Compact stars in Minimal Dilatonic Gravity

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Overview

1. Why do we need another theory?
   - Observational tests
   - Unsolved problems for compact stars
   - Ways to extend GR
   - Alternative gravity theories

2. The minimal dilatonic gravity

3. Applications to the case of compact stars
   - Neutron stars
   - White dwarfs

4. The COCAL implementation

5. Plans for future research
Tests of General Relativity (GR)

General relativity has been tested on many scales, but mostly in weak to moderate-field regime:

- Laboratory, Earth and Solar System scale \((v/c \ll 1 \text{ or } GM/c^2 R \ll 1)\), upper bound for violations by the Cassini mission – \(10^{-5}\)
- Binary pulsars: PSR B1913+16, PSR J0737-3039, PSR J0348+0432 – 0.05%
- Galaxies and galaxies cluster: Sloan Digital Sky Survey III Baryon Oscillations Spectroscopic Survey – 6%

The real probes for the strong field regime \((v/c > 0.1 \text{ or } GM/c^2 R \sim 1)\) are:

- Final stages of binary coalescence of compact objects (WD, NS, BH)
- Cosmological tests of the (early) Universe

Known problems: Classical theory (not renormalizable), Singularities, Cosmological constant problem, Vacuum fluctuations, Dark Energy, Dark Matter, The initial inflation and initial singularity problem etc.
Examples of what we don’t know:

- The rotation curves of disc galaxies [Corbelli & Salucci (2000)]
- Weak gravitational lensing results [Clowe et al. (2006), Huterer (2010)]
- An ongoing quest:
  - The Dark Energy Survey (operational),
  - Sloan Digital Sky Survey III (operational, 35% of the sky, with photometric observations of around 500 million objects and spectra for more than 1 million objects),
  - The Euclid Mission (2020, L2 space telescope)
  - HETDEX (2014), DESI (2018), BOSS (operational) etc.
White dwarfs (WD) and neutron stars (NS) – significant observational data and modelling efforts, but still inconsistencies:

- The ultra-massive white dwarfs: SNLS-03D3bb (Nature 443 (2006) 308) and SN2007if (ApJ 713 (2010)), type Ia SN with progenitor exceeding the $M_{Ch} = 1.4M_\odot$ (up to 2.4-2.8$M_\odot$)
- Stiff M(R) dependence for neutron stars or a dispersion in the observed masses?
- The question of the maximal NS mass and its relation to stellar black holes and astrophysical jets
- The Gamma-Ray Bursts mistery: huge energies, short characteristic time-scales, long life of the central engine

There are numerous approaches towards solving these problems – better MHD modeling, stronger and more complicated magnetic fields, better and richer equation of states etc.

One can also choose to go to a deeper level and extend the very GR.
Ways to extend GR

Requirements:

- reproduce the Minkowski spacetime in the absence of matter and cosmological constants,
- be constructed from only the Riemann curvature tensor and the metric,
- follow the symmetries and conservation laws of the stress-energy tensor of matter,
- reproduce Poisson's equation in the Newtonian limit.

Starting from the Einstein-Hilbert action, one can:

- increase the spacetime dimensions
- change the functional dependence of the Lagrangian density on the Ricci scalar $R$
- include other scalars generated from the Riemann curvature in the Lagrangian density,
- include additional scalar, vector, or tensor fields.
Alternative gravity theories

Some of the more popular alternatives of GR ($A_E = \int \frac{1}{2\kappa} R \sqrt{-g} \, d^4x$):

- **Gaus Bonnet theory** – includes a term of the form:
  
  $G = R^2 - 4R_{\mu\nu}R_{\mu\nu} + R_{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ in the action $A = \int d^Dx \sqrt{-g} \, G$. (no additional dynamical degrees of freedom)

- **Lovelock theory** – a natural generalization of GR to $D > 4$.
  
  $$\mathcal{L} = \sqrt{-g} \left( \alpha_0 + \alpha_1 R + \alpha_2 \left( R^2 + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} \right) + \alpha_3 \mathcal{O}(R^3) \right)$$

- **$f(R)$ theories** – a family of theories in which the arbitrary function $f(R)$ may lead to the accelerated expansion and structure formation of the Universe /dark energy or dark matter alternative/. $A = \int \frac{1}{2\kappa} f(R) \sqrt{-g} \, d^4x$

- **Brans-Dicke scalar-tensor theory** – the gravitational interaction is mediated by a scalar field ($\phi = 1/G$) – i.e. a varying $G$, as well as the tensor field of general relativity. Contain a tunable, dimensionless Brans-Dicke coupling constant $\omega$.
  
  $$A = \int d^4x \sqrt{-g} \left( \frac{\phi R - \omega \frac{\partial_a \phi \partial^a \phi}{16\pi}}{\phi} + \mathcal{L}_M \right)$$

- **Chameleon scalar-tensor theory** – Introduces a scalar particle (the chameleon) which couples to matter, with a variable effective mass, an increasing function of the ambient energy density $m_{\text{eff}} \sim \rho^\alpha$, where $\alpha \simeq 1$. ($m_{\text{eff}} \sim mm - pc$).
The action, following Fiziev, PRD 87, 044053 (2013)

\[ A_{g,\phi} = \frac{c}{2\kappa} \int d^4x \sqrt{|g|}(\Phi R - 2\Lambda U(\Phi)) \]

Here, \( \Phi \in (0, \infty) \) is the new scalar field called “dilaton”, \( \Lambda > 0 \) is the cosmological constant and \( \kappa = 8\pi G_N/c^2 \) is the Einstein constant.

Effects

Clearly, the introduction of the scalar \textbf{dilaton} \( \Phi \) leads to varying gravitational constant \( G(\Phi) = G_N/\Phi \), while the introduction of the cosmological potential \( U(\Phi) \) leads to a variable cosmological factor instead of a constant \( \Lambda \).

Note: In order to keep gravity as existing and attractive force \( \Phi > 0 \).
The action

\[ A_{g,\phi} = \frac{c}{2\kappa} \int d^4x \sqrt{|g|} (\Phi R - 2\Lambda U(\Phi)) \]

This action corresponds to the Brans-Dicke theory with \( \omega = 0 \). GR is recovered for \( \Phi = 1, U(1) = 1 \).

In general, the MDG model and the \( f(R) \) models are equivalent only locally. Only under additional conditions, the two models can be considered globally equivalent. Those conditions define the class of the potentials \( U(\Phi) \), for which one also avoids some of the well-known problems in the \( f(R) \) theories, like physically unacceptable singularities, ghosts, etc.

Some of the properties of the MDG model already demonstrated:

1. The inflation and the graceful exit to the present day accelerating de Sitter expansion of the Universe (\( U(\Phi) \) can be reconstructed from \( a(t) \)).
2. Avoids any conflicts with the existing solar system and laboratory gravitational experiments when \( m_\Phi \sim 10^{-3} \text{eV}/c^2 \).
3. The time of inflation as a reciprocal quantity to the mass of dilaton \( m_\Phi \).
The field equations of MDG

– variation of the MDG action with respect to $\Phi$ gives:

$$ R = 2\Lambda U,\Phi(\Phi) $$  \hspace{1cm} (1)

Note: this is an algebraic relation. It ensures that $\Phi$ has the same properties as $R$. (for example, $R = const$ leads to $\Phi = const$ and $G(\Phi) = const$.

– variation of the MDG action with respect to $g_{\alpha\beta}$ gives:

$$ \Phi G_{\alpha\beta} + \Lambda U(\Phi)g_{\alpha\beta} + \nabla_\alpha \nabla_\beta \Phi - g_{\alpha\beta} \Box \Phi = 0 $$  \hspace{1cm} (2)

– the trace of eq. 2 leads to:

$$ \Box \Phi + \Lambda V,\Phi(\Phi) = 0 $$  \hspace{1cm} (3)

Here $V,\Phi(\Phi) = 2/3(\Phi U,\Phi(\Phi) - 2U(\Phi))$ or $V(\Phi) = \frac{2}{3} \int_1^\Phi (\Phi U,\Phi(\Phi) - 2U) d\Phi$

– And the traceless part:

$$ \Phi \hat{R}^\beta_\alpha = -\nabla_\alpha \nabla^\beta \Phi $$  \hspace{1cm} (4)
The final form of the field equations:

If we include the standard action of the matter fields $\Psi$, based on the minimal interaction with gravity:

$$A_{\text{matt}} = \frac{1}{c} \int d^4x \sqrt{|g|} L_{\text{matt}}(\Psi, \nabla \Psi; g_{\alpha\beta})$$

we get the final form of the field equations in cosmological units $\Lambda = 1, \kappa = 1, c = 1$:

$$\Box \Phi + 2/3(\Phi U_{,\Phi}(\Phi) - 2U(\Phi)) = \frac{1}{3} T$$

$$\Phi R^\beta_\alpha = -\nabla_\alpha \nabla^\beta \Phi - \hat{T}^\beta_\alpha$$

Note: The dilaton $\Phi$ does not interact directly with the matter and thus it is a good candidate for the dark matter. Its interaction with the usual matter goes only through the gravitational interaction.
Properties of MDG

1. MDG and f(R) theories are related by the Legendre transform (i.e. there is a dictionary between the two models).

2. The withholding property: In order to guarantee that \( \Phi \in (0, \infty) \), we require that \( V(0) = V(\infty) = +\infty \), i.e. infinite potential barriers at the end of the interval.

3. From \( U(\Phi) = \frac{3}{2} \Phi^2 \int_1^\Phi \Phi^{-3} V,\Phi d\Phi + \Phi^2 \) (from \( U(1) = 1 \)), if we assume that \( V(\Phi) \sim \nu \Phi^n \), it follows that \( U(0) = U(\infty) = +\infty \).

4. Additional requirement: \( U(\Phi) > 0 \), for \( \Phi \in (0, \infty) \) (the cosmological term needs to have a definite positive sign).

5. From the convex condition \( U,\Phi\Phi > 0 \), for \( \Phi \in (0, \infty) \) (ensures the uniqueness of the Einstein vacuum).

6. The uniqueness of the deSitter vacuum is not guaranteed:

\[
V,\Phi\Phi = \frac{2}{3} (\Phi U,\Phi\Phi - U,\Phi), \quad V,\Phi\Phi\Phi = \frac{2}{3} \Phi U,\Phi\Phi\Phi
\]

Thus we can have \( V(\Phi) \) with several minima in the domain.
(e) Unique Einstein Vacuum and many deSitter vacuums: $U,\phi\phi > 0$

(f) Unique Einstein Vacuum and unique deSitter vacuums: $U,\phi\phi > 0, V,\phi\phi > 0$
If we postulate a unique deSitter vacuum, then the function $V(\Phi)$ will be convex for $\Phi \in (0, \infty)$ and the function $\frac{2}{3}(\Phi U,_{\Phi\Phi} - U,_{\Phi}) > 0$ is strictly positive.

A simple example of such pair of withholding potentials is:

$$V(\Phi) = \frac{1}{2} p^{-2}(\Phi + 1/\Phi - 2) \quad (7)$$

$$U(\Phi) = \Phi^2 + \frac{3}{16} p^{-2}(\Phi - 1/\Phi)^2 \quad (8)$$

where $p$ is a small parameter related with the dilaton mass.

We are going to use these withholding potentials in our study of compact stars.
Some bibliography

Some of the works where details on the MDG model have been worked out.

A theory in development

Plamen P. Fiziev, arXiv:gr-qc/0202074

The pioneering work on the MDG model is by O'Hanlon, Phys. Rev. Lett. 29 137 (1972).
We follow the first application to the case of neutron stars published in [Fiziev (2013)]:

Let us consider a static, spherically symmetric metric of the type:

\[
\begin{align*}
    ds^2 &= e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2, \\
    \Box \Phi + 2/3(\Phi U,\Phi(\Phi) - 2U(\Phi)) &= \frac{1}{3} T
\end{align*}
\]

(9)

where \( r \) is the luminosity distance to the center of symmetry, and \( d\Omega^2 \) describes the space-interval on the unit sphere.

The equations are the MDG field equations:

Then, if we assume the perfect fluid stress-energy tensor 

\[
T^{\mu\nu} = \text{diag}(\epsilon, p, p, p) / \epsilon = 1 \] 

we obtain:
The equations

1. For the inner domain $r \in [0, r_*]$:

\[
\frac{dm}{dr} = 4\pi r^2 \epsilon_{\text{eff}} / \Phi
\]

Additionally, we have:

\[
\epsilon_{\Lambda} = -p_{\Lambda} - \frac{\Lambda}{12\pi} \Phi,
\]

\[
\epsilon_{\Phi} = p - \frac{1}{3} \epsilon + \frac{\Lambda}{8\pi} V'(\Phi) + \frac{p_{\Phi}}{2} \Pi
\]

\[
\epsilon = \epsilon(p)
\]

where

\[
\Delta = r - 2m - \frac{1}{3} \Lambda r^3, \quad \epsilon_{\text{eff}} = \epsilon + \epsilon_{\Phi} + \epsilon_{\Lambda}, \quad p_{\text{eff}} = p + p_{\Phi} + p_{\Lambda}, \quad \Pi = \frac{m + 4\pi r^3 p_{\text{eff}} / \Phi}{\Delta - 2\pi r^3 p_{\Phi} / \Phi}
\]

and

\[
\epsilon_{\Lambda} = \frac{\Lambda}{8\pi} (U(\Phi) - \Phi), \quad p_{\Lambda} = \frac{\Lambda}{8\pi} (U(\Phi) - \frac{1}{3} \Phi)
\]

The 4 unknown functions are $m(r), p(r), \Phi(r), p_{\Phi}(r)$. 
The Initial and Boundary conditions:

\[ m(0) = m_c = 0, \Phi(0) = \Phi_c, p(0) = p_c \]

\[ p_\Phi(0) = p_{\Phi c} = \frac{2}{3} \left( \frac{\epsilon(p)}{3} - p_c \right) - \frac{\Lambda}{12\pi} V'(\Phi_c) \]

On the star’s edge \((p(r^*) = 0)\) we have

\[ m^* = m(r^*; p_c, \Phi_c), \Phi^* = \Phi(r^*, p_c, \Phi_c), p^*_\Phi = p_{\Phi c}(r^*, p_c, \Phi_c) \]

2. For the outer domain: a boundary value problem for \(\Phi\):

\[ p = 0, \epsilon = 0, \Phi_\Delta = 1 \]

After introducing the EOS, we solve the ODE system + the initial and boundary conditions for the unknown functions \(m(r), p(r), \Phi(r), p_\Phi(r)\).
The case of a TOV neutron star

If one uses the Tolman-Oppenheimer-Volkov (TOV) model for EOS (ideal Fermi neutron gas at zero temperature):

\[
\epsilon = \frac{1}{4\pi} K (\sinh(t) - t), \quad p = \frac{1}{12\pi} K (\sinh(t) - 8 \sinh(t/2) + 3t)
\]

Here \( K = \pi \frac{m^4 c^5}{4\hbar^2} \), \( t = 4 \log \left( \frac{p_F}{mc} + \left( 1 + \left( \frac{p_F}{mc} \right)^2 \right)^{1/2} \right) \) and

\[
p = \sqrt{\Lambda \hbar / cm_\Phi} = 10^{-21} \text{ (the dilaton mass parameter, for observational consistency, } p < 10^{-30}), \quad \Lambda \sim 10^{-44} \text{ km}^{-2},
\]

EOS in the original notations of [Oppenheimer & Volkoff (1939)], see also [Rezzola & Zanotti (2013)].

We use MAPLE to solve the ODE system using the shooting method for the BC and the rosenbrock method for the integration.
FIG. 1: The specific MDG-curve $F_N(p_c, \Phi_c) = 0$

FIG. 2: The MDG-SSSS interior in accord with MEOS

FIG. 3: The SSSS-disphere-mass-dependence on $r$

FIG. 7: Mass - radius relations for IFNG0T in GR $(m_{max}^* \approx 0.7051 \, m_\odot, \, r_{max}^* \approx 9.209 \, \text{km})$ and in MDG $(m_{max}^* \approx 0.5073 \, m_\odot, \, r_{max}^* \approx 7.092 \, \text{km})$
The TOV equations for White Dwarfs

In the case of GR, the white dwarfs are described well even in the polytropic approximation:

\[
M_{WD} = 0.17 - 1.3 M_\odot
\]

\[
R_{WD} = 0.008 - 0.02 R_\odot
\]

\[
\rho_{WD} = 10^5 - 10^9 \text{gr/cm}^3
\]

Composition:

He, C, O

The ODE system:

\[
\frac{dM(r)}{dr} = \beta r^2 \epsilon
\]

\[
\frac{dp(r)}{dr} = \alpha \epsilon M(r) r^2
\]

\[
\epsilon = \left( \frac{p(r)}{K} \right)^{1/\nu}
\]

Here the integration has been performed using Maple. \(M(r)\) is in \(M_\odot\), \(r\) in \([\text{km}]\)/
The WD case in MDG (for $A/Z = 2.15$)

In the case of white dwarfs, we use the polytropic EOS in the two regimes – the relativistic case ($k_F >> m_e$) and the non-relativistic case $k_F << m_e$:

\[ p_{\text{nonrel}} = K_{\text{nonrel}} \epsilon^{\frac{5}{3}}, \quad p_{\text{rel}} = K_{\text{rel}} \epsilon^{\frac{4}{3}}, \]

where

\[ K_{\text{nonrel}} = \frac{\hbar^2}{15\pi^2 m_e} \left( \frac{3\pi^2 Z}{Am_N c^2} \right)^{5/3}, \quad K_{\text{rel}} = \frac{\hbar c}{12\pi^2} \left( \frac{3\pi^2 Z}{Am_N c^2} \right)^{4/3} \]

We make the equation dimensional following [Sibar and Reddy (2004)].

<table>
<thead>
<tr>
<th>Model</th>
<th>$r_{MDG}$</th>
<th>$m_{MDG}$</th>
<th>$r_{GR}$</th>
<th>$m_{GR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relativistic WD ($p_0 = 10^{-14}$)</td>
<td>4 947</td>
<td>1.2406</td>
<td>4840</td>
<td>1.2431</td>
</tr>
<tr>
<td>Relativistic WD ($p_0 = 10^{-15}$)</td>
<td>8 799</td>
<td>1.2419</td>
<td>8600</td>
<td>1.2432</td>
</tr>
<tr>
<td>Relativistic WD ($p_0 = 10^{-16}$)</td>
<td>15 648</td>
<td>1.2427</td>
<td>15 080</td>
<td>1.2430</td>
</tr>
<tr>
<td>Non-Relativistic ($p_0 = 10^{-15}$)</td>
<td>10 603</td>
<td>0.3929</td>
<td>10 620</td>
<td>0.3941</td>
</tr>
<tr>
<td>Non-Relativistic ($p_0 = 10^{-16}$)</td>
<td>13 349</td>
<td>0.1969</td>
<td>13 360</td>
<td>0.1974</td>
</tr>
</tbody>
</table>
The WD case (M in solar masses, r in km, p in ergs/cm$^3$ $\times$ 10$^{38}$)

(l) Non-relativistic case

(m) Relativistic case

(n) Non-relativistic case

(o) Relativistic case

(p) Non-relativistic case

(q) Relativistic case
The two dimensionless pressures and some FORTRAN Rosenbrock fun

Denitsa Staicova, Plamen Fiziev (JINR)
Summary of the results

1. The MDG equations recover GR to a good precision ($\Phi = 1, U(1) = 1, \Lambda = 0$)
2. For a massive dilaton, the $M(R)$ curves are consistent with GR
3. In the NS case, the total mass of the dilasphere is 30% of that of the NS
4. In the case of polytropic WD, the mass of the dilasphere is $\sim 27\%$ of that of the star
5. The WD radius and the mass increase with the introduction of the dilaton
6. The current value of the dilaton mass with which we are working ($d \sim 10^{-20}$) is well above the one required by observations $\sim 10^{-30}$.

The results for the cases of those simplistic EOS-es are promising, but we need new tools to solve for really light dilaton!
As part of this COST visit at the ITP, the MDG static equations were implemented in the Compact Object CALculator (Tsokaros et al. in prep (2014)).

Some preliminary results for the NS case $\gamma = 2$/:

The TOV solver is stable up to 150 NS radii!
The desired course of action:

1. To use more realistic EOS for both the WD case and the NS case (for example using the online database cococubed)
2. To get nearer to the cosmological horizon.
3. To use the TOV solver in Cocal (with Antonios Tsokaros)
4. A 3 + 1 formulation of the field equations
5. To use the cocal implementation for rotating neutron stars
6. Why not even for binary systems

The final goal is to see if we can obtain more massive compact stars in MDG without complex EOS or at least, if we can get out of the shadow of the stiff $M(R)$. 

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The final goal is to see if we can obtain more massive compact stars in MDG without complex EOS or at least, if we can get out of the shadow of the stiff $M(R)$. 
That’s all!

Thank you
Fiziev
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Fiziev
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The GRB mystery

1. Energy \( \sim 10^{53} \text{erg} \), two types – Short and Long
2. Different variability time-scales – ms, sec, hundreds of seconds
3. X-ray plateaus – continued injection of energy (\( \sim 100s \))
4. X-ray flares – multiple rebrightening, happening at up to \( 10^5 s \)
6. Extended high energy emission (GeV scale, example GRB130427A)
7. All those properties call for a long-lasting, extremely powerful central engine
8. Figure credit: Gehrels et. al (2009), Gendre et al. (2012), ApJ 766, 30, 2013