

# A microscopic approach to cosmic structure formation

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with

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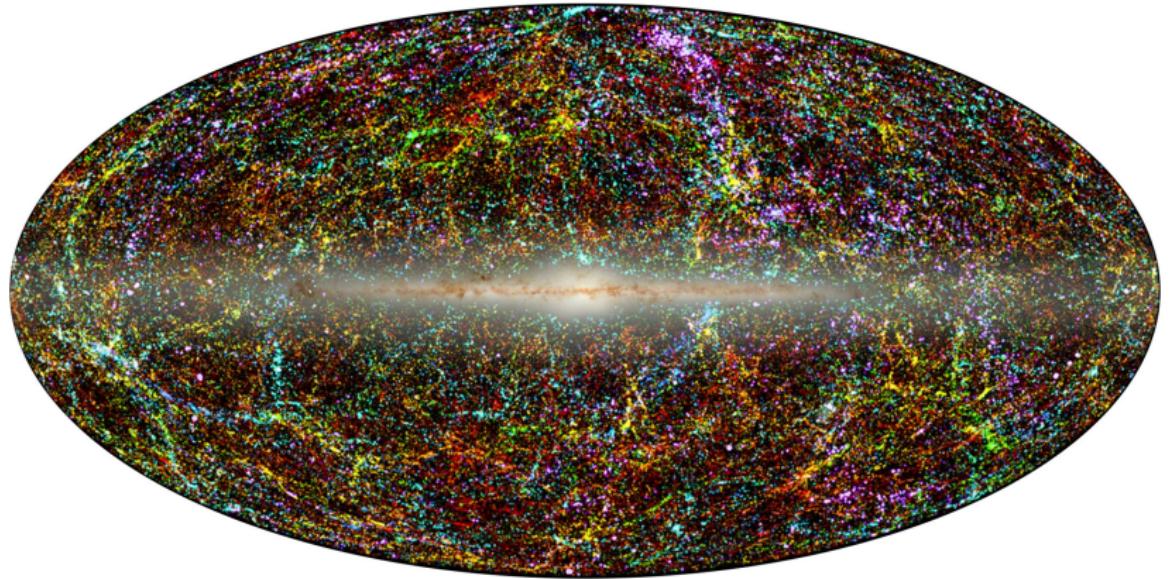
Heidelberg University, ZAH, ITA

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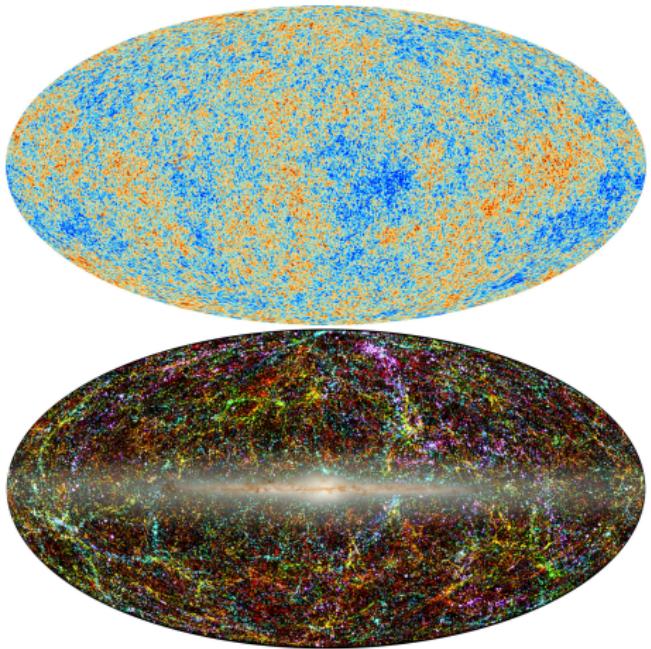
# Problems in cosmic structure formation



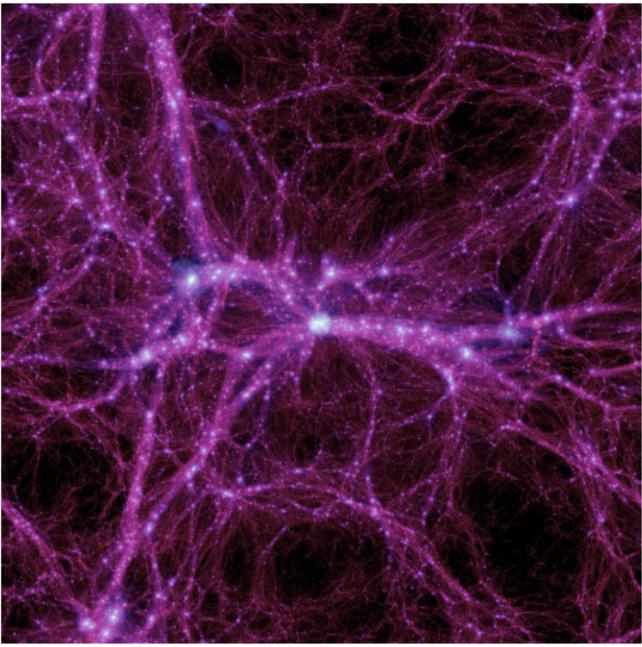
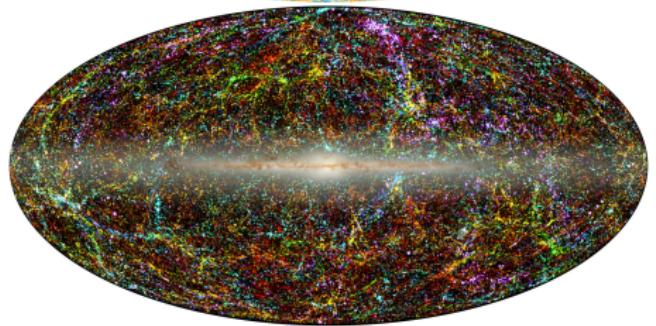
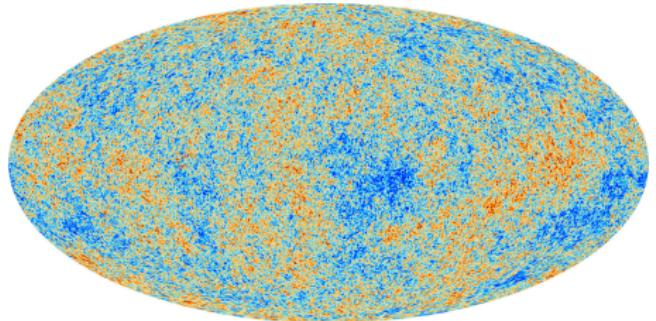
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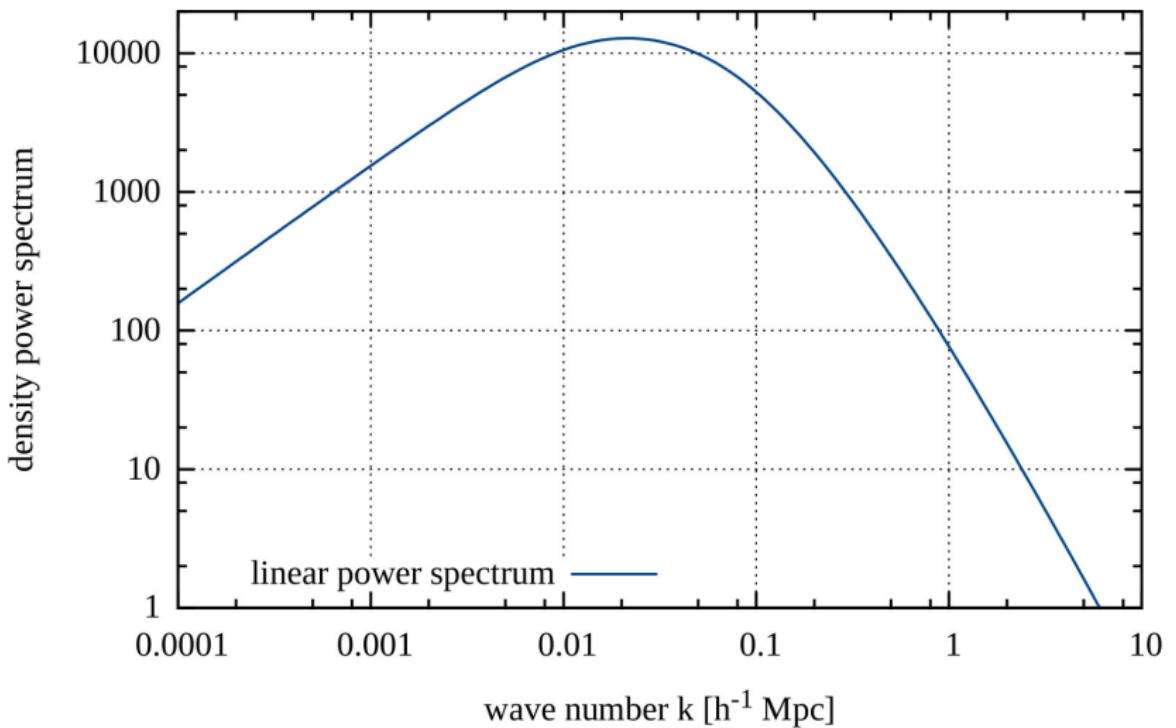
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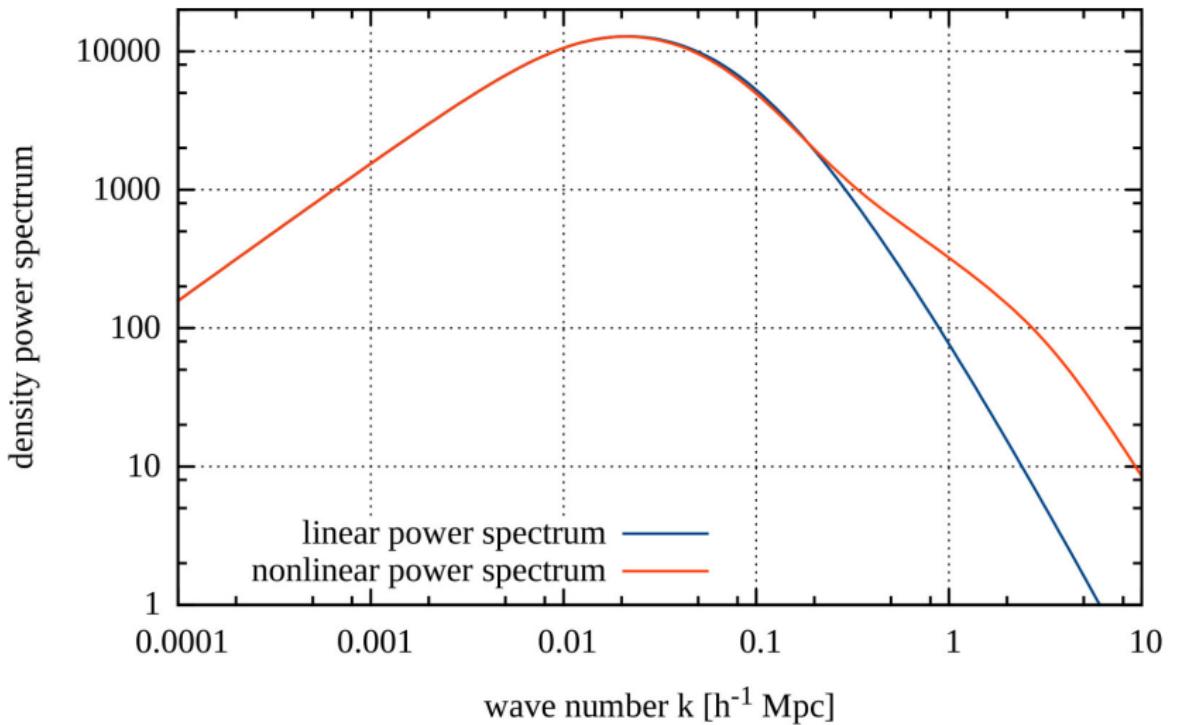
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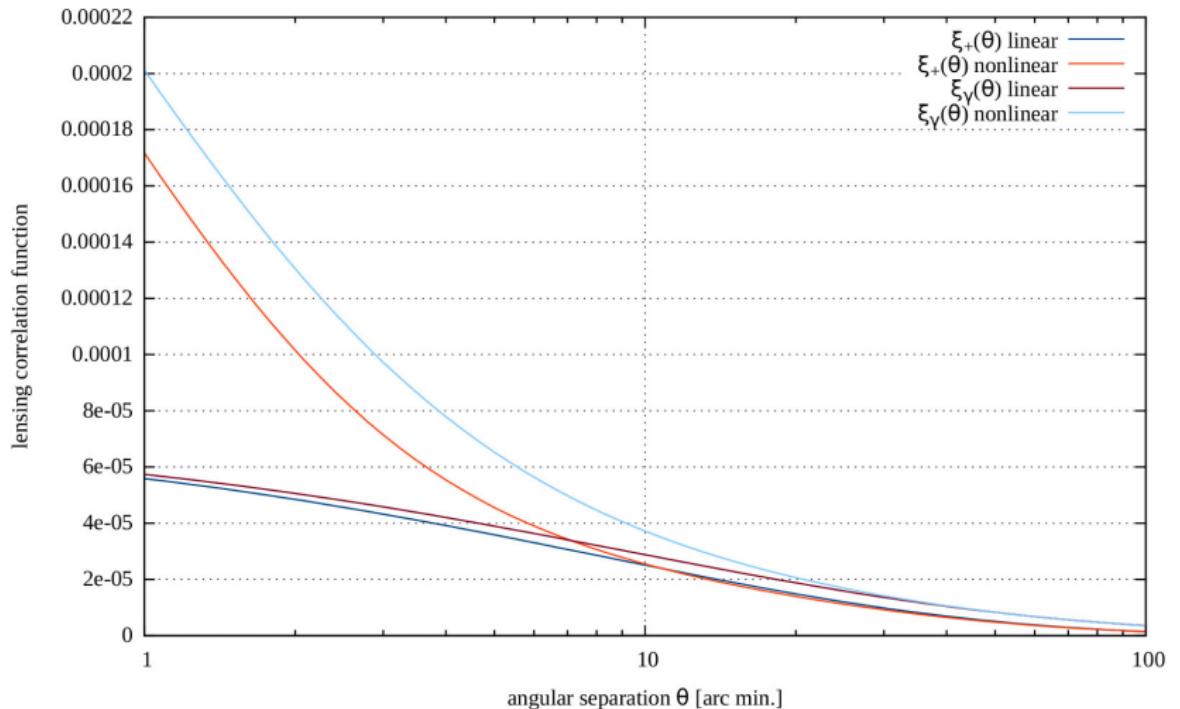
# Problems in cosmic structure formation



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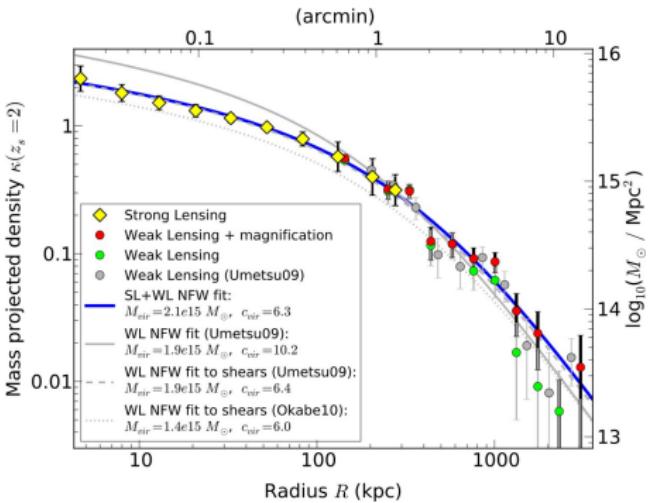
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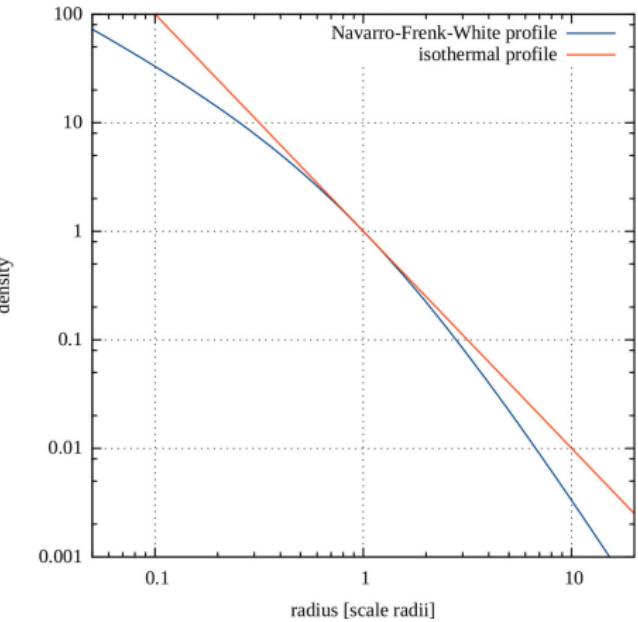
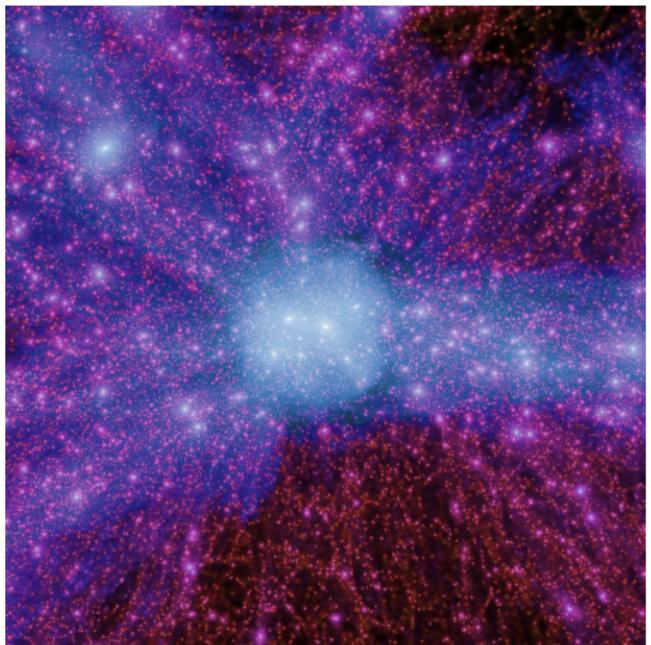
# Problems in cosmic structure formation



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# Problems in cosmic structure formation



## Conventional:

- Hydrodynamical equations:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$

$$\dot{\vec{u}} + 2H\vec{u} = -\vec{\nabla}\phi$$

$$\vec{\nabla}^2\phi = 4\pi G\bar{\rho}\delta$$

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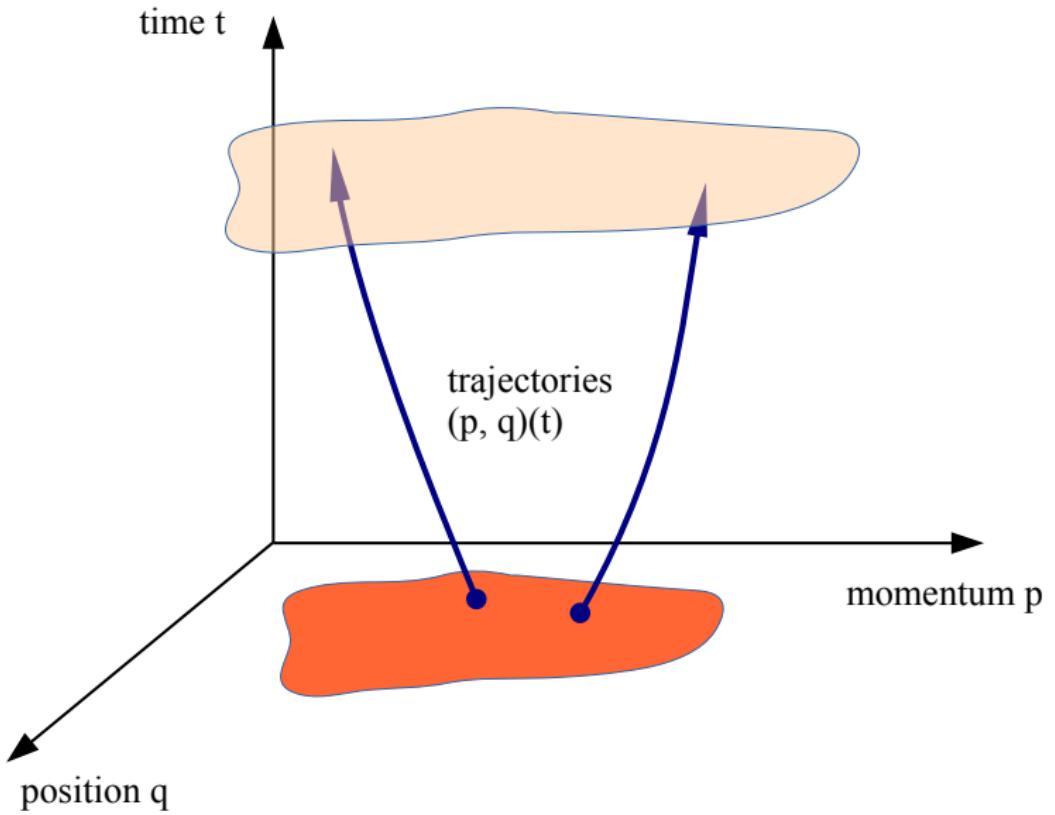
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## New approach:

- Non-equilibrium statistics of  $N$  classical particle trajectories
- Describe particle ensemble by partition sum (generating functional)  $Z$
- Derive statistical properties by functional derivatives

# Phase-space trajectories



# Phase-space trajectories

- Classical particles follow Hamiltonian equations of motion,

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}, \quad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}, \quad x = (q, p)$$

- Trajectories are described by (retarded) Green's function  $G_R(t, t')$ ,

$$\bar{x}(t) = \underbrace{G_R(t, 0)x^{(i)}}_{\text{free motion}} - \underbrace{\int_0^t G_R(t, t')K(t')dt'}_{\text{interaction}}$$

- In static space, with potential  $v$ ,

$$G_R(t, t') = \begin{pmatrix} 1 & (t - t')/m \\ 0 & 1 \end{pmatrix}, \quad K = \begin{pmatrix} 0 \\ \nabla v \end{pmatrix}$$

Equilibrium thermodynamics:

$$Z_{\text{gc}} = \int d\Gamma e^{-\beta(H(x)-\mu N)}$$

$$\langle N \rangle = \frac{1}{\beta} \frac{\partial \ln Z_{\text{gc}}}{\partial \mu}$$

Non-equilibrium, statistical quantum field theory:

$$Z_0[J] = \text{Tr} \int \mathcal{D}\varphi e^{i \int (\mathcal{L}_0 + J\varphi) d^4x}$$

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$$\langle \rho(1)\rho(2) \rangle = \frac{\delta}{i\delta H_\rho(1)} \frac{\delta}{i\delta H_\rho(2)} Z$$

# Some detail

- Generating functional:

$$Z[H, J, K] = e^{i\hat{S}} e^{iH\hat{\Phi}} \int d\Gamma^{(i)} e^{i \int \langle J, \bar{x} \rangle dt}$$

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- Interaction operator:

$$\hat{S}_I = - \int d1 \int d2 \frac{\delta}{\delta H_B(1)} v(12) \frac{\delta}{\delta H_\rho(2)}$$

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- Perturbation theory:  $e^{i\hat{S}} = 1 + i\hat{S} - \frac{1}{2}\hat{S}^2 + \dots$

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# Specialisation to cosmology

- ① Choose initial phase-space measure

$$d\Gamma^{(i)} = P(q, p) d^{3N}q d^{3N}p$$

fully specified by initial power spectrum

- ② Change time coordinate

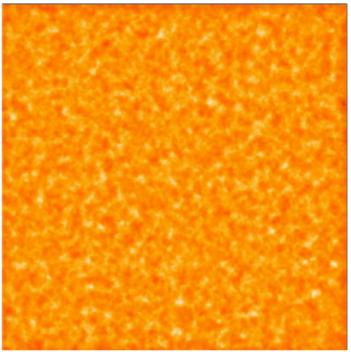
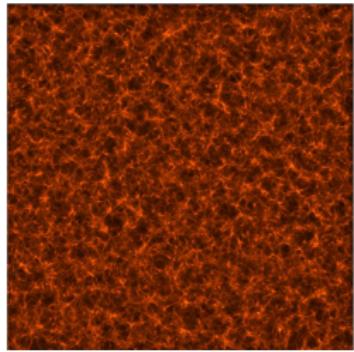
$$t \rightarrow \tau = D_+(t) - D_+(t_i)$$

- ③ Adapt Green's function to expanding universe

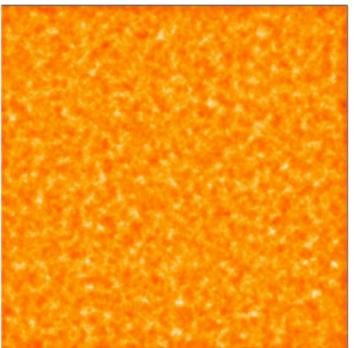
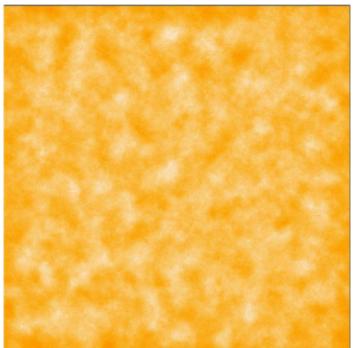
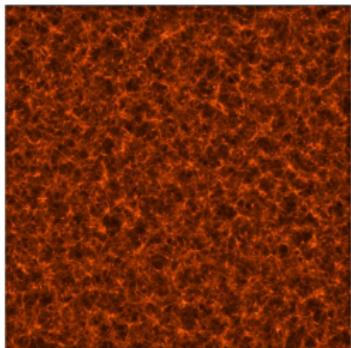
# Improved Zel'dovich trajectories



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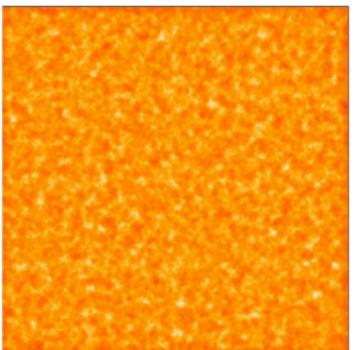
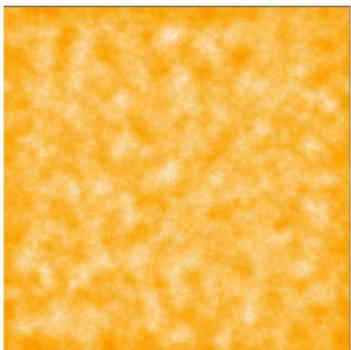
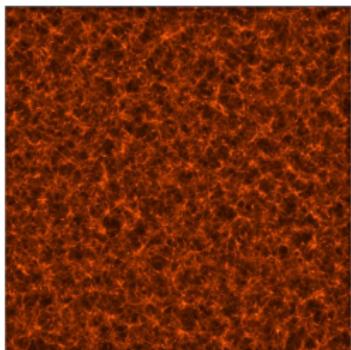


# Improved Zel'dovich trajectories



$$\vec{q}(\tau) = \vec{q}^{(i)} + \vec{p}^{(i)} \int_0^\tau \exp(h(\tau')) d\tau'$$

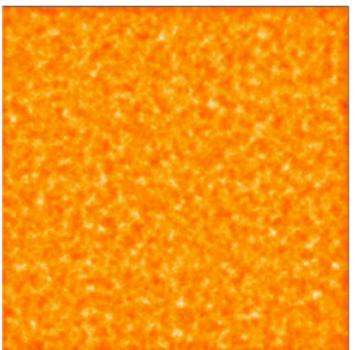
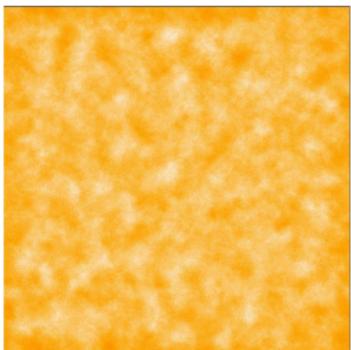
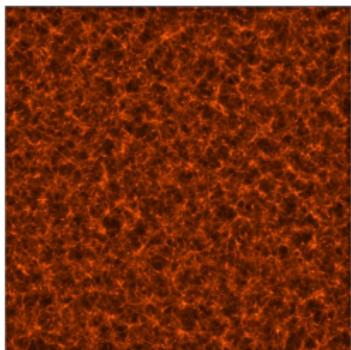
# Improved Zel'dovich trajectories



$$\vec{q}(\tau) = \vec{q}^{(i)} + \vec{p}^{(i)} \int_0^\tau \exp(\textcolor{red}{h}(\tau')) d\tau'$$

$$\textcolor{red}{h}(\tau) = g^{-1}(\tau) - 1$$

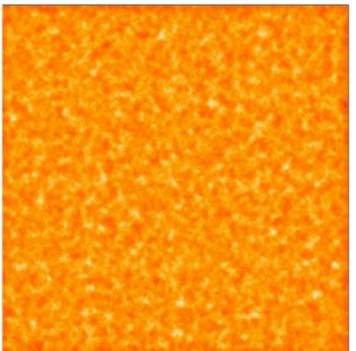
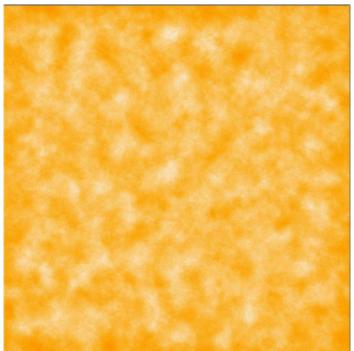
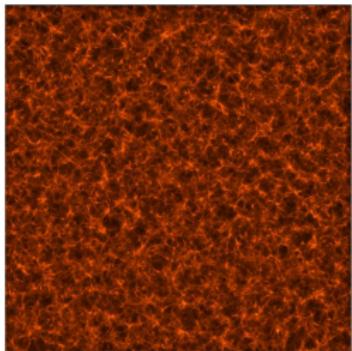
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# Improved Zel'dovich trajectories

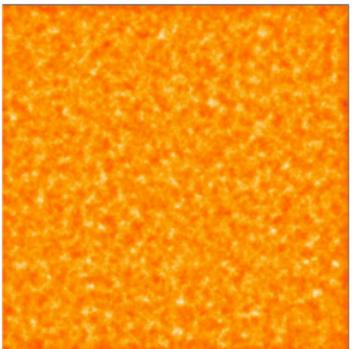
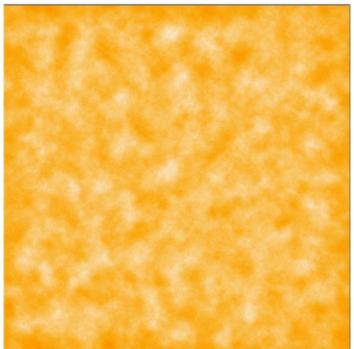
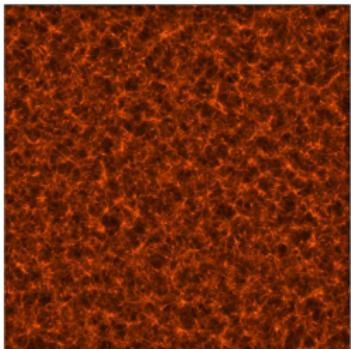


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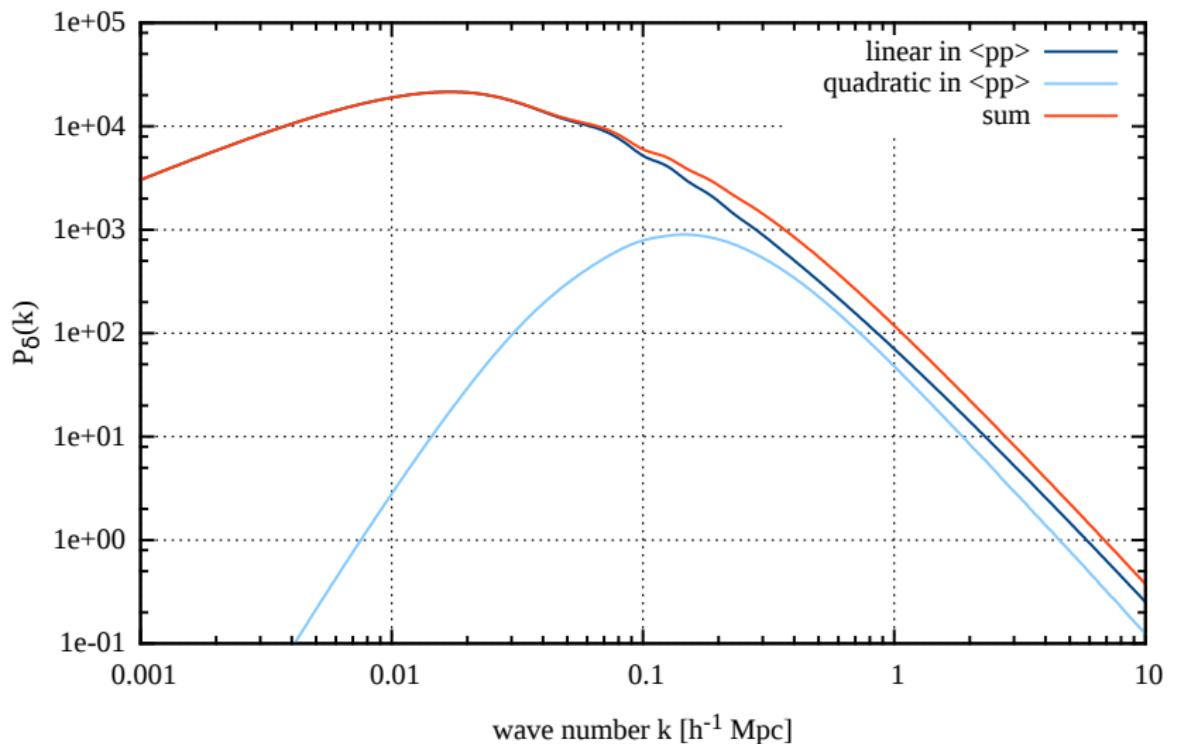


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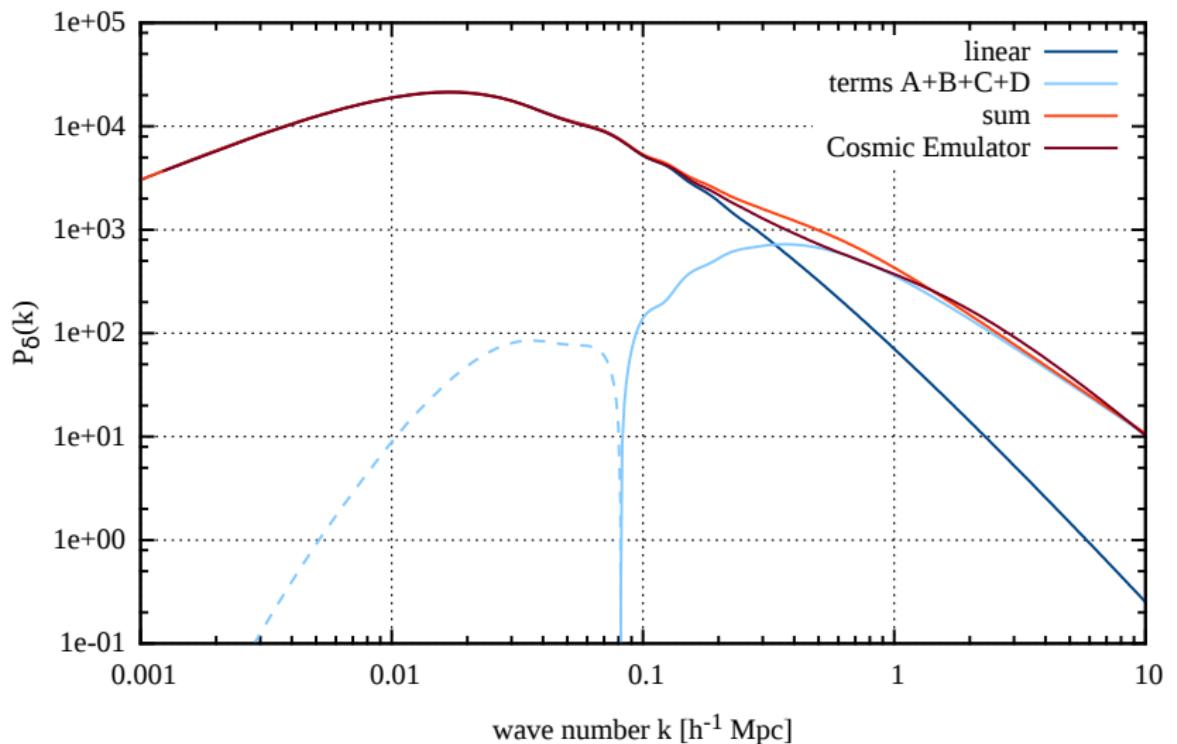
$$\hat{v}(k) = -\frac{3}{2k^2} \frac{a}{g^2} \propto a^{-2} \text{ (EdS)}$$

# Density power spectrum



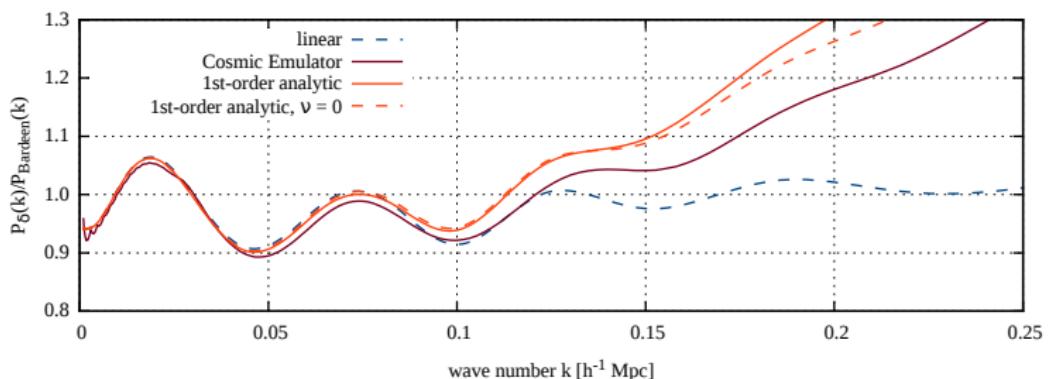
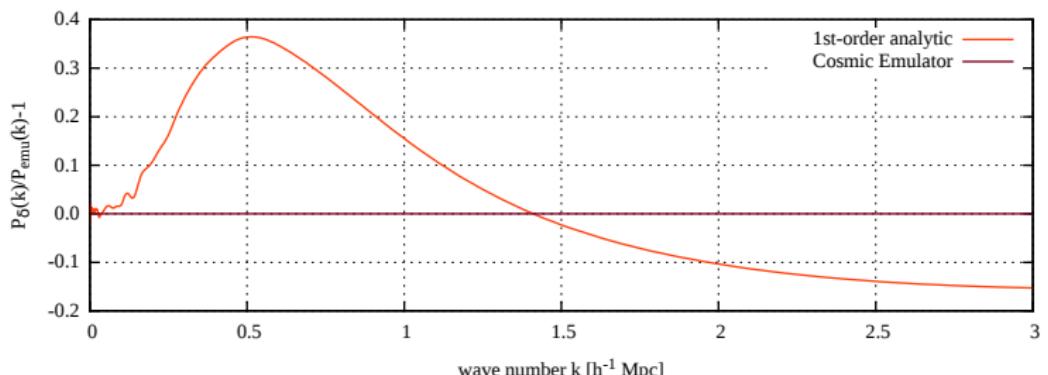
Improved Zel'dovich trajectories, no interaction

# Density power spectrum



Improved Zel'dovich trajectories plus first-order interaction

# Density power spectrum



Improved Zel'dovich trajectories plus first-order interaction

- ① Non-equilibrium statistical field theory for dark-matter particles set up
- ② Hamiltonian equations of motion, simple Green's function
- ③ Expansion parameter is deviation from unperturbed (improved Zel'dovich) trajectories
- ④  $n$ -point statistics for collective fields obtained by functional derivatives
- ⑤ First-order perturbation theory reproduces numerical results already quite well

# Further applications

