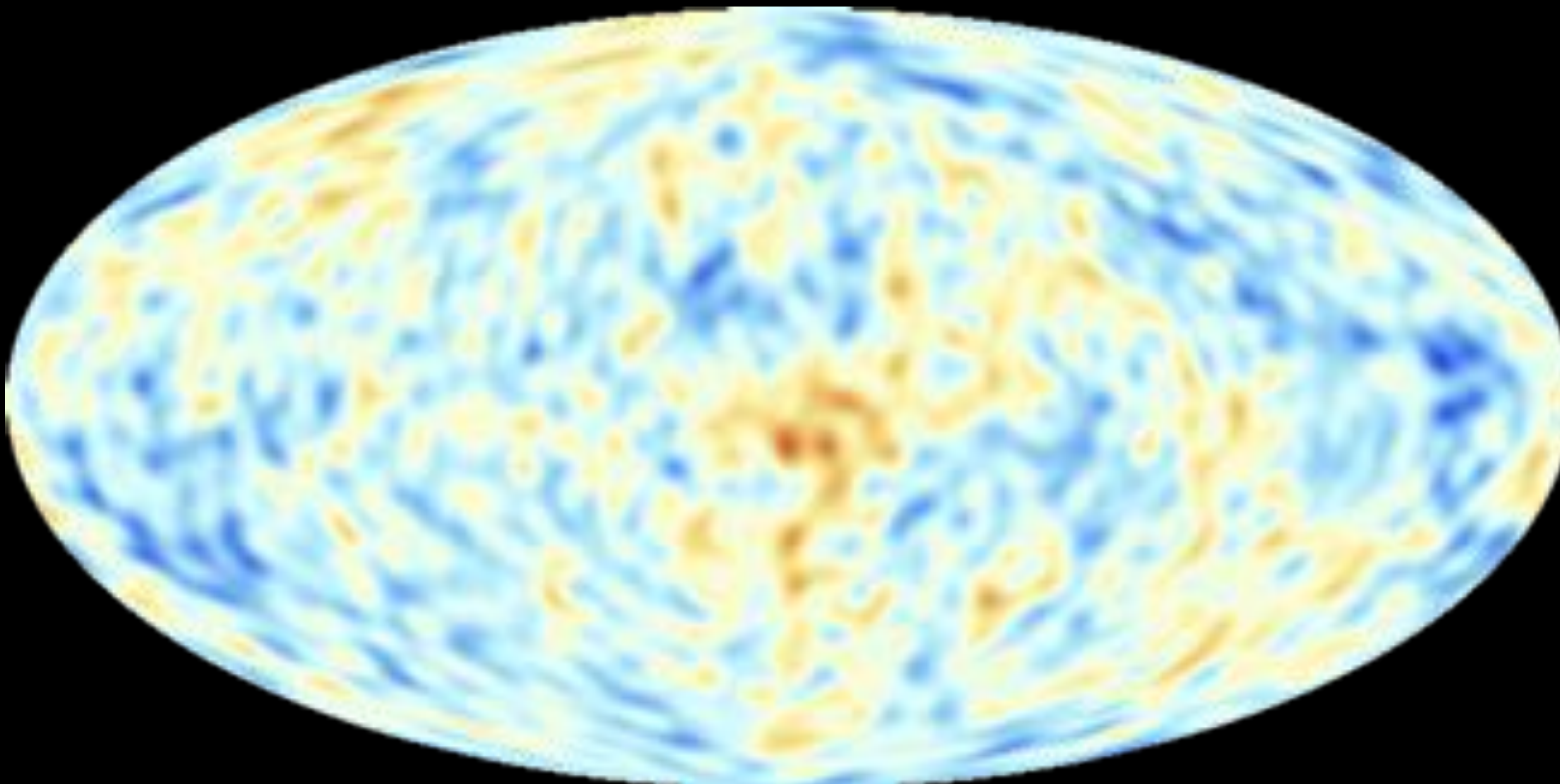


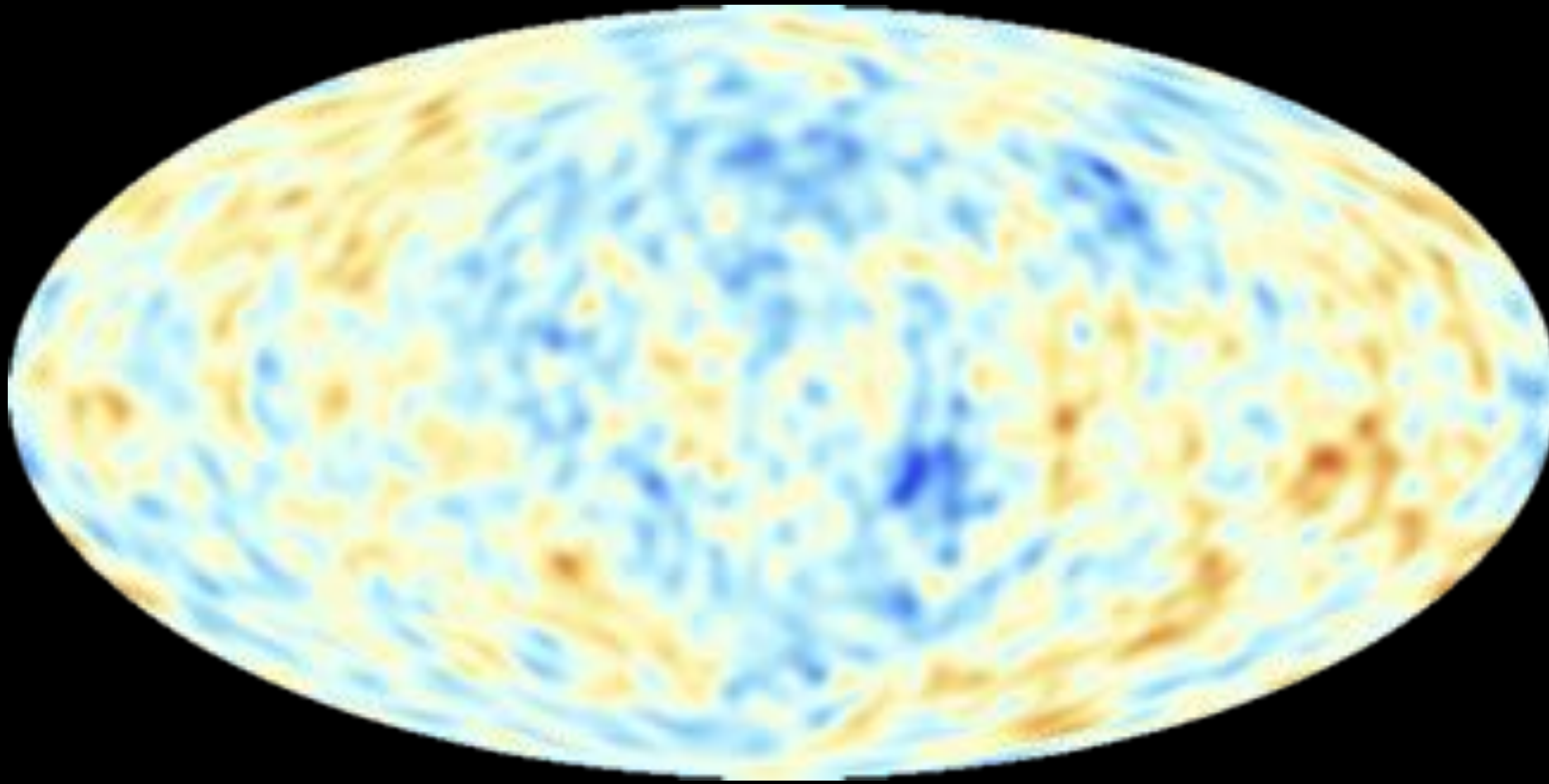
$$n_s \simeq 0.96$$

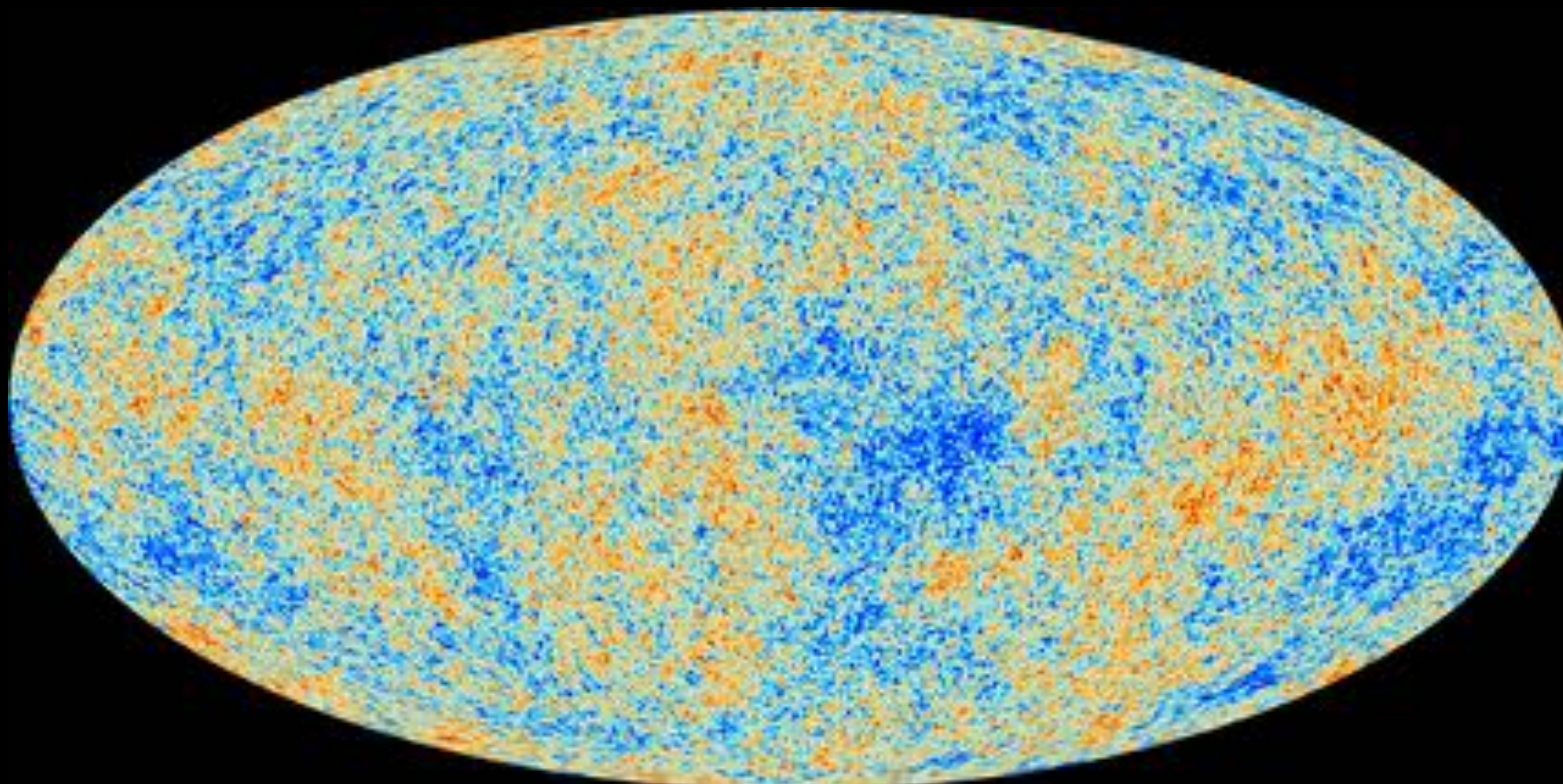




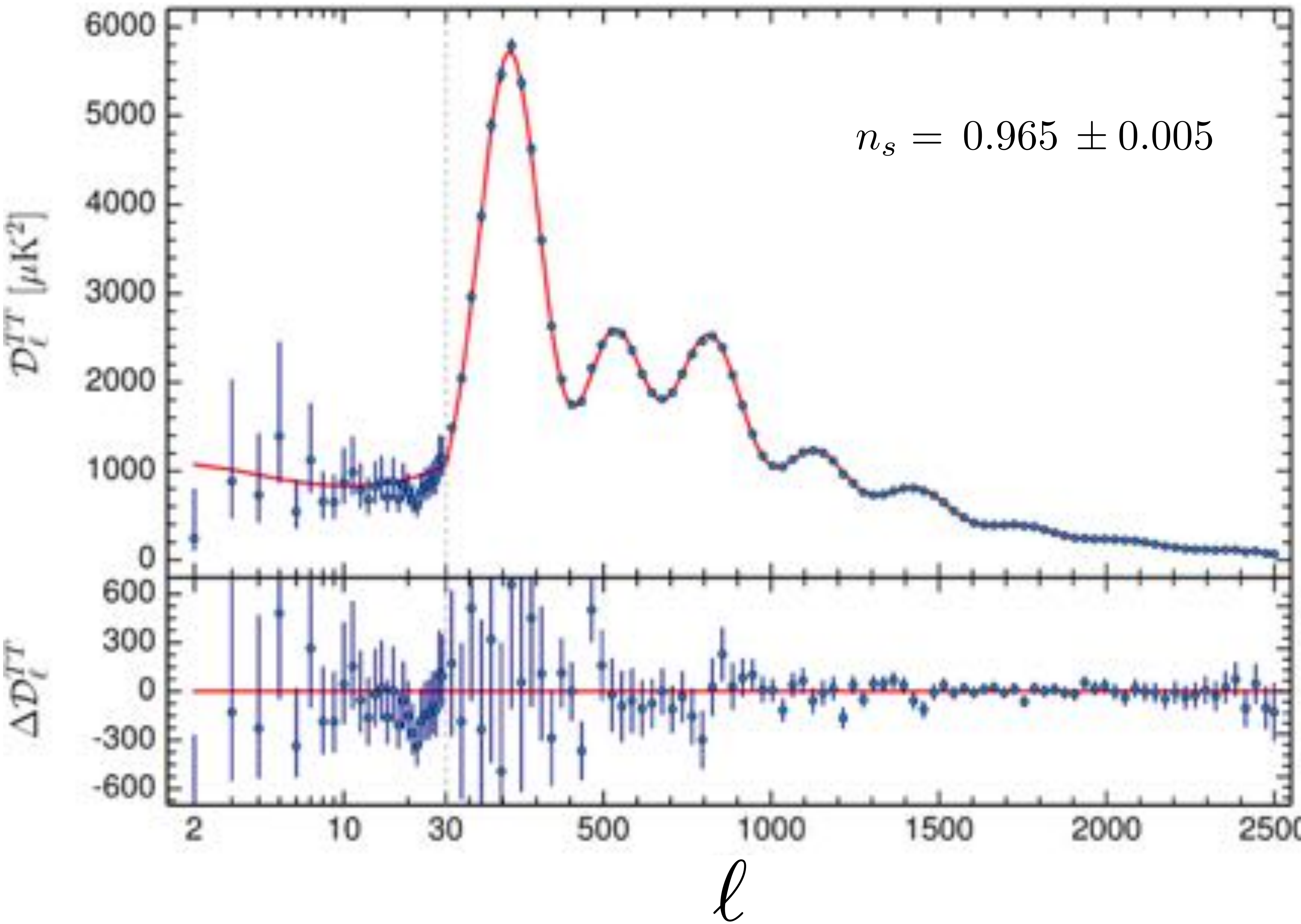




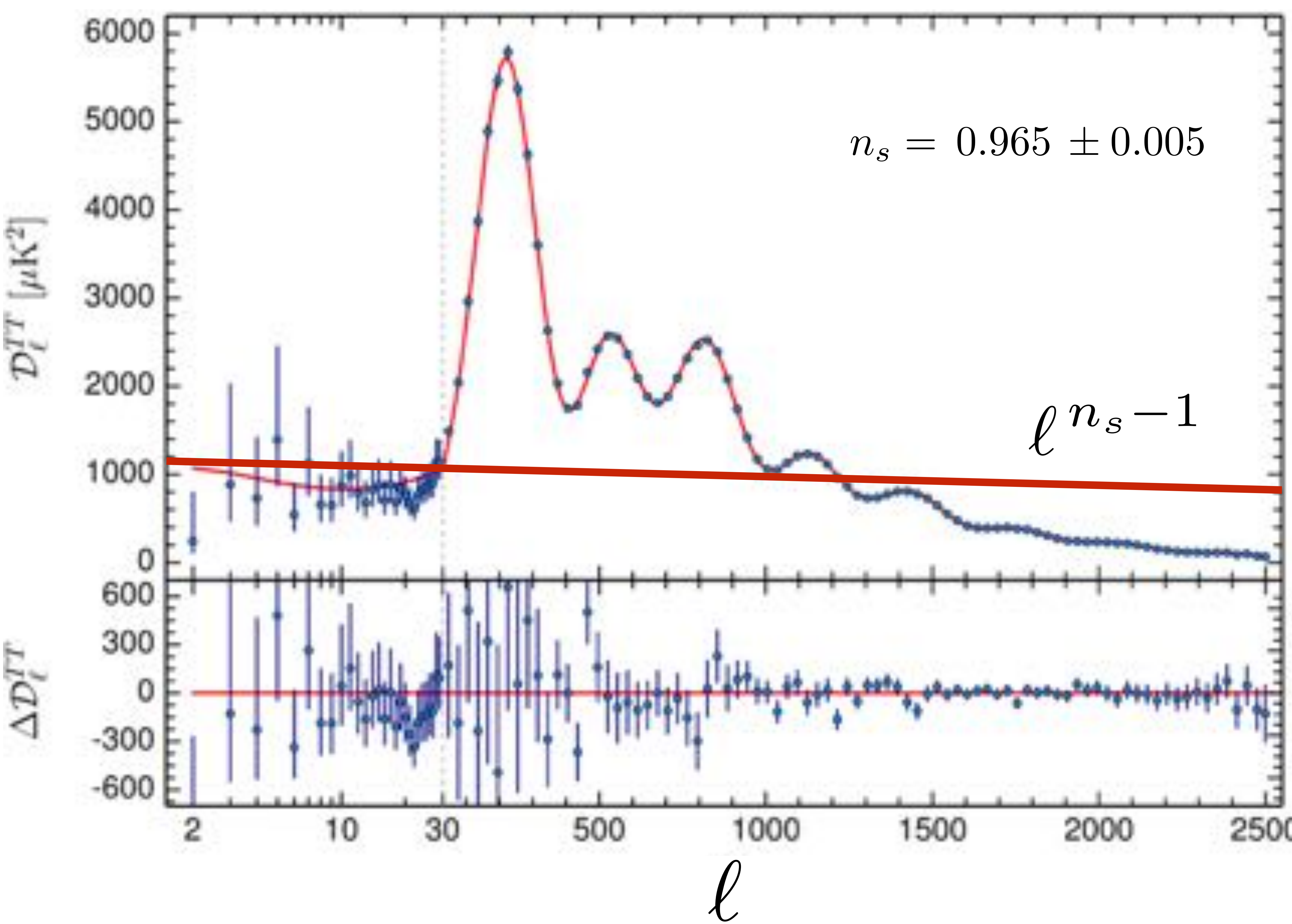




The anisotropies of the Cosmic Microwave Background
as observed by Planck



Planck Collaboration, arxiv.org/abs/1502.02114
``Planck 2015 results. XX. Constraints on inflation"



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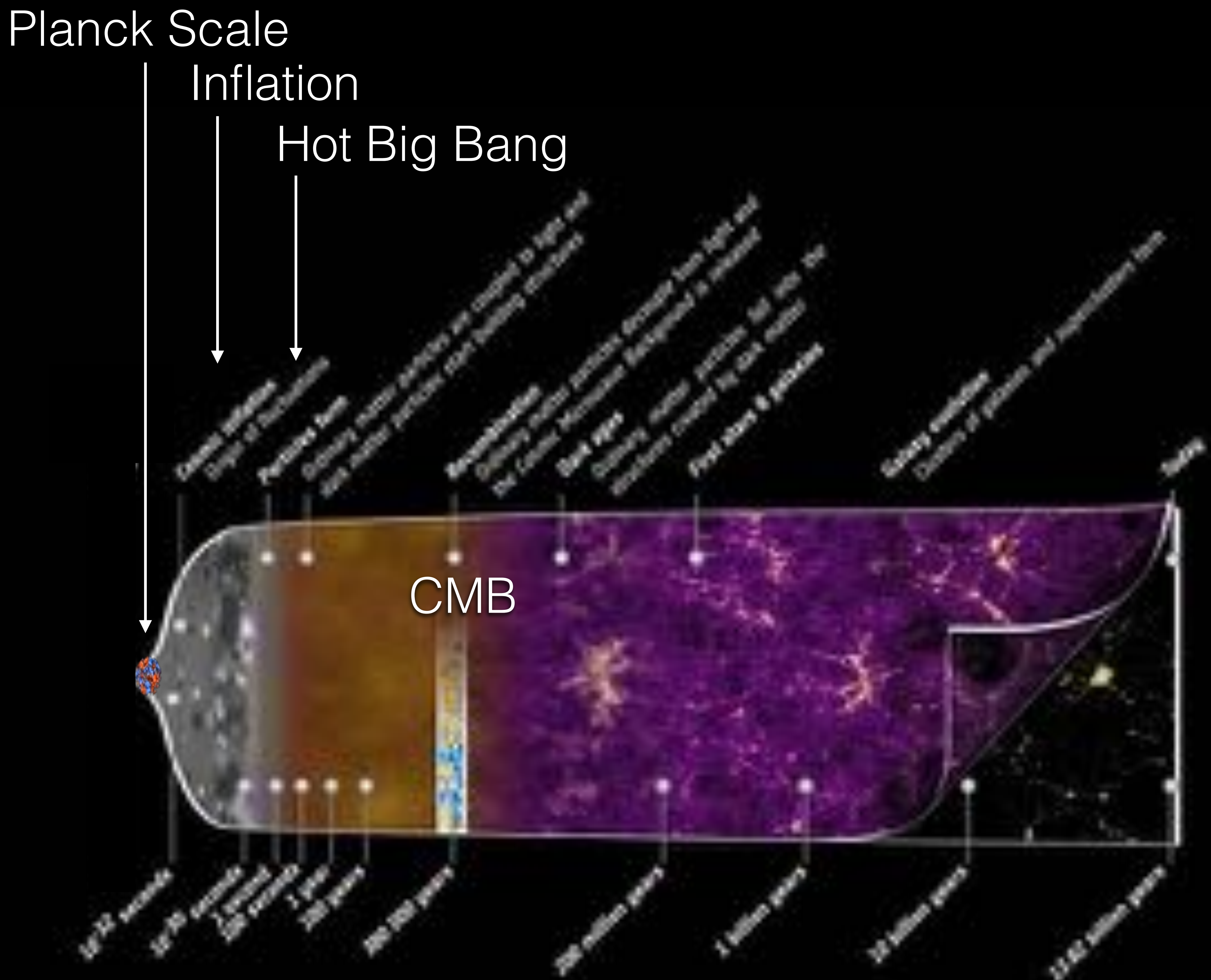
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Origin of primordial entanglement

BKL conjecture (Belinsky-Khalatnikov-Lifshitz 1970)

In classical General Relativity, the spatial coupling of degrees of freedom is suppressed in the approach to a space-like singularity

Quantum BKL conjecture (Bianchi-Hackl-Yokomizo 1512.08959)

In quantum gravity, correlations between spatially separated degrees of freedom are suppressed in the approach to a Planck curvature phase

$$\begin{cases} \hat{H}\,\Psi[g_{ij}(x),\varphi(x)] = 0 \\ \lim_{K\to 1/\ell_P^4} \Psi[a,\phi,\delta g_{ij}(x),\delta\varphi(x)] = \prod_{\vec{x}} \psi\big(\phi,\delta g_{ij}(x),\delta\varphi(x)\big) \end{cases}$$

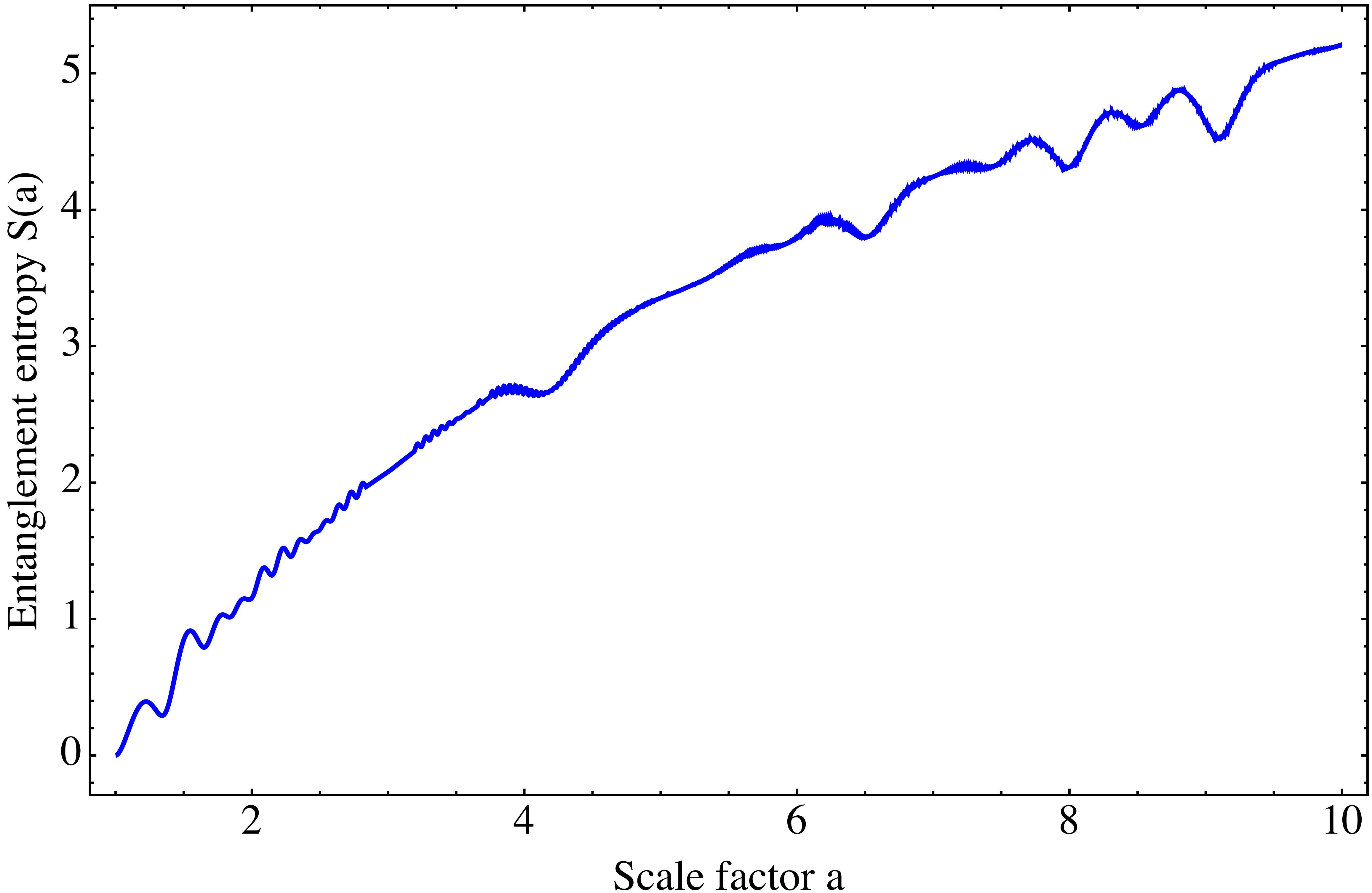
Scenario: correlations present at the beginning of inflation are produced during a pre-inflationary BKL phase via vacuum squeezing

Motivated by the "architecture conjecture" (Bianchi-Myers, *CQG* 2013)

Origin of primordial entanglement

Pre-inflationary scenario:

BKL phase with entanglement entropy growing from a zero law to an area law



EB, Hackl and Yokomizo 1512.08959

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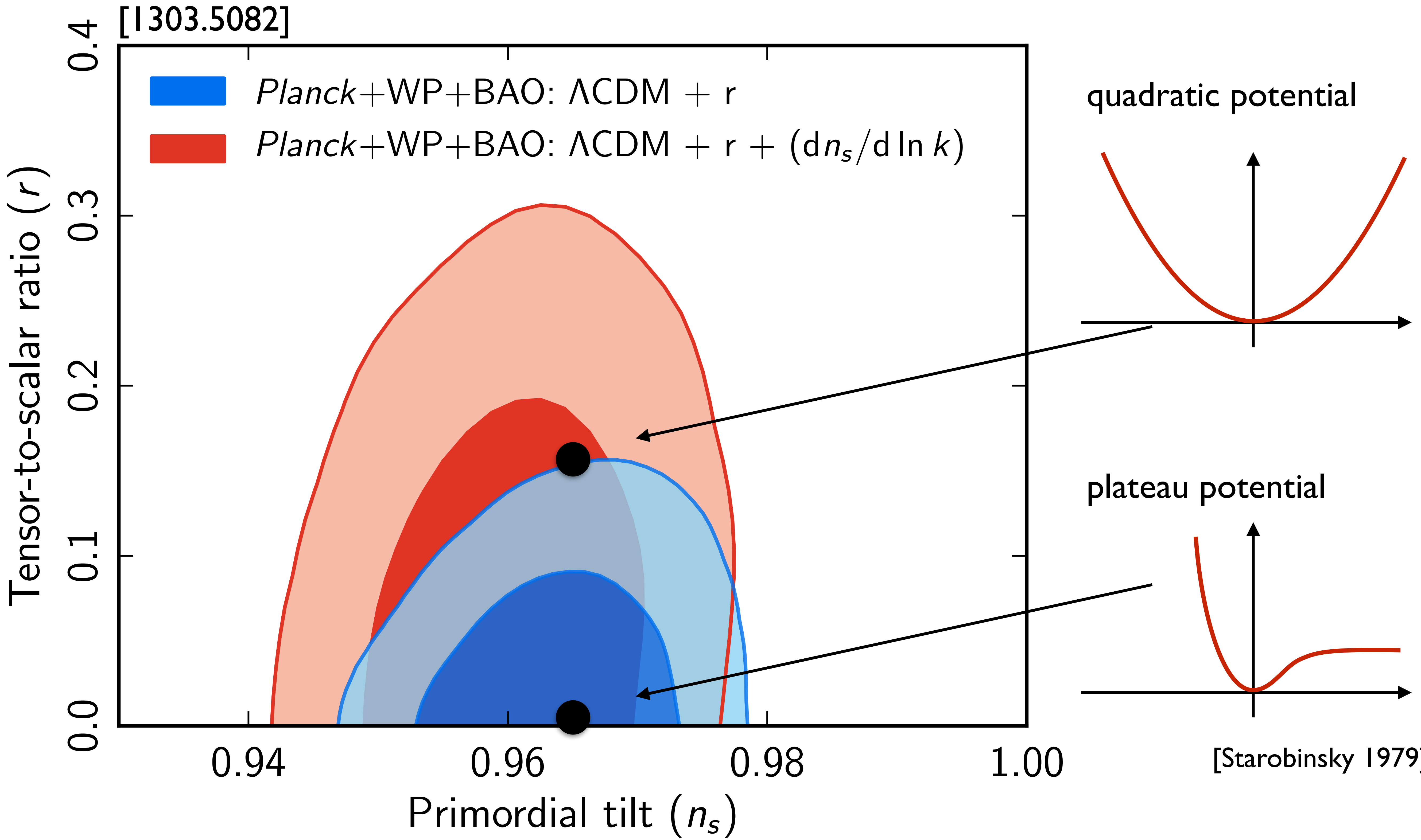
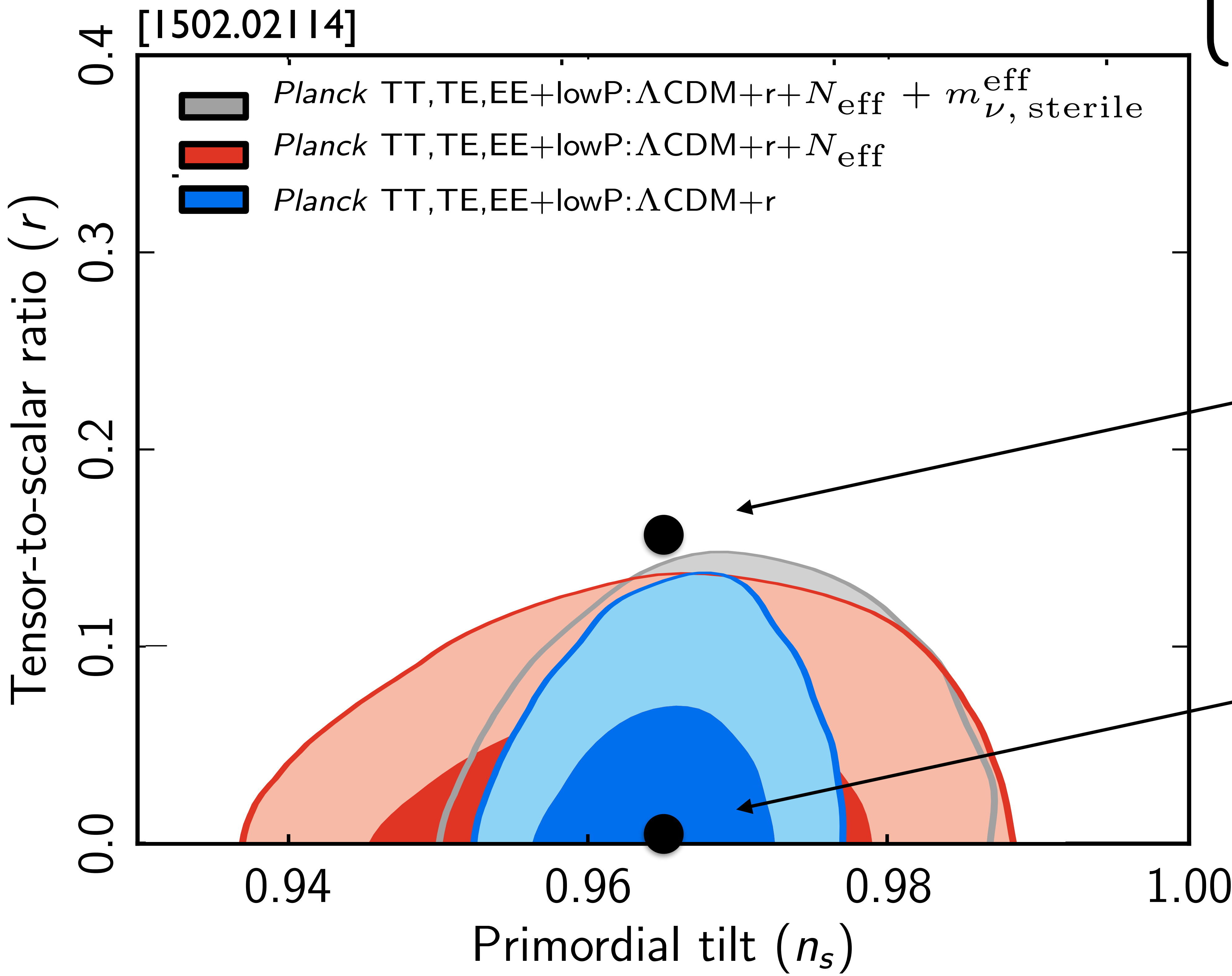
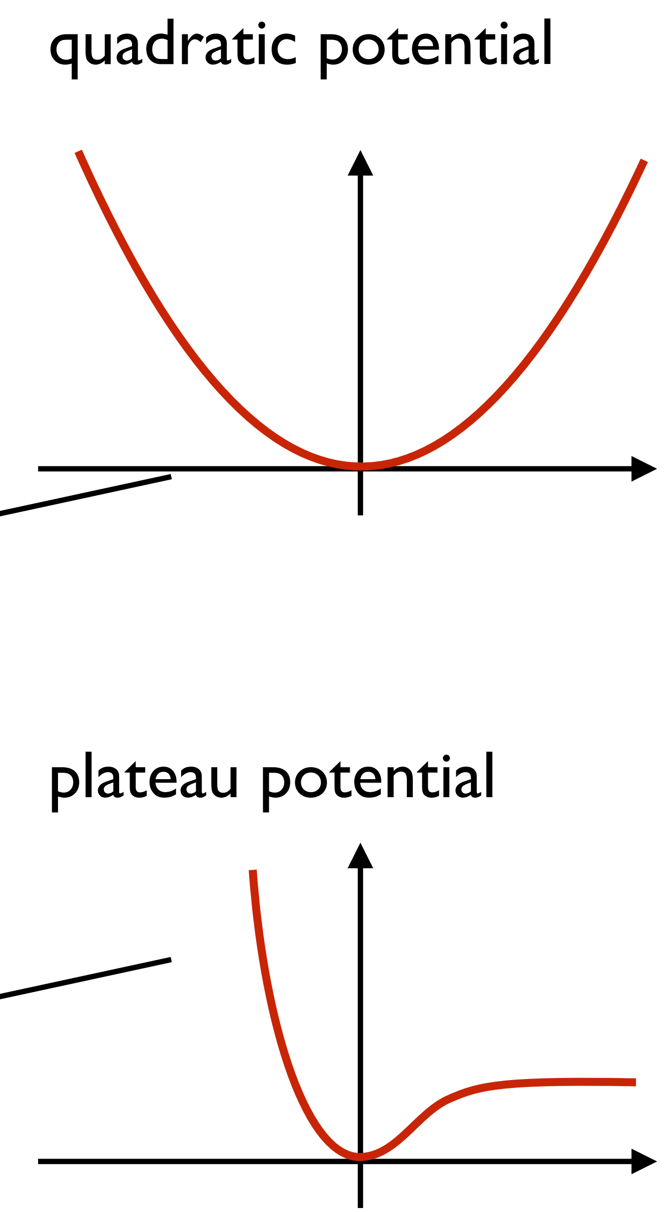
PLANCK 2013 constraints on inflationary models



PLANCK 2015 constraints on inflationary models

$$\begin{cases} A_s = (2.196 \pm 0.05) \times 10^{-9} \\ n_s = 0.9603 \pm 0.0073 \\ r < 0.11 \quad \text{at } k_* = (20\,\mathrm{Mpc})^{-1} \end{cases}$$





[Starobinsky 1979]

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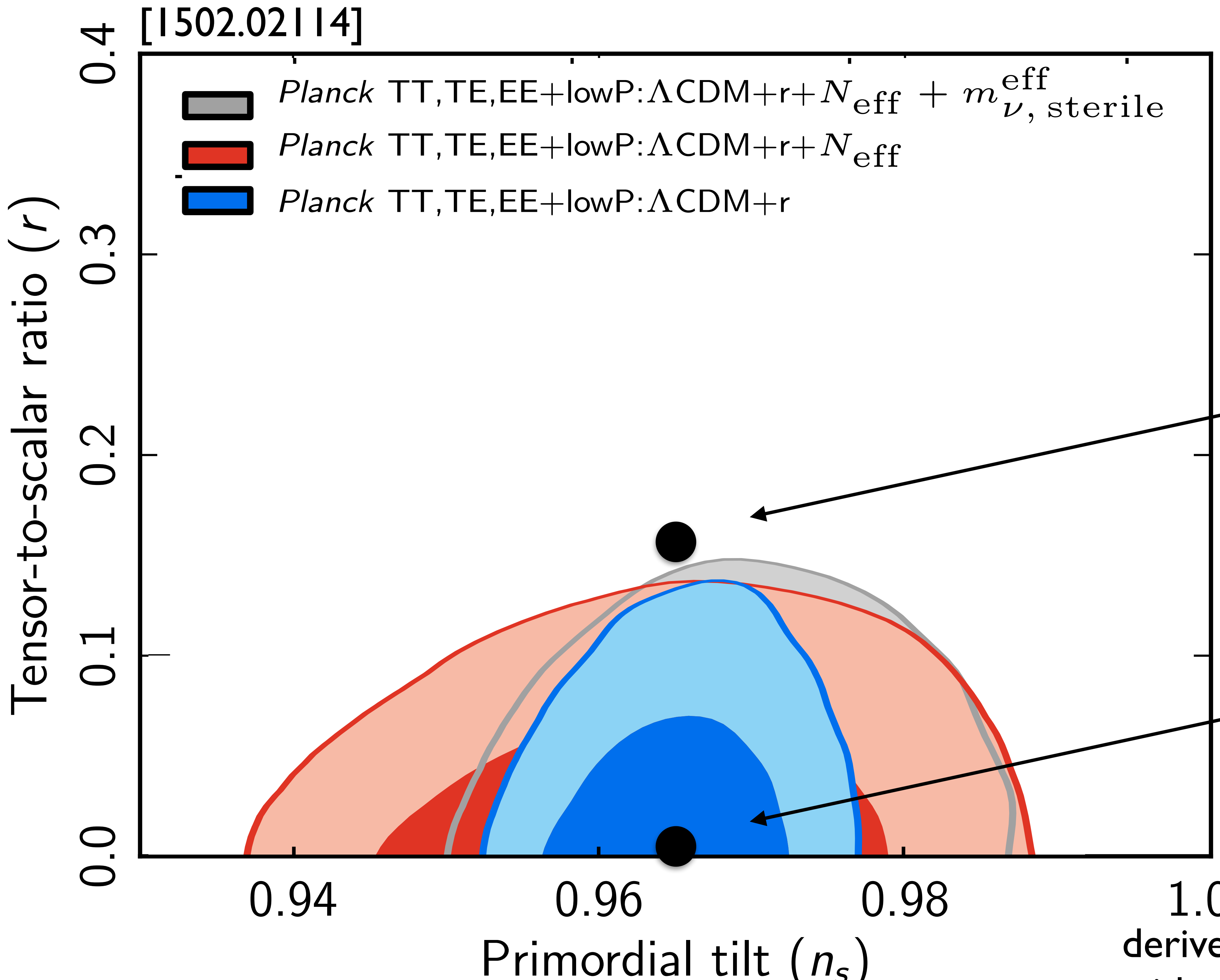
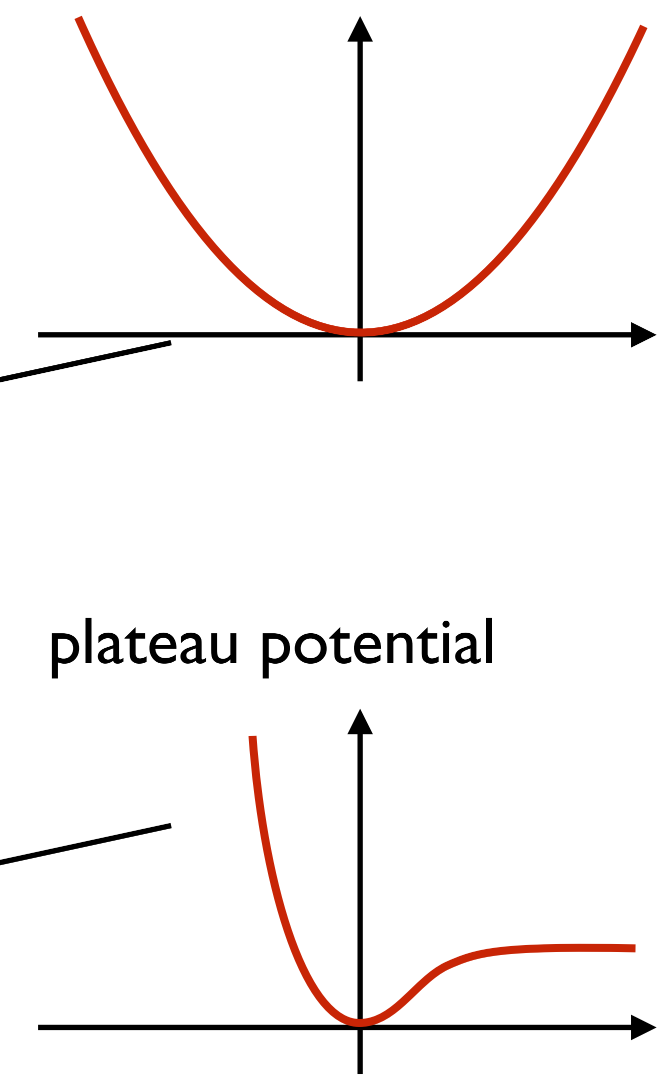




[Starobinsky 1979]

derived from General Relativity with quantum corrections

$$S = \frac{1}{16\pi G}\int (R + \alpha\, R^2)\sqrt{-g}\, d^4x$$

Slow-roll initial conditions in $R + \alpha R^2$ gravity

Action $$S = \frac{1}{16\pi G}\int \left(R + \alpha\, R^2\right)\sqrt{-g}\, d^4x$$

Classical FLRW background + quantum perturbation

$$g_{\mu\nu}(\vec{x},t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

Einstein equations

$$G_{\mu\nu} + \alpha\,\mathcal{H}_{\mu\nu} = 0$$

Gravity-driven inflation

$$\begin{cases} A_s = \dfrac{1}{18\pi^2}\dfrac{\ell_P^2}{\alpha}N^2 \\ n_s = 1 - \dfrac{2}{N} \\ r = \dfrac{12}{N^2} \end{cases}$$

Planck 2015

$$\begin{cases} A_s = (2.196 \pm 0.05) \times 10^{-9} \\ n_s = 0.9603 \pm 0.0073 \\ r < 0.11 \quad \text{at } k_* = (20\,\mathrm{Mpc})^{-1} \end{cases}$$

Compatible with data with $N \simeq 60$ and $\alpha \simeq (10^5\,\ell_P)^2$

Pre-inflationary initial conditions in $R + \alpha R^2$ gravity

EB-Fernandez-Satz, *in progress*

Initial condition in post-Planckian phase

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 1/\ell_0^4 \qquad \text{with} \qquad \ell_P < \ell_0 \ll 10^5\,\ell_P$$

Background: select initial conditions s.t. $N \geq 60$ e-folds of slow-roll

Quantum perturbation: scalar and tensor modes of the geometry
local vacuum has low entanglement entropy at the curvature scale

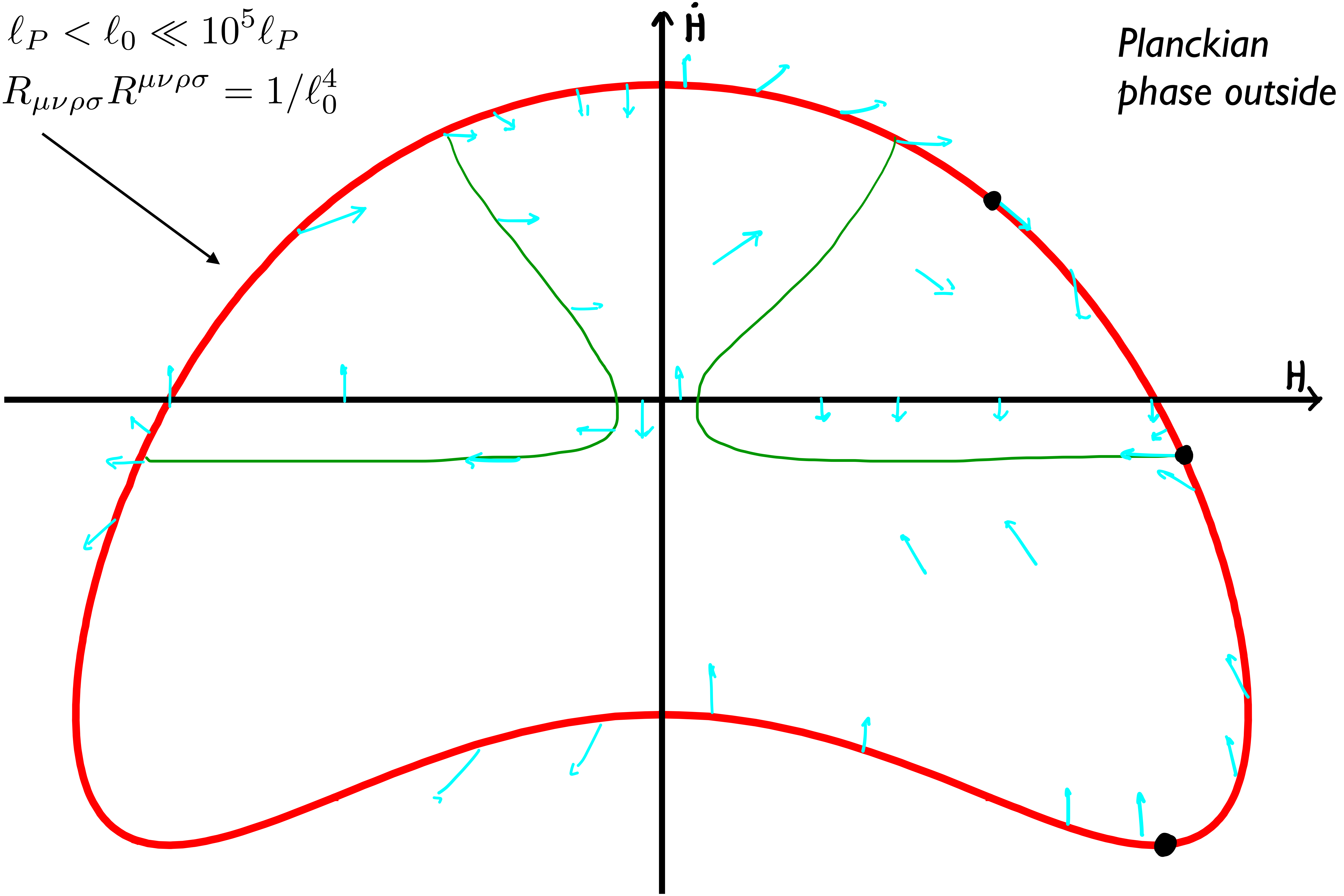
Pre-slowroll phase produces Minkowskian correlations + squeezing

Expected enhancement of power in tensor modes at low ℓ

Pre-inflationary initial conditions in $R + \alpha R^2$ gravity

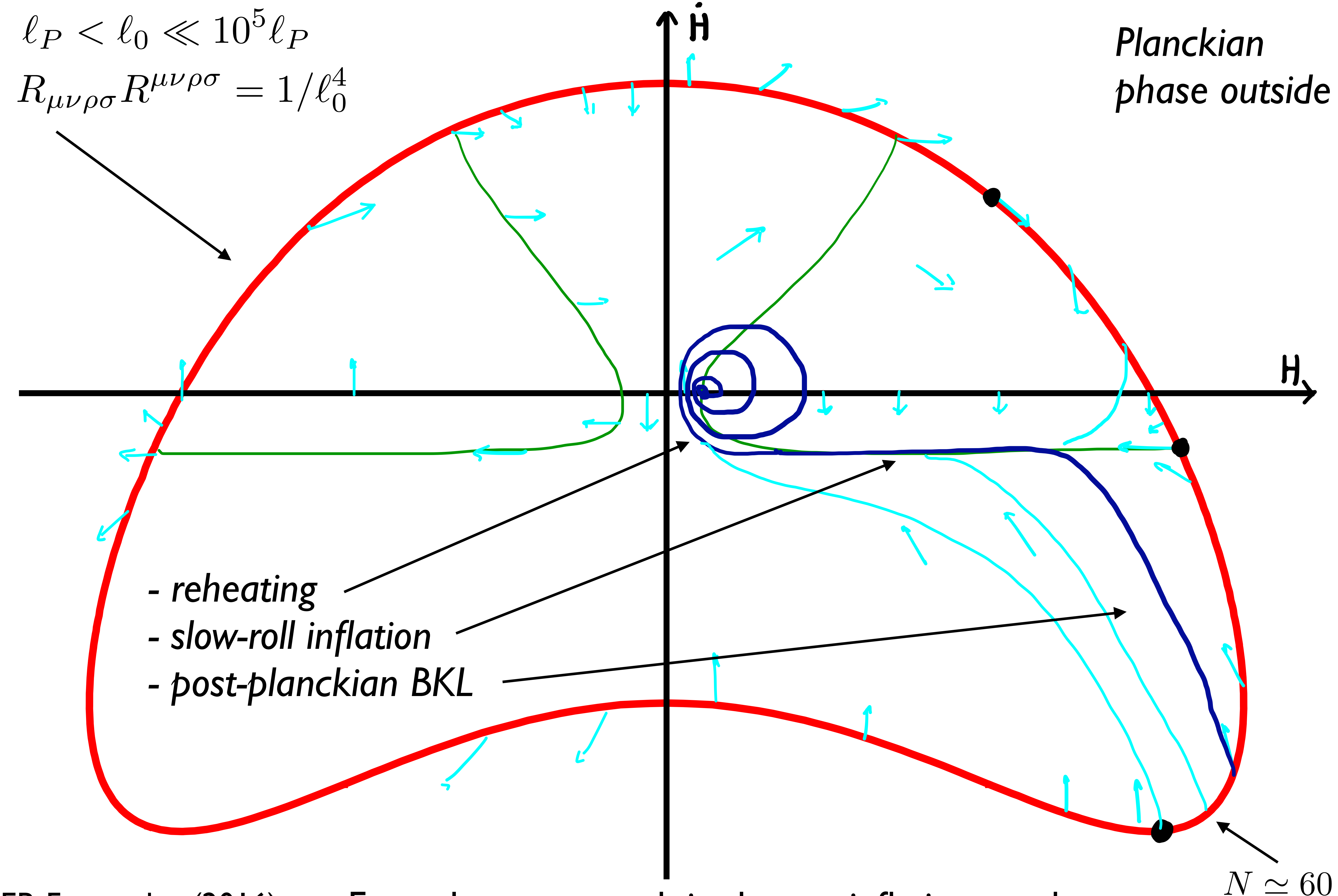
$\ell_P < \ell_0 \ll 10^5 \ell_P$

$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 1/\ell_0^4$



EB-Fernandez (2016)

Pre-inflationary initial conditions in $R + \alpha R^2$ gravity



EB-Fernandez (2016)

Entanglement growth in the pre-inflationary phase

Entanglement in the sky

Summary:

* The vacuum

  - entanglement, squeezing and instabilities
  - origin of primordial entanglement

* Inflation and pre-inflationary initial conditions

  - PLANCK constraints on models
  - gravity-driven inflation and entanglement