Results from Gauge Theory of Gravity

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Overview

1. Aim: Derive the theory of gravity from first principles along the line of non-Abelian gauge theories.
2. Method: Canonical transformations ensure by construction that the action principle is preserved. Then, an action form-invariant under the diffeomorphism group implements the General Principle of Relativity.
3. Key results:
   - the connection coefficients are the gauge fields of gravity
   - gauge theory determines the coupling of base fields and gauge fields
   - each type of base field (scalar, vector, tensor, spinor; massive or massless) has its particular coupling term
   - the Lagrangian/Hamiltonian for the "free" spacetime dynamics must be postulated
   - Einstein's General Relativity is the particular case of
     (i) Hilbert Lagrangian (Ricci scalar) for the "free" spacetime dynamics
     (ii) scalar or massless vector base fields

Minimal Set of Basic Principles

1. Action Principle: The system dynamics follows from the variation of its action S, namely $\delta S \stackrel{!}{=} 0$.
2. Special Principle of Relativity: The action integrand must be form-invariant (symmetric) under (global) Lorentz transformations.
3. General Principle of Relativity: The action integrand must be form-invariant (symmetric) under local Lorentz transformations.
4. Gauge Principle: Promoting a global symmetry of a given system to a local symmetry by adding appropriate gauge fields yields a theory which is realized in nature.
5. Quadratic Momentum: The action integrand should contain a quadratic momentum term for the dynamics of the gravitational field — in analogy to the dynamics of all other fields.

Hamiltonian action principle for **static** spacetime

Action principle for a system of real scalar and vector fields (static metric)

$$S = \int_\Omega \left(\pi^\alpha \frac{\partial \phi}{\partial x^\alpha} + p^{\beta\alpha} \frac{\partial a_\beta}{\partial x^\alpha} - \mathcal{H}(\pi^\mu, \phi, p^{\nu\mu}, a_\nu, x^\mu) \right) \mathrm{d}^4 x$$

with

$$\delta S \stackrel{!}{=} 0, \quad \delta\phi\big|_{\partial\Omega} = \delta a_\mu\big|_{\partial\Omega} \stackrel{!}{=} 0.$$

Calculus of variations: $\delta S = 0$ holds exactly for the solutions of the

Covariant canonical field equations

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = \frac{\partial H}{\partial p^i} \quad \to \quad \frac{\partial \phi}{\partial x^\mu} = \frac{\partial \mathcal{H}}{\partial \pi^\mu}, \qquad \frac{\partial a_\nu}{\partial x^\mu} = \frac{\partial \mathcal{H}}{\partial p^{\nu\mu}}$$

$$\frac{\mathrm{d}p^i}{\mathrm{d}t} = -\frac{\partial H}{\partial q_i} \quad \to \quad \frac{\partial \pi^\alpha}{\partial x^\alpha} = -\frac{\partial \mathcal{H}}{\partial \phi}, \qquad \frac{\partial p^{\nu\alpha}}{\partial x^\alpha} = -\frac{\partial \mathcal{H}}{\partial a_\nu}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\partial H}{\partial t}\bigg|_{\mathrm{expl}} \quad \to \quad \frac{\partial \theta_\mu{}^\alpha}{\partial x^\alpha} = \frac{\partial \mathcal{H}}{\partial x^\mu}\bigg|_{\mathrm{expl}}, \qquad \theta_\mu{}^\alpha : \text{can. E-M tensor}$$

Hamiltonian action principle for **dynamic** spacetime

Action principle generalized for a dynamic metric

$$S=\int_\Omega\left(\tilde{\pi}^\alpha\frac{\partial\phi}{\partial x^\alpha}+\tilde{p}^{\beta\alpha}\frac{\partial a_\beta}{\partial x^\alpha}+\tilde{k}^{\beta\lambda\alpha}\frac{\partial g_{\beta\lambda}}{\partial x^\alpha}-\tilde{\mathcal{H}}(\tilde{\pi},\phi,\tilde{p},a,\tilde{k},g,x)\right)\mathrm{d}^4x$$

with $\sqrt{-g}\,\mathrm{d}^4x$ the invariant volume form and the tensor densities

$$\tilde{\pi}^\mu=\pi^\mu\sqrt{-g},\quad \tilde{p}^{\mu\nu}=p^{\mu\nu}\sqrt{-g},\quad \tilde{k}^{\mu\lambda\nu}=k^{\mu\lambda\nu}\sqrt{-g},\quad \tilde{\mathcal{H}}=\mathcal{H}\sqrt{-g}.$$

$g_{\beta\lambda}(x)$ denotes the system's metric and g the metric's determinant.

Extended set of covariant canonical field equations

$$\frac{\partial\phi}{\partial x^\mu}=\frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{\pi}^\mu},\qquad \frac{\partial a_\nu}{\partial x^\mu}=\frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{p}^{\nu\mu}},\qquad \frac{\partial g_{\nu\lambda}}{\partial x^\mu}=\frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{k}^{\nu\lambda\mu}}$$

$$\frac{\partial\tilde{\pi}^\alpha}{\partial x^\alpha}=-\frac{\partial\tilde{\mathcal{H}}}{\partial\phi},\qquad \frac{\partial\tilde{p}^{\nu\alpha}}{\partial x^\alpha}=-\frac{\partial\tilde{\mathcal{H}}}{\partial a_\nu},\qquad \frac{\partial\tilde{k}^{\nu\lambda\alpha}}{\partial x^\alpha}=-\frac{\partial\tilde{\mathcal{H}}}{\partial g_{\nu\lambda}}=-\tfrac{1}{2}\tilde{T}^{\lambda\nu}.$$

Klein-Gordon Hamiltonian in a dynamic spacetime

Example (Hamiltonian for the dynamics of a real scalar field)

The Klein-Gordon Hamiltonian with spacetime-dependent metric $g_{\alpha\beta}(x)$ is

$$\tilde{\mathcal{H}}_{\mathrm{KG}}(\tilde{\pi},\phi,g)=\tfrac{1}{2}\tilde{\pi}^\alpha\tilde{\pi}^\beta\,g_{\alpha\beta}\frac{1}{\sqrt{-g}}+\tfrac{1}{2}m^2\,\phi^2\sqrt{-g},$$

the field equations emerge as

$$\frac{\partial\phi}{\partial x^\nu}=\frac{\partial\tilde{\mathcal{H}}_{\mathrm{KG}}}{\partial\tilde{\pi}^\nu}=g_{\nu\beta}\frac{\tilde{\pi}^\beta}{\sqrt{-g}}=\pi_\nu,\qquad \phi_{;\nu}=\pi_\nu$$

$$\frac{\partial\tilde{\pi}^\alpha}{\partial x^\alpha}=-\frac{\partial\tilde{\mathcal{H}}_{\mathrm{KG}}}{\partial\phi}=-m^2\phi\sqrt{-g},\qquad \tilde{\pi}^\alpha{}_{;\alpha}=-m^2\phi\sqrt{-g}+2\tilde{\pi}^\alpha s^\beta{}_{\alpha\beta}$$

$$T^{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{KG}}}{\partial g_{\nu\mu}}=\pi^\mu\pi^\nu-\tfrac{1}{2}g^{\mu\nu}\left(\pi^\alpha\pi^\beta g_{\alpha\beta}-m^2\,\phi^2\right)$$

Requirement of form-invariance for the action principle

For a gauge theory that includes a general mapping of spacetime $x\mapsto X$, we need the connection coefficients $\gamma^\eta{}_{\alpha\xi}$ as additional dynamic quantities

Condition for canonical transformations under a dynamical spacetime

$$S=\int_\Omega\left(\tilde{\pi}^\beta\frac{\partial\phi}{\partial x^\beta}+\tilde{p}^{\alpha\beta}\frac{\partial a_\alpha}{\partial x^\beta}+\tilde{k}^{\alpha\lambda\beta}\frac{\partial g_{\alpha\lambda}}{\partial x^\beta}+\tilde{q}_\eta{}^{\alpha\xi\beta}\frac{\partial\gamma^\eta{}_{\alpha\xi}}{\partial x^\beta}-\tilde{\mathcal{H}}-\frac{\partial\tilde{\mathcal{F}}_1^\beta}{\partial x^\beta}\right)\mathrm{d}^4x$$

$$=\int_{\Omega'}\left(\tilde{\Pi}^\beta\frac{\partial\Phi}{\partial X^\beta}+\tilde{P}^{\alpha\beta}\frac{\partial A_\alpha}{\partial X^\beta}+\tilde{K}^{\alpha\lambda\beta}\frac{\partial G_{\alpha\lambda}}{\partial X^\beta}+\tilde{Q}_\eta{}^{\alpha\xi\beta}\frac{\partial\Gamma^\eta{}_{\alpha\xi}}{\partial X^\beta}-\tilde{\mathcal{H}}'\right)\mathrm{d}^4X$$

- The integrands must be world scalar densities in order to be form-invariant under general spacetime transformations.
- ⇝ The partial derivatives must be promoted to covariant derivatives.
- ⇝ The connection coefficients γ, Γ are the gauge quantities.
- $\tilde{\mathcal{F}}_1^\beta$ is the generating function of the canonical transformation $x\mapsto X$.

General CT rules under dynamic spacetime

Legendre transf.: $\tilde{\mathcal{F}}_1^\mu(\phi,\Phi,a,A,g,G,\gamma,\Gamma,x)\mapsto\tilde{\mathcal{F}}_3^\mu(\tilde{\pi},\Phi,\tilde{p},A,\tilde{k},G,\tilde{q},\Gamma,x)$

$$\tilde{\Pi}^\mu=-\frac{\partial\tilde{\mathcal{F}}_3^\kappa}{\partial\Phi}\frac{\partial X^\mu}{\partial x^\kappa}\left|\frac{\partial x}{\partial X}\right|\qquad \delta^\mu_\nu\phi=-\frac{\partial\tilde{\mathcal{F}}_3^\mu}{\partial\tilde{\pi}^\nu}$$

$$\tilde{P}^{\nu\mu}=-\frac{\partial\tilde{\mathcal{F}}_3^\kappa}{\partial A_\nu}\frac{\partial X^\mu}{\partial x^\kappa}\left|\frac{\partial x}{\partial X}\right|\qquad \delta^\mu_\nu a_\alpha=-\frac{\partial\tilde{\mathcal{F}}_3^\mu}{\partial\tilde{p}^{\alpha\nu}}$$

$$\tilde{K}^{\xi\lambda\mu}=-\frac{\partial\tilde{\mathcal{F}}_3^\kappa}{\partial G_{\xi\lambda}}\frac{\partial X^\mu}{\partial x^\kappa}\left|\frac{\partial x}{\partial X}\right|\qquad \delta^\mu_\nu g_{\alpha\beta}=-\frac{\partial\tilde{\mathcal{F}}_3^\mu}{\partial\tilde{k}^{\alpha\beta\nu}}$$

$$\tilde{Q}_\eta{}^{\xi\lambda\mu}=-\frac{\partial\tilde{\mathcal{F}}_3^\kappa}{\partial\Gamma^\eta{}_{\xi\lambda}}\frac{\partial X^\mu}{\partial x^\kappa}\left|\frac{\partial x}{\partial X}\right|\qquad \delta^\mu_\nu\gamma^\eta{}_{\alpha\beta}=-\frac{\partial\tilde{\mathcal{F}}_3^\mu}{\partial\tilde{q}_\eta{}^{\alpha\beta\nu}}$$

$$\left|\frac{\partial x}{\partial X}\right|:=\frac{\partial\left(x^0,\ldots,x^3\right)}{\partial\left(X^0,\ldots,X^3\right)}=\frac{\sqrt{-G}}{\sqrt{-g}}\qquad \tilde{\mathcal{H}}'=\left(\tilde{\mathcal{H}}+\left.\frac{\partial\tilde{\mathcal{F}}_3^\alpha}{\partial x^\alpha}\right|_{\mathrm{expl}}\right)\left|\frac{\partial x}{\partial X}\right|$$

Generating function

The generating function $\tilde{\mathcal{F}}_3^{\mu}$ is devised to define the required mappings

$$\Phi(X) = \phi(x), \qquad A_\mu(X) = a_\alpha(x)\frac{\partial x^\alpha}{\partial X^\mu}, \qquad G_{\nu\mu}(X) = g_{\alpha\lambda}(x)\frac{\partial x^\alpha}{\partial X^\nu}\frac{\partial x^\lambda}{\partial X^\mu}$$

and

$$\Gamma^{\kappa}{}_{\alpha\beta}(X) = \gamma^{\xi}{}_{\eta\tau}(x)\frac{\partial x^\eta}{\partial X^\alpha}\frac{\partial x^\tau}{\partial X^\beta}\frac{\partial X^\kappa}{\partial x^\xi} + \frac{\partial^2 x^\xi}{\partial X^\alpha \partial X^\beta}\frac{\partial X^\kappa}{\partial x^\xi}.$$

It is given by

$$\tilde{\mathcal{F}}_3^{\mu}\Big|_x = -\tilde{\pi}^{\mu}\,\Phi - \tilde{p}^{\alpha\mu}\,A_\beta\frac{\partial X^\beta}{\partial x^\alpha} - \tilde{k}^{\alpha\beta\mu}G_{\xi\lambda}\frac{\partial X^\xi}{\partial x^\alpha}\frac{\partial X^\lambda}{\partial x^\beta} - \tilde{q}_\eta{}^{\alpha\beta\mu}\left(\Gamma^{\tau}{}_{\xi\lambda}\frac{\partial x^\eta}{\partial X^\tau}\frac{\partial X^\xi}{\partial x^\alpha}\frac{\partial X^\lambda}{\partial x^\beta} + \frac{\partial x^\eta}{\partial X^\tau}\frac{\partial^2 X^\tau}{\partial x^\alpha \partial x^\beta}\right).$$

$\rightsquigarrow$ $\tilde{\mathcal{F}}_3^{\beta}$ simultaneously defines the transformation rules for the conjugate momentum fields $\tilde{\pi}$, $\tilde{p}$, $\tilde{k}$, $\tilde{q}$ and for the Hamiltonian $\tilde{\mathcal{H}}$.

Generally invariant action principle

We thus encounter the "gauged" Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{G}}$ (after "some algebra"!)

$$\tilde{\mathcal{H}}_{\mathrm{G}} = \tilde{\mathcal{H}} + \left(\tilde{p}^{\alpha\beta}a_\xi + \tilde{k}^{\alpha\lambda\beta}g_{\xi\lambda} + \tilde{k}^{\lambda\alpha\beta}g_{\lambda\xi}\right)\gamma^{\xi}{}_{\alpha\beta} + \tfrac{1}{2}\tilde{q}_\eta{}^{\alpha\xi\beta}\left(\frac{\partial\gamma^{\eta}{}_{\alpha\xi}}{\partial x^\beta} + \frac{\partial\gamma^{\eta}{}_{\alpha\beta}}{\partial x^\xi} + \gamma^{\tau}{}_{\alpha\beta}\gamma^{\eta}{}_{\tau\xi} - \gamma^{\tau}{}_{\alpha\xi}\gamma^{\eta}{}_{\tau\beta}\right)$$

Inserting the Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{G}}$ into the above action functionals yields

$$S = \int_\Omega \left(\tilde{\pi}^{\beta}\phi_{;\beta} + \tilde{p}^{\alpha\beta}\,a_{\alpha;\beta} + \tilde{k}^{\alpha\lambda\beta}\,g_{\alpha\lambda;\beta} - \tfrac{1}{2}\tilde{q}_\eta{}^{\alpha\xi\beta}R^{\eta}{}_{\alpha\xi\beta} - \tilde{\mathcal{H}}\right)\mathrm{d}^4x.$$

- The partial derivatives of the fields ϕ, a_μ, and $g_{\nu\mu}$ in the original action functional are indeed converted into covariant derivatives.
- In contrast, the partial derivatives of the non-tensorial gauge fields $\gamma^{\eta}{}_{\alpha\xi}$ cannot be converted into covariant derivatives.
- Miraculously, the terms of the calculated gauge Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{G}}$ complement these derivatives to the Riemann curvature tensors $R^{\eta}{}_{\alpha\xi\beta}$

The "free" gauge field Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$

As common to all gauge theories,

- the gauge formalism yields merely the coupling terms of the fields of the given system $\tilde{\mathcal{H}}$ to the gauge fields,
- the gauge formalism does not provide the Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ describing the dynamics of the "free" gauge fields, here: the dynamics of the $\gamma^{\xi}{}_{\alpha\beta}(x)$ in classical vacuum,
- the Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ for the dynamics of the "free" gauge fields must be added "by hand", based on physical reasoning,
- the "free gauge field Hamiltonian" $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ accounts for the residual indeterminacy of any gauge theory, here: the gauge theory of gravity.

Final action functional is a world scalar ⇔ general principle of relativity

$$S = \int_\Omega \left(\tilde{\pi}^{\beta}\phi_{;\beta} + \tilde{p}^{\alpha\beta}\,a_{\alpha;\beta} + \tilde{k}^{\alpha\lambda\beta}\,g_{\alpha\lambda;\beta} - \tfrac{1}{2}\tilde{q}_\eta{}^{\alpha\xi\beta}R^{\eta}{}_{\alpha\xi\beta} - \tilde{\mathcal{H}} - \tilde{\mathcal{H}}_{\mathrm{Dyn}}\right)\mathrm{d}^4x$$

with a form-invariant Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}(\tilde{k},\tilde{q},g) = \tilde{\mathcal{H}}_{\mathrm{Dyn}}(\tilde{K},\tilde{Q},G)$.

Canonical field equations for given $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$

$$\phi_{;\,\mu} = \frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{\pi}^{\mu}}, \qquad \tilde{\pi}^{\beta}{}_{;\,\beta} = -\frac{\partial\tilde{\mathcal{H}}}{\partial\phi} + 2\tilde{\pi}^{\beta}s^{\alpha}{}_{\beta\alpha}$$

$$a_{\nu;\,\mu} = \frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{p}^{\nu\mu}}, \qquad \tilde{p}^{\nu\beta}{}_{;\,\beta} = -\frac{\partial\tilde{\mathcal{H}}}{\partial a_\nu} + 2\tilde{p}^{\nu\beta}s^{\alpha}{}_{\beta\alpha}$$

$$g_{\xi\lambda;\,\mu} = \frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial\tilde{k}^{\xi\lambda\mu}}, \qquad \tilde{k}^{\xi\lambda\beta}{}_{;\,\beta} = -\frac{\partial\left(\tilde{\mathcal{H}} + \tilde{\mathcal{H}}_{\mathrm{Dyn}}\right)}{\partial g_{\xi\lambda}} + 2\tilde{k}^{\xi\lambda\beta}s^{\alpha}{}_{\beta\alpha}$$

$$-\frac{R^{\eta}{}_{\xi\lambda\mu}}{2} = \frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial\tilde{q}_\eta{}^{\xi\lambda\mu}}, \qquad \tilde{q}_\eta{}^{\xi\lambda\beta}{}_{;\,\beta} = -\tilde{p}^{\xi\lambda}a_\eta - 2\tilde{k}^{\beta\xi\lambda}g_{\beta\eta} + \tilde{q}_\eta{}^{\xi\beta\alpha}s^{\lambda}{}_{\beta\alpha} + 2\tilde{q}_\eta{}^{\xi\lambda\beta}s^{\alpha}{}_{\beta\alpha}$$

- Throughout tensor equations $\rightsquigarrow$ form-invariant in any reference frame.
- $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ must be postulated, $\tilde{\mathcal{H}}_{\mathrm{Dyn}} \equiv 0 \Leftrightarrow$ flat metric comp. spacetime.
- It includes possible torsion ($s^{\lambda}{}_{\beta\alpha} \neq 0$) and non-metricity ($g_{\xi\lambda;\,\mu} \neq 0$).
- $\rightsquigarrow$ Most general set for the coupled dynamics of fields and spacetime.

Consistency relation of the canonical field equations

From the canonical equations, one directly derives the

Energy-momentum balance relation

$$2\tilde{k}^{\nu\beta\lambda}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial\tilde{k}^{\mu\beta\lambda}} - \tilde{q}_{\mu}{}^{\tau\beta\lambda}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial\tilde{q}_{\nu}{}^{\tau\beta\lambda}} + \tilde{q}_{\tau}{}^{\nu\beta\lambda}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial\tilde{q}_{\tau}{}^{\mu\beta\lambda}} - 2g_{\mu\beta}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{Dyn}}}{\partial g_{\nu\beta}}$$
$$= a_\mu \frac{\partial\tilde{\mathcal{H}}}{\partial a_\nu} - \tilde{p}^{\nu\beta}\frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{p}^{\mu\beta}} + 2g_{\mu\beta}\frac{\partial\tilde{\mathcal{H}}}{\partial g_{\nu\beta}} = \tilde{\theta}_\mu{}^\nu \quad\leftarrow\quad \text{canonical E-M tensor.}$$

It can be shown to have the equivalent Lagrangian representation

$$\frac{\partial\mathcal{L}_{\mathrm{Dyn}}}{\partial g^{\alpha\beta}{}_{;\nu}} g^{\alpha\beta}{}_{;\mu} + 2\frac{\partial\mathcal{L}_{\mathrm{Dyn}}}{\partial R^{\eta}{}_{\alpha\beta\nu}} R^{\eta}{}_{\alpha\beta\mu} - \delta^\nu_\mu\mathcal{L}_{\mathrm{Dyn}} = -\theta_\mu{}^\nu$$
$$\theta_\mu{}^\nu = \frac{\partial\mathcal{L}}{\partial\left(\frac{\partial\phi}{\partial x^\nu}\right)}\frac{\partial\phi}{\partial x^\mu} + \frac{\partial\mathcal{L}}{\partial a_{\alpha;\nu}}a_{\alpha;\mu} - \delta^\nu_\mu\mathcal{L},$$

which actually represents a generalized Einstein equation.

Generalized Einstein equation

The Einstein equation proper is thus the particular case if:

- The dynamics of the "free" gravitational field is described by the Hilbert Lagrangian $\mathcal{L}_{\mathrm{Dyn}} \propto R \equiv R^{\beta}{}_{\alpha\beta\mu}g^{\mu\alpha}$
- The system satisfies the metric compatibility condition $g_{\alpha\beta;\mu} = 0$
- The system is torsion-free with a symmetric Ricci tensor $R_{\mu\nu}$
- The system Lagrangian $\mathcal{L}$ describes the dynamics of a scalar field (and possibly a massless vector field).

The source term is then given by Hilbert's metric energy-momentum tensor $T_\mu{}^\nu$

$$T_\mu{}^\nu = \frac{2}{\sqrt{-g}}\frac{\partial(\mathcal{L}\sqrt{-g})}{\partial g^{\mu\beta}}g^{\nu\beta}.$$

Yet, if the given system comprises a massive vector field, the source term of the generalized Einstein equation is given by the canonical E-M tensor $\theta_\mu{}^\nu$, which then differs from the metric E-M tensor $T_\mu{}^\nu$ by a divergence:

$$\theta_\mu{}^\nu = T_\mu{}^\nu - \frac{\partial\mathcal{L}}{\partial a_\nu}a_\mu - \frac{\partial\mathcal{L}}{\partial a_{\nu;\beta}}a_{\mu;\beta} = T_\mu{}^\nu - \left(\frac{\partial\mathcal{L}}{\partial a_{\nu;\beta}}a_\mu\right)_{;\beta}.$$

Conclusions and Outlook

1. Key results of the gauge theory of gravity:
   - the connection coefficients $\gamma^{\xi\alpha\beta}$ are the gauge fields of gravity
   - the Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ for the "free gauge fields", i.e. for the gravity dynamics in classical vacuum must be postulated
   - gauge theory provides us with the coupling of fields and gauge fields
   - each type of field (scalar, vector, tensor, spinor; massive, massless) has its particular spacetime coupling — which is beyond the Einstein theory
   - e.g.: spacetime coupling of spin fields yields an effective mass term $\rightarrow$ analogy to the Pauli coupling term of a spinor in a magnetic field
2. Actual work:
   - Discussion of physically reasonable options for $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ and their respective cosmological consequences
   - Discussion of the cosmological consequences of the modified coupling of massive vector (spin-1) fields to spacetime
   - Discussion of the emerging effective mass term of spin-$^1/_2$ fields with respect to e.g. the neutrino mass issue

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