Hydrodynamics and Magnetohydrodynamics

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Abstract

Hydrodynamics is an incredibly successful framework to describe the dynamics of matter from scales as small as those of colliding elementary particles, up to the largest scales in the universe. This course will provide an introduction to the mathematical and physical properties of hydrodynamics and its extension to magnetized fluids, i.e. magnetohydrodynamics or MHD. Starting from an initial kinetic-theory description, the equations of hydrodynamics will be derived and their most important properties will be discussed. The course will also discuss the nonlinear nature of the hydrodynamics equations and the occurrence of nonlinear waves such as shocks and rarefaction waves. The final part of the course will deal with neutrally charged and magnetized plasmas and discuss the basic features of ideal magnetohydrodynamics and the associated nonlinear waves. A series of exercises will parallel the course. The content of the lectures can be found in a series of books [1, 2, 3].

Syllabus and plan of the lectures

1. On the fluid approximation, Newtonian kinetic theory, The Boltzmann equation.
2. The H-theorem, The moment equations, The Maxwell-Boltzmann equilibrium distribution.
3. The zero-order approximation: perfect fluids, The first-order approximation: non-perfect fluids, Relativistic kinetic theory.
4. The relativistic Boltzmann equation, The relativistic moment equations, The general-relativistic hydrodynamic equations, Relativistic equilibrium distributions, The laws of thermodynamics.
5. Equations of state, Kinematic properties of fluids, Evolution laws of the kinematic quantities, Mass current and energy-momentum of perfect fluids.
6. Hydrodynamics equations of perfect fluids, Stationary flows, Bernoulli's theorem, Vorticity, Irrotational flows, Kelvin-Helmholtz theorem, Isentropic flows.
7. Hyperbolic systems of partial differential equations, Quasi-linear formulation.
8. Conservative formulation, Linear and nonlinear behaviour, Characteristic equations for linear systems, Characteristic curves and caustics.
9. Domain of determinacy and region of influence, Linear hydrodynamic waves, Sound waves, Nonlinear hydrodynamic waves.
10. Simple waves and discontinuous waves, Rarefaction waves, Shock waves, Contact discontinuities, The Riemann problem.
11. Introduction to plasmas, The magnetohydrodynamic equations, Flux-freezing condition.
12. Plasma orbit theory, magnetic mirrors, Debye shielding, plasma parameter.
13. Magnetohydrostatic solutions, Hydromagnetic waves.
14. Magnetic reconnection. Magnetic helicity, Force-free solutions.
15. Magnetohydrodynamic instabilities.

References

[1] Rezzolla L and Zanotti O 2013 *Relativistic Hydrodynamics* (Oxford University Press, Oxford UK)

[2] Choudhuri A R 1998 *The physics of fluids and plasmas: an introduction for astrophysicists* (Cambridge University Press, Cambridge UK)

[3] Goedbloed J and Poedts S 2004 *Principles of Magnetohydrodynamics: With Applications to Laboratory and Astrophysical Plasmas* (Cambridge University Press, Cambridge, UK)