Numerical Relativity

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Abstract

The course will provide an introduction to the mathematical and numerical techniques presently employed for the accurate solution of the Einstein equations together with those of relativistic hydrodynamics. The first part of the course will concentrate on the mathematical aspects of numerical relativity. These include the formulation of the equations, be it the field equations or those of relativistic hydrodynamics, the definition of hyperbolic system of partial differential equations and the development of nonlinear waves in hydrodynamics. The second part of the course, instead, will concentrate on the numerical aspects and the most advanced techniques for the numerical solution of these equations. The students are expected to be familiar with the theory of General Relativity and to be proficient in differential geometry and tensor calculus. A series of exercises will parallel the course. The content of the lectures can be found in a series of books [1, 2, 3, 4, 5].

Syllabus and plan of the lectures

1.a The 3+1 decomposition of spacetime
1.b Formulations of the Einstein equations. Lagrangian formulations

2.a The ADM formulation
2.b Conformal traceless formulations

3.a Gauge conditions in 3+1 formulations
3.b Constraint equations. initial data and constrained evolution

4.a Hyperbolic systems of partial differential equations
4.b Quasi-linear formulation. Conservative formulation

5.a Characteristic equations for linear systems. Riemann invariants
5.b Characteristics and caustics. Domain of determinacy. region of influence

6.a Linear hydrodynamic waves. Sound waves
6.b Nonlinear hydrodynamic waves. Rarefaction waves. Shock waves

7.a Contact discontinuities. The Riemann problem
7.b Solution of the one-dimensional Riemann problem

8.a Formulations of the hydrodynamic equations. The Wilson formulation
8.b The importance of conservative formulations. The ”Valencia” formulation

9.a Finite-Difference Methods. The discretisation process
9.b Numerical errors. Consistency. convergence and stability

10.a The upwind scheme. The FTCS scheme. The Lax-Friedrichs scheme
10b The leapfrog scheme. The Lax-Wendroff scheme. Kreiss-Oliger dissipation. Artificial-viscosity approaches

11.a HRSC Methods and Conservative schemes
11.b Rankine-Hugoniot conditions

12.a Finite-volume conservative numerical schemes
12.b Finite-difference conservative numerical schemes

13.a Upwind methods
13.b Monotone methods. Total variation diminishing methods

14.a Godunov methods. Reconstruction techniques
14.b Slope-limiter methods

15.a Approximate Riemann solvers
15.b HLLE. Roe Riemann solvers

16.a The method of lines. Explicit Runge-Kutta methods
16.b Implicit-explicit Runge-Kutta methods

References

[1] Rezzolla L and Zanotti O 2013 *Relativistic Hydrodynamics* (Oxford University Press, Oxford UK)

[2] Alcubierre M 2008 *Introduction to* 3+1 *Numerical Relativity* (Oxford, UK: Oxford University Press)

[3] Baumgarte T W and Shapiro S L 2010 *Numerical Relativity: Solving Einstein's Equations on the Computer* (Cambridge University Press, Cambridge UK)

[4] Toro E F 2009 *Riemann Solvers and Numerical Methods for Fluid Dynamics* (Springer-Verlag)

[5] Leveque R J 2002 *Finite Volume Methods for Hyperbolic Problems* (New York: Cambridge University Press)