The disk-magnetosphere interaction and the limiting spin period of accreting neutron stars

Student: Stefano Ascenzi

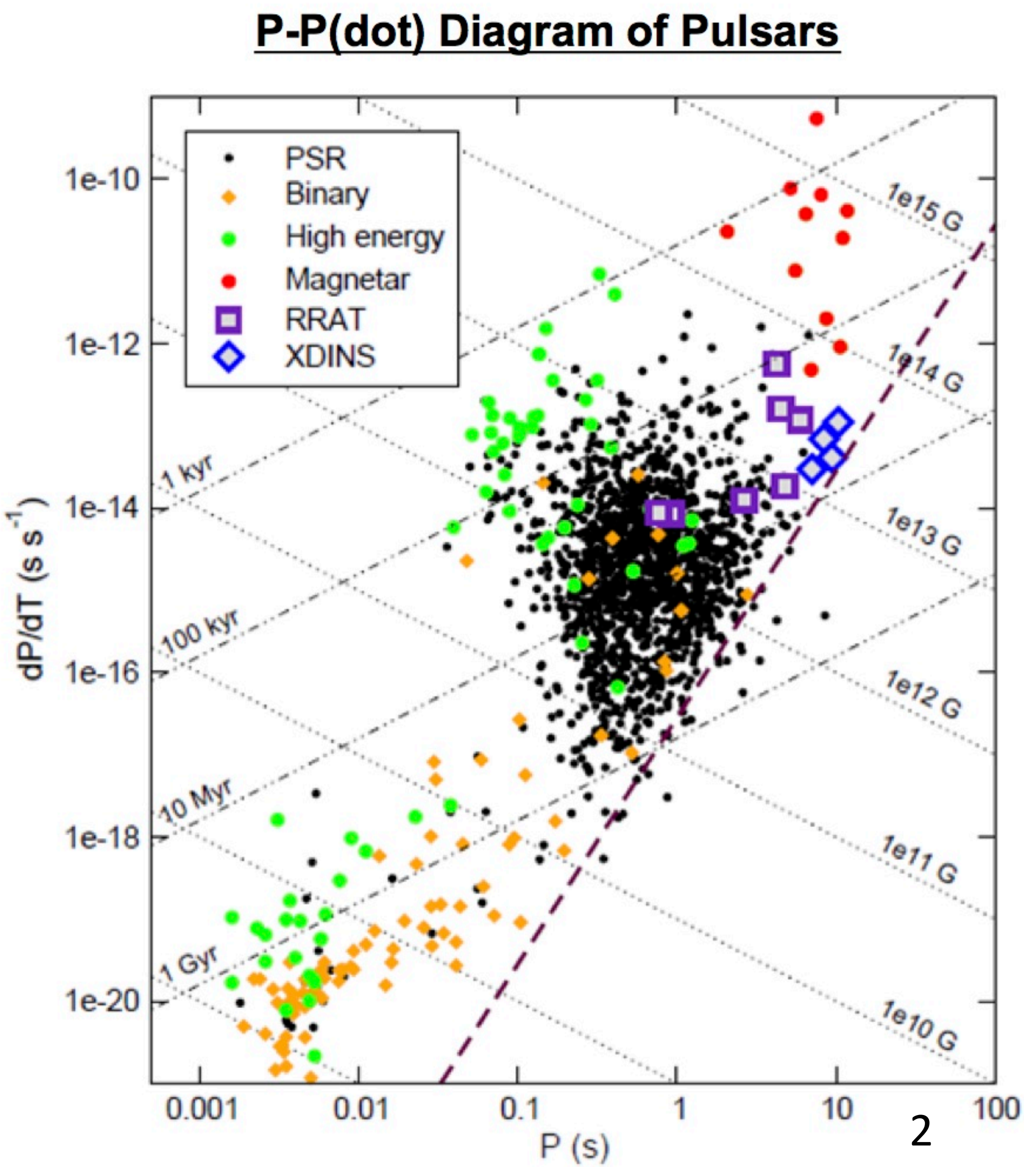
Advisor: Luigi Stella-INAF-OAR

Co-advisor: Simone Dall'Osso

Outline

- Pulsar: populations and distribution
- Accretion onto magnetized compact objects
- The Ghosh & Lamb model for magnetically-threaded accretion disks
- Beyond the Ghosh & Lamb model

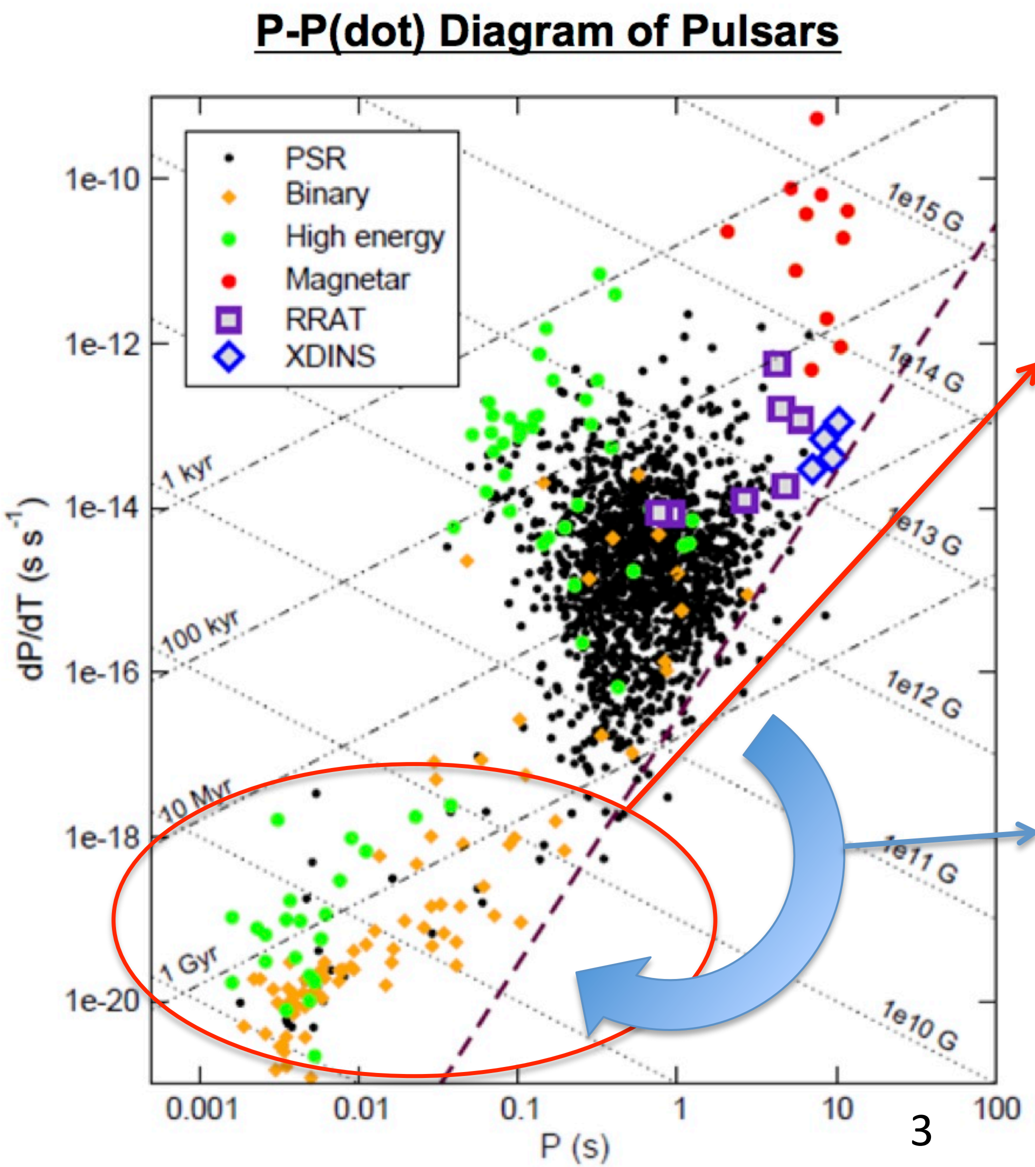
Pulsar: Populations and Distribution



Neutron Stars (NS) are often observed as Pulsar: pulsating sources in radio, X or γ wavebands with stable periods.

The P-P(dot) diagram is one of the best tool to study the different radio pulsar populations.

Pulsar: Populations and Distribution



The oldest NSs are in binary systems and have millisecond periods (millisecond pulsars).

Their evolution is described by the **recycling scenario**

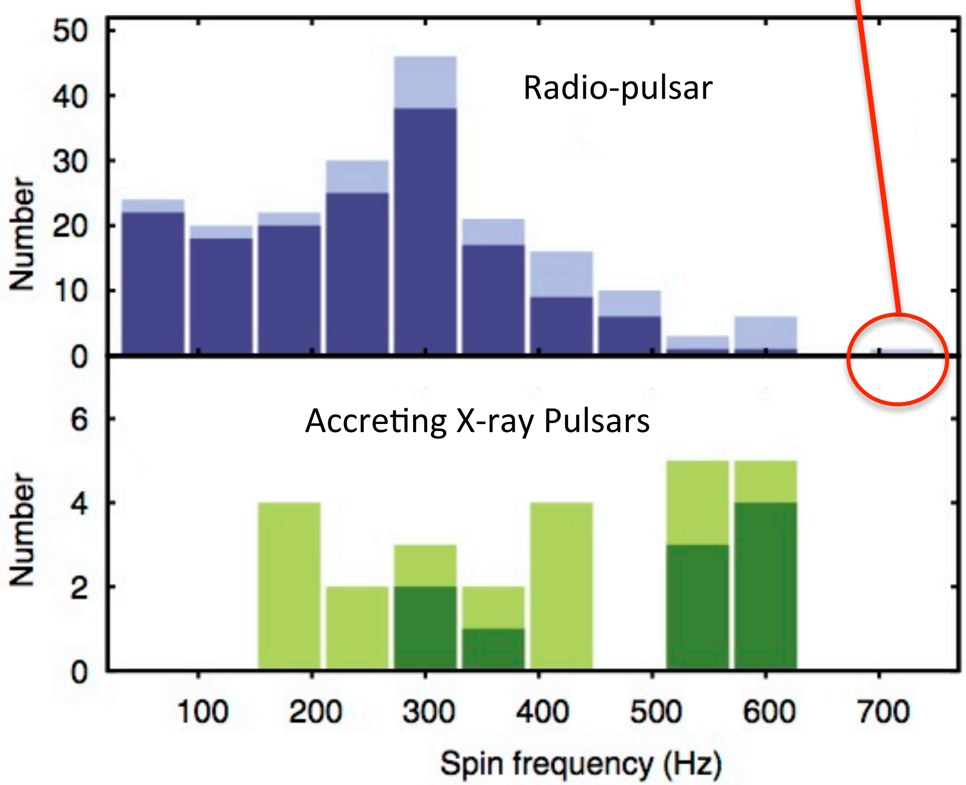
Pulsar: Populations and Distribution

ACCRETING MILLISECOND PULSARS (AMXPs)

- Transient Sources, powered by accretion outbursts
- Usually faint outburst luminosities $\approx 10^{36} erg / s$
- Ultra-compact binaries are common. 40% of the total AMXP population.
- Very small donors preferred, with masses below $0.2 M_{Sun}$

Pulsar: Populations and Distribution

PSR J1748-2446ad



Papitto et al. (2014)

MP spin frequency distribution

Cutoff at 730 Hz. Why?

Rouled-out explanations

Intrinsic limit of the NS structure

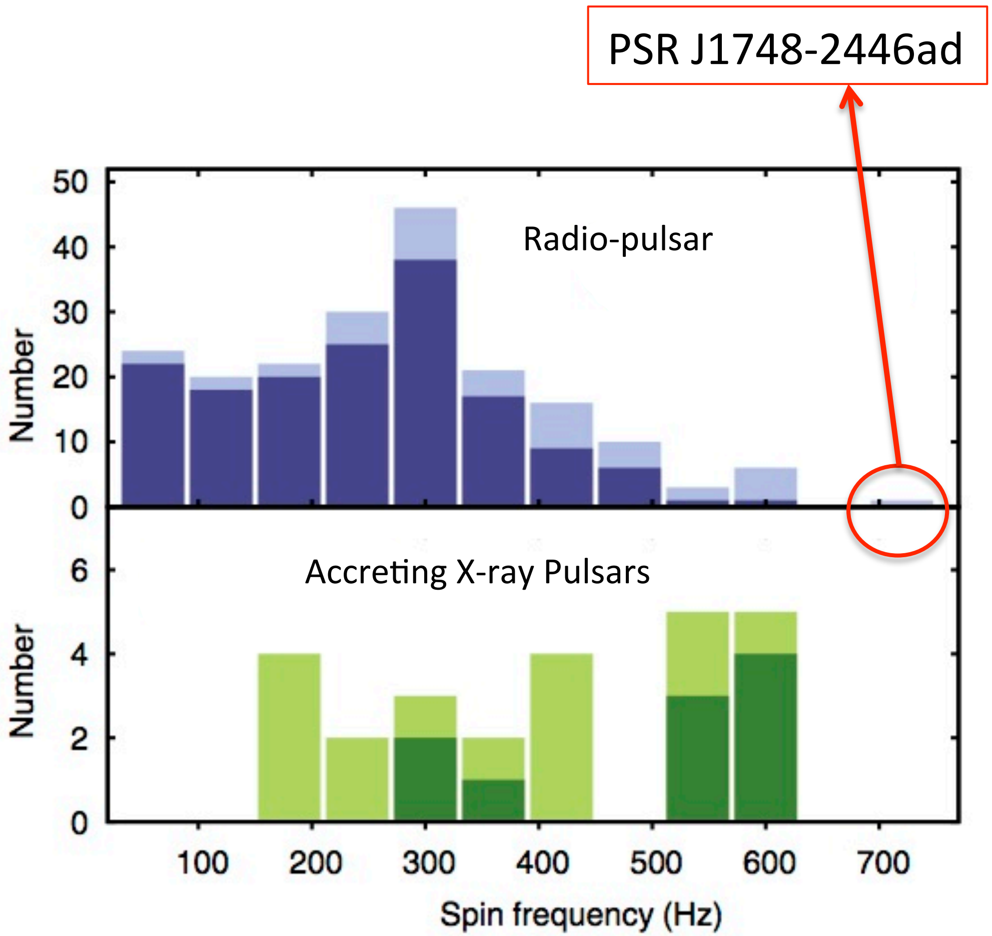
NSs sustain frequencies up to 2000 Hz for reasonable EOS

Instrumentation Bias

X-ray observatories (as RXTE) are not effected by loss of sensitivities at these frequencies

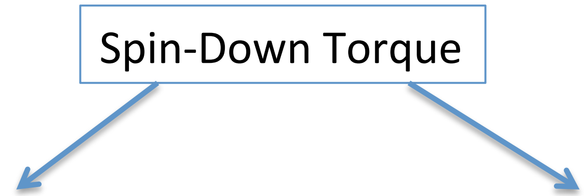
The upper limit may be due to a spin-equilibrium, which the NS attains when a spin-down torque balances the accretion spin-up

Pulsar: Populations and Distribution



Papitto et al. (2014)

MP spin frequency distribution



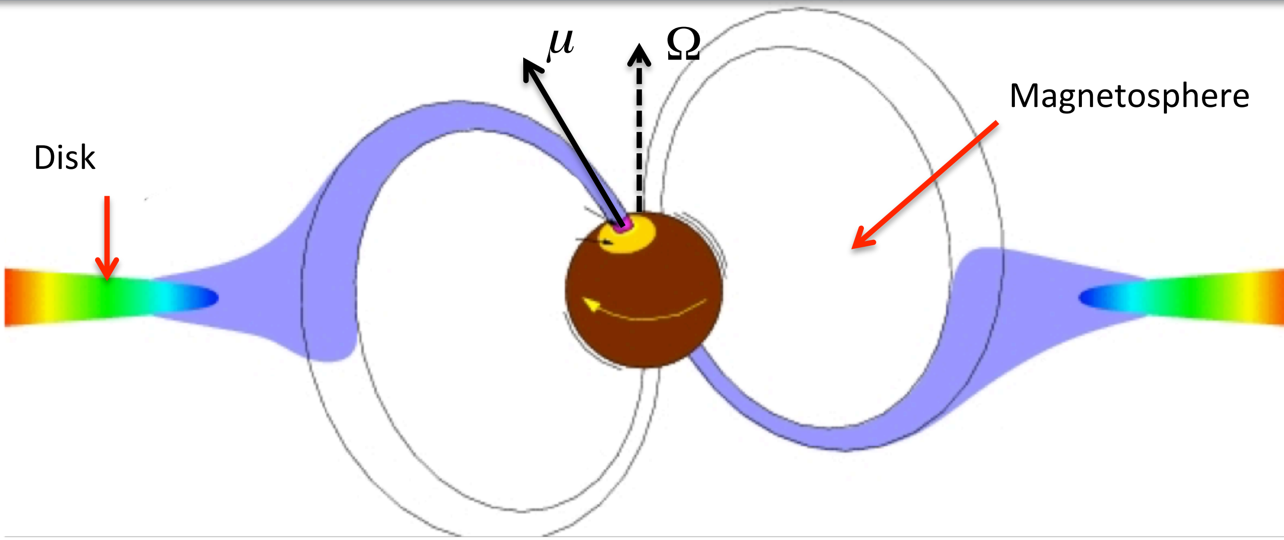
Gravitational Waves (GW) (Bildsten 1998)

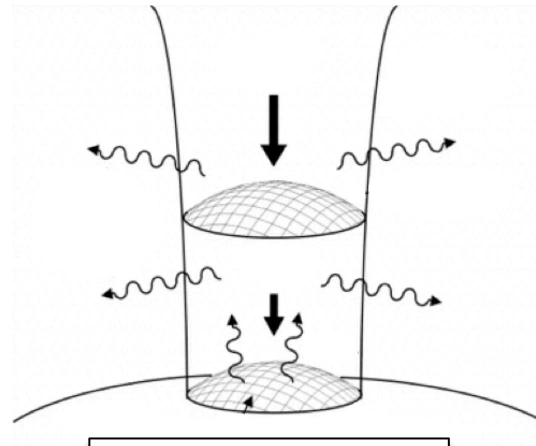
Requires too high deformation of the NS (Haskell & Patruno 2011)

Disk-Magnetosphere interaction (White-Zhang 1997)

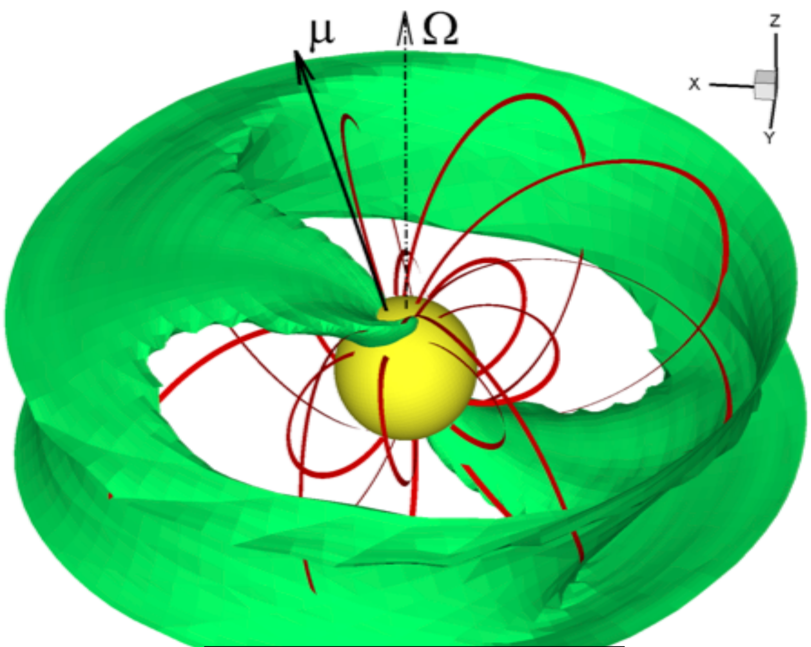
Requires a relation between magnetic field and mass accretion rate

Accretion onto Neutron Stars





Accretion Column



Romanova et. al 2008

Magnetospheric Radius

$$r_m \approx r_A^{(0)} = \dot{M}^{-2/7} \mu^{4/7} (2GM)^{-1/7}$$

Spin-up Torque

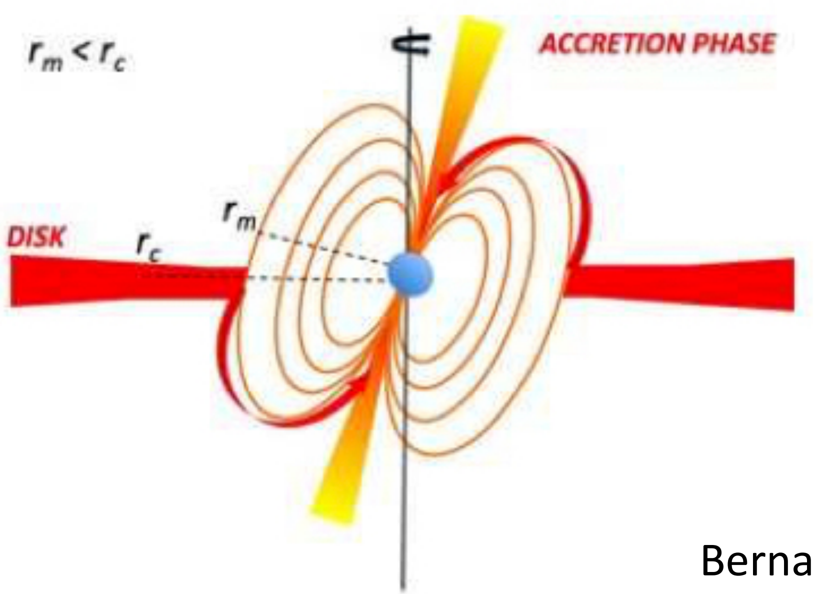
$$N_0 = \dot{M} \sqrt{GMr_m}$$

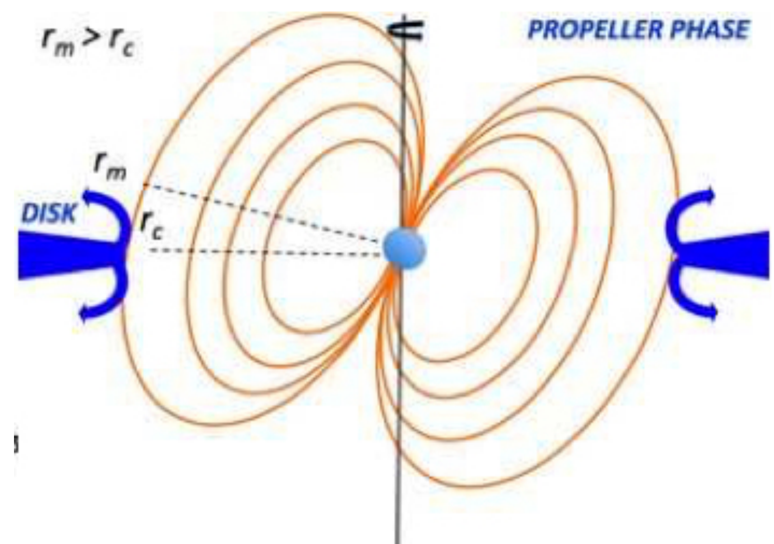
Accretion onto Neutron Stars

Corotation Radius → $r_c = (GM\Omega_s^{-2})^{1/3}$

Accretion → $r_m < r_c$

Propeller → $r_m > r_c$

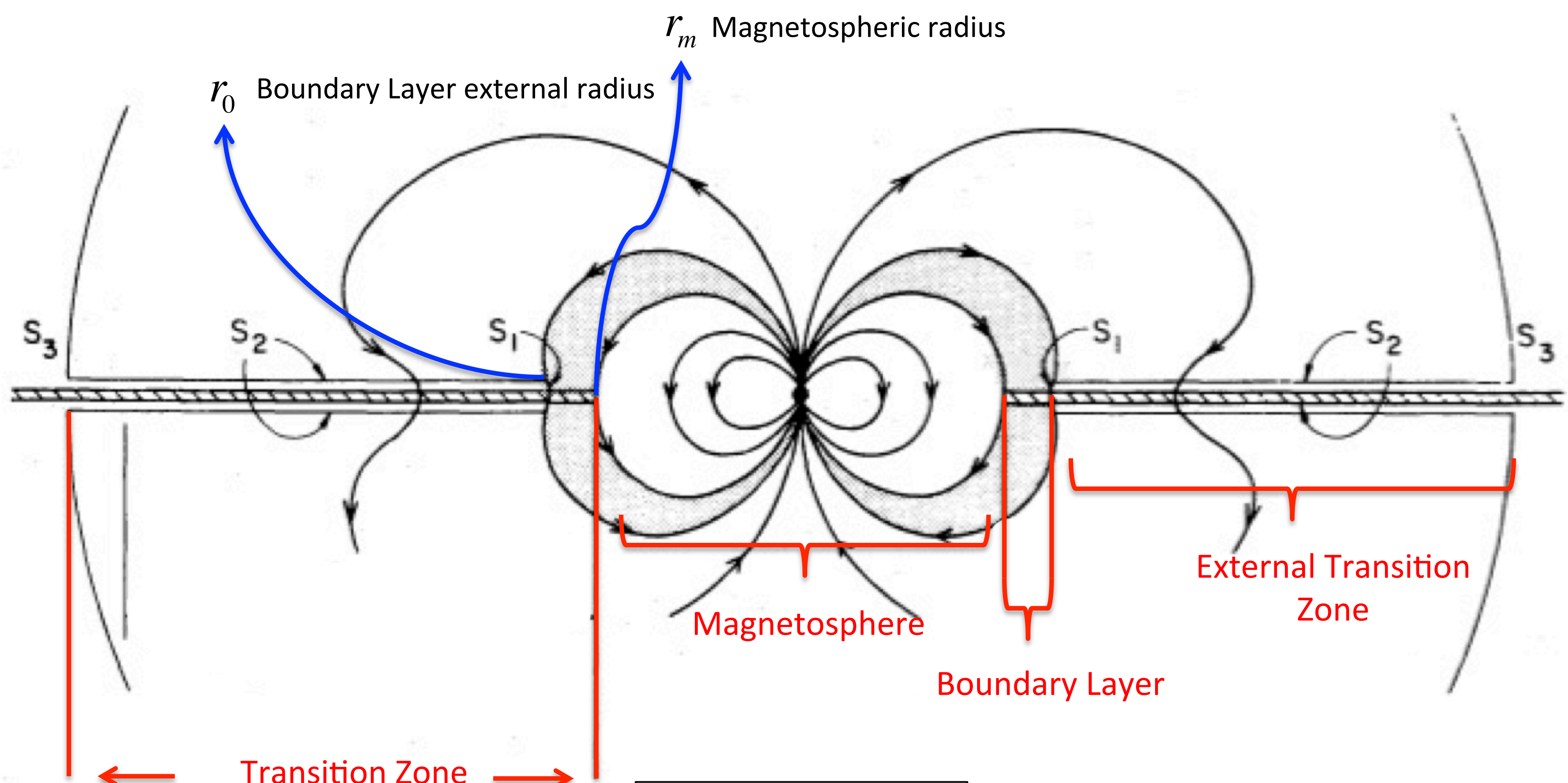




Bernardini et al. 2013

The Ghosh & Lamb Model

The Ghosh & Lamb model (1979) is an advanced description of the disk-magnetosphere coupling



Ghosh & Lamb 1979

The Ghosh & Lamb Model

Boundary Layer

HP:

- Steady-state and axial symmetry
- Dominant Azimuthal velocity
- Neglecting Radial Magnetic Field
- Purely radial electric Field
- Thin Disk
- Narrow Boundary-layer, i.e. $\varepsilon \equiv \delta_0 / r_0 << 1$

The Ghosh & Lamb Model

Boundary layer structure

Vertical structure

$$p = C_p \rho \left(\frac{h}{r}\right)^2 \left(\frac{GM}{r}\right)$$

$$\frac{1}{3}\frac{caT^4}{\rho h^2 k} = \frac{J^2}{\sigma_{eff}}$$

$$p = \rho\left(\frac{2k_B T}{m_p}\right)$$

$$\dot{M}_d = 4\pi r h |v_r| \rho$$

$$\sigma_{eff} = \left(\frac{c^2}{4\pi}\right) h^{-1} r^{-1} \left(\Omega - \Omega_s\right)^{-1} \gamma_\phi$$

$$\gamma_\phi = \frac{B_\phi}{B_z}$$

Radial Structure

$$\frac{d}{dr}(\dot{M}_d r^2 \Omega) = r^2 \Omega \left(\frac{d\dot{M}_d}{dr}\right) + B_\phi B_z r^2$$

$$v_r\left(\frac{dv_r}{dr}\right) = -\frac{GM}{r^2} + \Omega^2 r - \frac{1}{\rho}\frac{dp}{dr} + \frac{1}{4\pi\rho}\{(\vec{\nabla} \wedge \vec{B}) \wedge \vec{B}\} \cdot \hat{r}$$

$$\vec{\nabla} \wedge \vec{B} = \frac{4\pi}{c}\vec{J}$$

$$\vec{J} = \sigma_{eff}\left(\vec{E} + \frac{\vec{v}}{c} \wedge \vec{B}\right)$$

$$\frac{d\dot{M}_d}{dr} = 4\pi r \rho g(r)$$

Dimensionless equations

$$\frac{d\omega}{dx} = C_\omega \frac{b^2}{F}$$

$$u_r \frac{du_r}{dx} = -\frac{1-\omega^2}{2} + C_\omega \frac{(1+\gamma_\phi^2) u_r^2 b^2}{F(\omega - \omega_s)}$$

$$\frac{db}{dx} = -C_b \frac{b^{3/4} u_r^{9/8}}{F^{1/8} (\omega - \omega_s)^{9/8}}$$

$$\frac{dF}{dx} = 0.125\left(\gamma_\phi^{-8/27} C_b^{8/27} C_\omega^{1/27} C_p^{19/54}\right) g(x) \frac{F}{u_r}$$

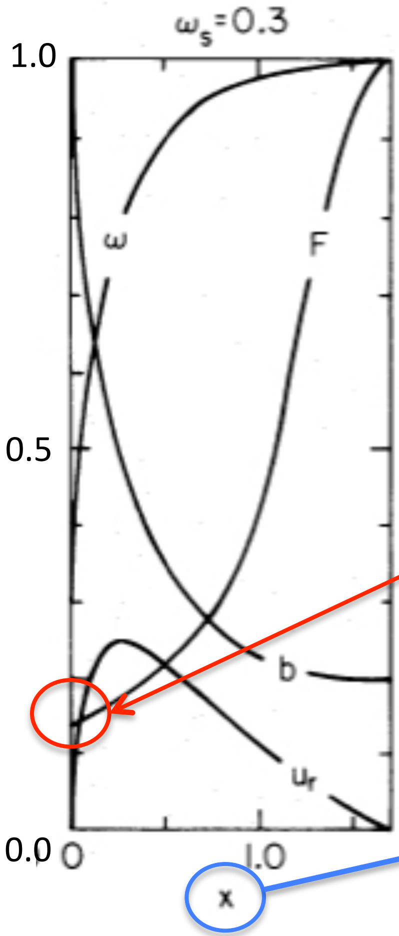
Gate function

$$g(x) = \begin{cases} 1 & 0 \le x \le x_m \\ (x_0 - x)^2 (x_0 - x_m)^{-2} & x_m \le x \le x_0 \end{cases}$$

$$x_0 - x_m = 0.6 \qquad x_0 = \delta / \delta_0$$

The Ghosh & Lamb Model

Ghosh & Lamb gate function



Ghosh & Lamb 1979

$$\delta_0 = 0.031\left(\gamma_\phi^{-16/27} C_b^{16/27} C_\omega^{2/27} C_p^{-8/27}\right) r_0$$

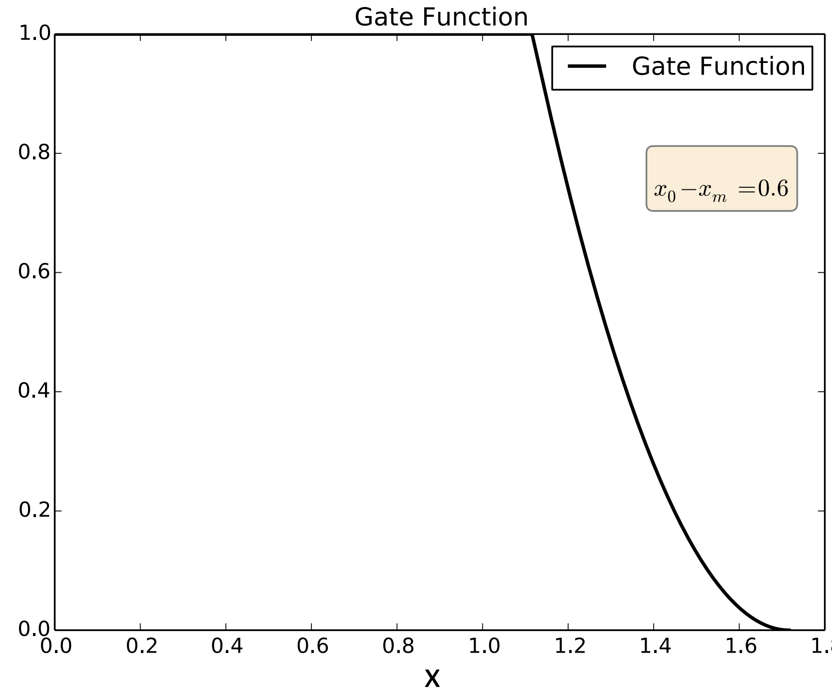
Ghosh & Lamb gate function

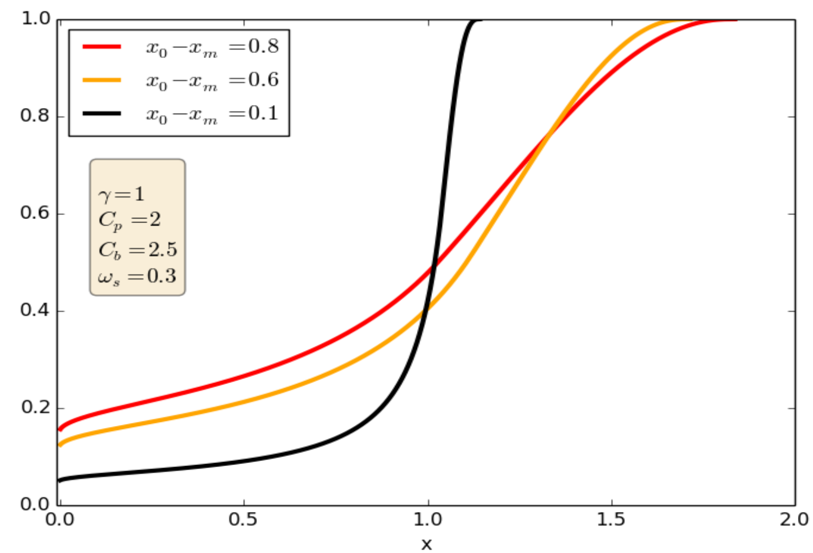
Radial flux does not vanish!

$$F \propto u_r$$

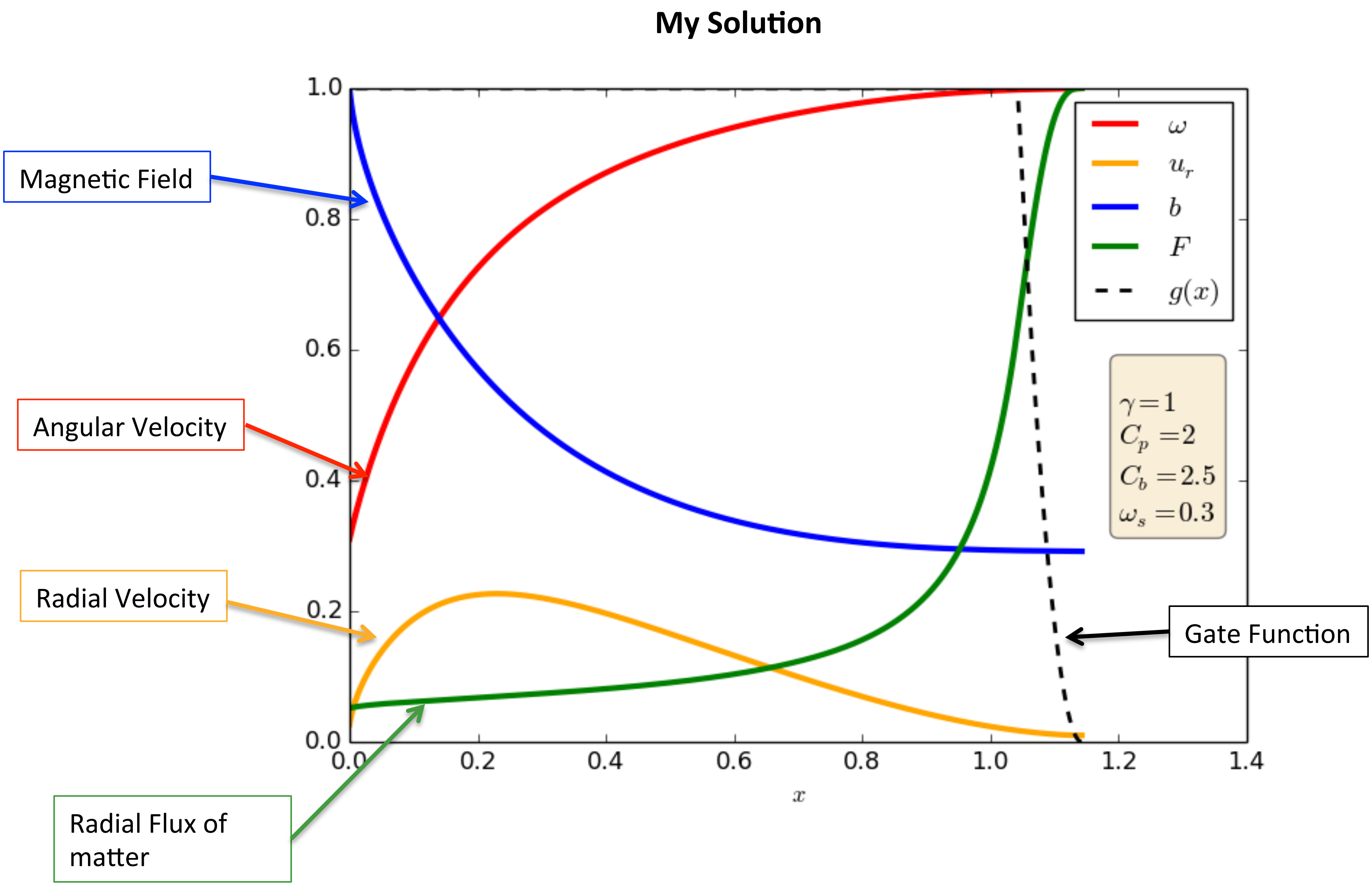
Dimensionless radial variable

Flux for different gate functions

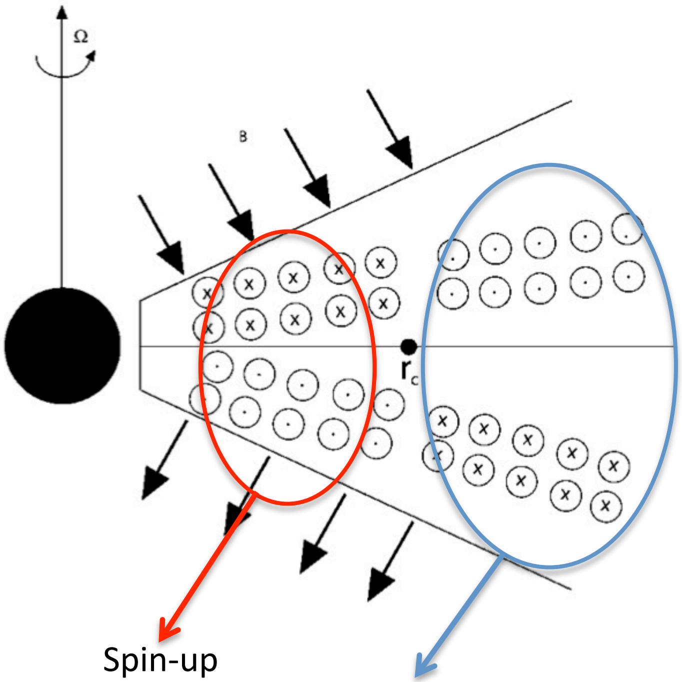




The Ghosh & Lamb Model



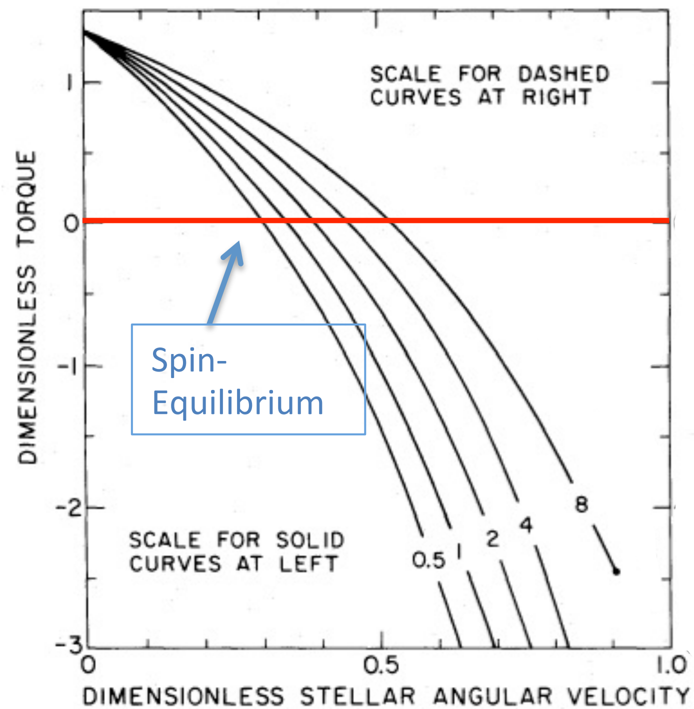
The Ghosh & Lamb Model



Vietri 2008

Accretion Torque

$$N = N_0 + N_{out} = n(\omega_s)N_0$$
$$N_0 \approx \dot{M}\sqrt{GMr_0}$$
$$N_{out} = \int_{r_0}^{r_s} \gamma_\phi B_z^2 r^2 \, dr$$



Ghosh & Lamb 1979

The accretion torque vanish at a precise frequency called “spin equilibrium” frequency

$\omega_c \approx 0.35$ ← Ghosh & Lamb

$$P_{eq} \approx 2.67\dot{M}_{17}^{-3/7}\mu_{30}^{6/7}$$

Beyond the Ghosh & Lamb model

Accretion disks close to the stellar surface

$$\varepsilon = \delta_0 / r_0 << 1$$ ← Ghosh & Lamb hypothesis

Ghosh & Lamb model → $$\varepsilon < 1\%$$ $$r_0 \approx 10^3 \, km$$

Are they satisfied for accreting millisecond pulsars?

If the spin-equilibrium holds → $$r_0 = (\omega_c)^{2/3} r_c = 0.5 r_c$$

$$r_c = 1683 \left(\frac{M}{1.4 M_{sun}} \right)^{1/3} \nu_s^{-2/3} km$$ → $$\nu_s = 600 - 700 \, Hz$$ → $$r_c \approx 23 km$$

Comparable with NS typical radius

Beyond the Ghosh & Lamb model

A new generalized model

HP:
Same of Ghosh & Lamb model,
but $\varepsilon \approx 10^{-1}$
First order terms included

$$r_0 \approx 10km$$

$$r_{in} = r_0\left(1 - x_0\varepsilon\right)$$
$$r = r_0\left[1 - (x_0 - x)\varepsilon\right]$$

$$\frac{d\omega}{dx} = C_\omega \frac{b^2}{F} + \left(6C_\omega \frac{x_0 b^2}{F} - 2\omega\right)\varepsilon$$

$$u_r \frac{du_r}{dx} = -\frac{1-\omega^2}{2x_0} + C_\omega \frac{\left(\gamma_\phi^2 + 1\right)u_r^2 b^2}{F\left(\omega - \omega_s\right)} - \frac{\varepsilon}{2x_0}\left[\left(x_0 - 2x\right) + \omega^2\left(2x_0 - x\right)\right] + 12\frac{C_\omega x_0 u_r^2 b^2}{F\left(\omega - \omega_s\right)}\varepsilon$$

$$\frac{db}{dx} = -C_b \frac{b^{3/4} u_r^{9/8}}{F^{1/8}\left(\omega - \omega_s\right)^{9/8}} x_0^{9/16}\left[1 + \left(\frac{37}{16}x_0 - \frac{5}{2}x\right)\varepsilon\right]$$

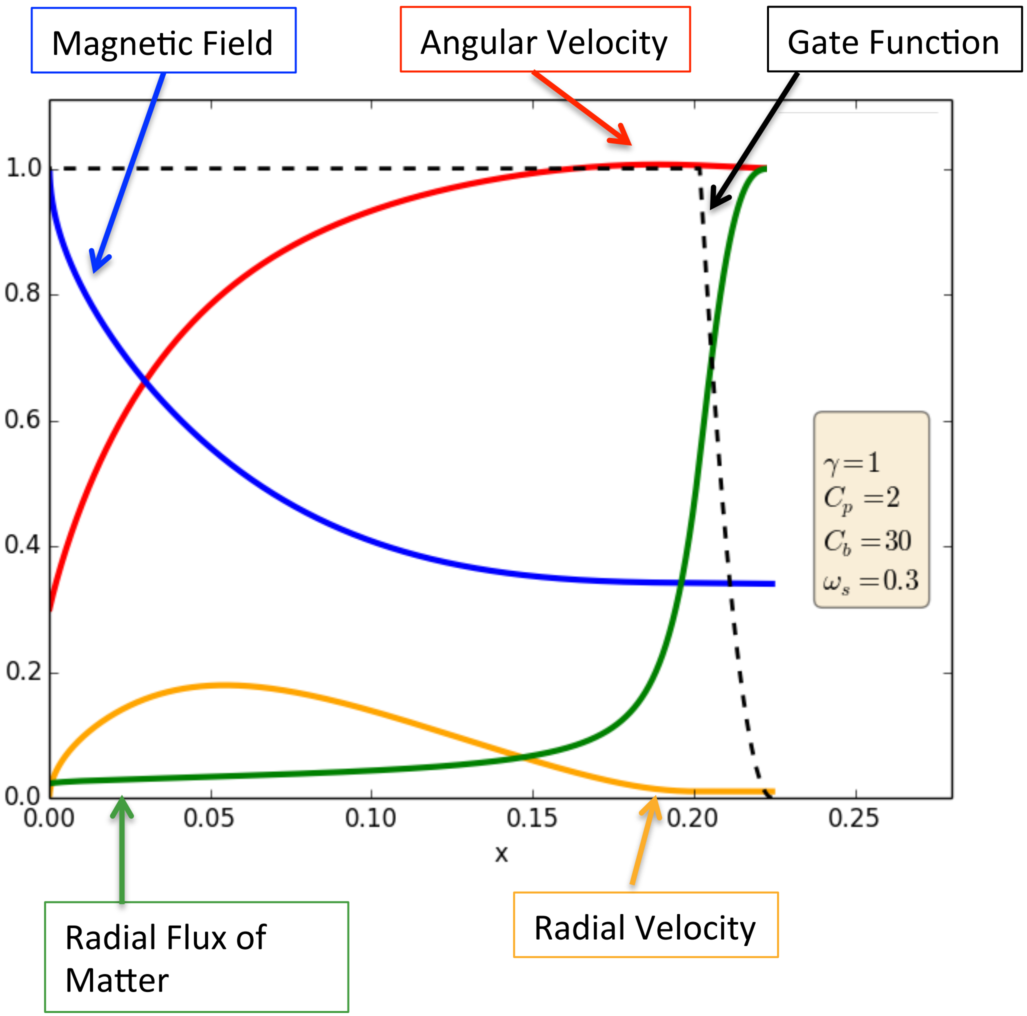
$$\frac{dF}{dx} = 0.114\left(\gamma_\phi^{-8/27} C_b^{8/27} C_\omega^{1/27} C_p^{19/54}\right) g(x) \frac{F}{x_0^{1/2} u_r}\left[1 + \left(x_0 - \frac{3}{2}x\right)\varepsilon\right]$$

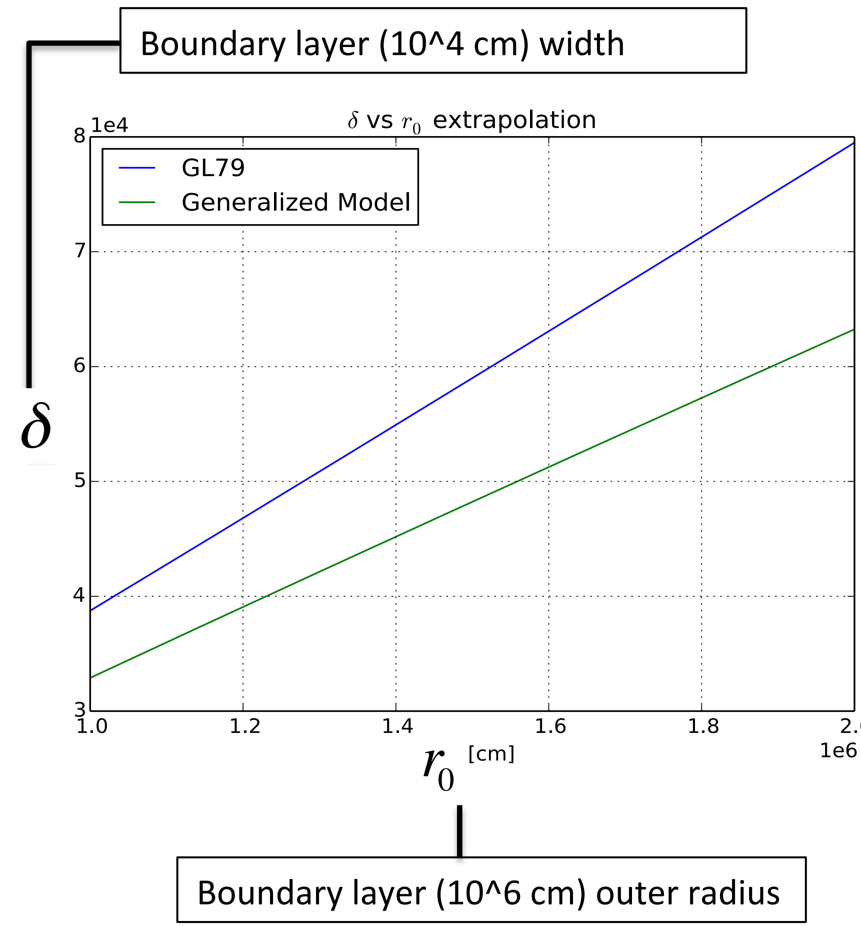
$$\delta_0 = 0.026\left(\gamma_\phi^{-16/27} C_b^{16/27} C_\omega^{2/27} C_p^{-8/27}\right) r_0$$

$$r_0 = 0.39\left(\gamma_\phi^{11/27} C_b^{16/27} C_\omega^{-25/27} C_p^{-8/27}\right)^{2/7} r_A^{(0)}$$

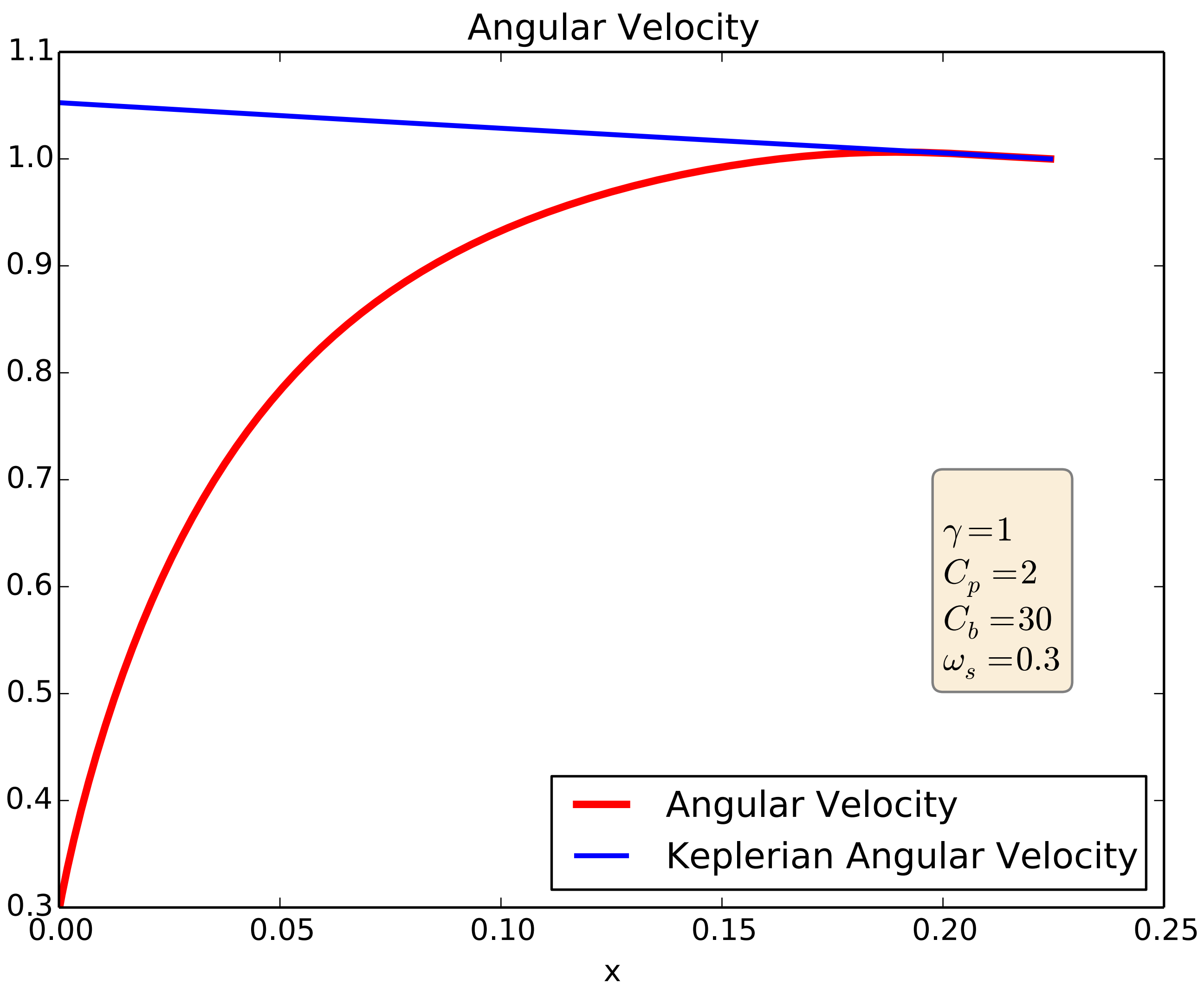
Beyond the Ghosh & Lamb model

My Solution





Beyond the Ghosh & Lamb model



The angular velocity is sub-keplerian!

Conclusions

RESULTS

- Extension to higher perturbative order of the Ghosh and Lamb model (needed for application to AMXPs).
- Appropriate shape of the gate function

- Weaker magnetic screening, well-behaved radial flux of matter.
- Thinner BL than in GL model → first order approx. realisitc given that $\varepsilon < 0.1$

FUTURE DEVELOPMENTS

- Calculation of the torque function in the current expansion
- Derivation of the corresponding equilibrium spin in the expanded GL model
- Higher orders in ε to confirm the trends

THANK YOU FOR THE ATTENTION!

Bibliography

- Bildsten, L., 1998, Ap. J., *501*, L89-L93
- Chakrabarty, D. *et al.*, 2003, Nature, *424*, 42-44
- Elsner, R.F., Lamb, F.K. 1976, Nature, *262*, 356. 1977, Ap.J., *215*, 897
- Ghosh,P., Lamb, F. K., and Pethick, C.J. 1977, Ap. J., *217*, 578
- Ghosh, P., Lamb, F. K. 1979, Ap. J., *232*, 259-276
- Ghosh, P., Lamb, F. K. 1979, Ap. J., *234*, 296-316
- Haskell, B., Patruno, A., 2011, Ap. J., *738*, L14

- Papitto, A. *et al.*, 2014, Nature, *501*, 517-520
- Patruno, A., 2010, Ap. J., *722*, 909-918
- Patruno, A., Watts, A.L., 2013, arXiv:1206.2727
- White, N.E., Zhang, W., 1997, Ap. J., *490*, L87, L90

Appendix A: Structure Constants

The Ghosh & Lamb boundary-layer has 4 structure constants:

γ_ϕ Magnetic Pinch

C_p Deviation from the hydrostatic equilibrium

$$C_\omega = \sqrt{2}\gamma_\phi \left(\frac{r_0}{r_A^{(0)}}\right)^{-7/2} \left(\frac{\delta_0}{r_0}\right)$$

$$C_b = 6^{1/8}\gamma_\phi^{7/8} C_p^{1/2} \left(\frac{r_0}{r_A^{(0)}}\right)^{7/16} \left(\frac{\delta_0}{r_0}\right)^{25/16} \psi^{-5/4}$$

$$\psi \equiv \left[\left(\frac{9\dot{M}^2}{32\pi^2}\right)\left(\frac{k}{ac}\right)(GMr_0)^{1/2}\left(\frac{2k_B}{GMm_p}\right)^4\right]^{1/10} \propto r_0^{1/20}$$

Constrain on the Structure Constants

Ghosh & Lamb

$$\gamma_\phi^{8/27} C_b^{-8/27} C_\omega^{25/54} C_p^{4/27} = 0.125 b_0^{-1}$$

My model

$$\gamma_\phi^{8/27} C_b^{-8/27} C_\omega^{25/54} C_p^{4/27} + 0.148\gamma_\phi^{-8/27} C_b^{8/27} C_\omega^{29/54} C_p^{-4/27} = 0.114 b_0^{-1}$$

Appendix B: Dimensionless Variables

$$x \equiv \frac{r - r_m}{\delta_0}$$

Dimensionless radial variable

Boundary layer typical scale

$$\omega \equiv \frac{\Omega}{\Omega_K(r_0)}$$

Keplerian velocity at the boundary-layer outer radius

$$u_r \equiv -v_r \left(\frac{2GM}{r_0}\right)^{-1/2} \varepsilon^{-1/2}$$

Free fall velocity

$$b \equiv \frac{B_z}{B_z(r_m)}$$

Magnetic field at the magnetosphere

$$F \equiv \frac{\dot{M}_d}{\dot{M}}$$

Mass accretion rate

Appendix C: Numerical Integration