Neutrino-Driven Turbulent Convection in Stalled Supernova Cores

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The Supernova Problem

Core-Collapse Supernovae:

• End of massive stars
• Birthplace of heavy elements, neutron stars, black holes …
• Regulate star formation
• …

Problem: how do they explode?
Core-Collapse Supernovae

From Janka et al. 2012
Shock Revival by Neutrinos

From Janka 2001
The Roles of Turbulence

- Regulates accretion
- Turbulent pressure
- Transports heat
- Increase dwelling time

Difficult to simulate!
Turbulent Pressure

Rankine-Hogoniot jump condition:
\[ \rho_d v_d^2 + p_d = \rho_u v_u^2 + p_u \]

EOS:
\[ p_d = (\gamma_{th} - 1) \rho_d \epsilon_d \quad \gamma_{th} \approx \frac{4}{3} \]

Effect of downstream turbulence (Murphy et al. 2013):
\[ \rho_d v_d^2 + p_d \rightarrow \rho_d \bar{v}_d^2 + \rho_d (\delta v)_d^2 + p_d \]

Turbulence can be modeled with an effective EOS
\[ \rho_d (\delta v)_d^2 \leftrightarrow (\gamma_{turb} - 1) \rho_d \epsilon_{turb} \quad \gamma_{turb} \approx 2 \]

Jump conditions for a shock with downstream turbulence:
\[ \rho_d \bar{v}_d^2 + (\gamma_{turb} - 1) \rho_d \epsilon_{turb} + (\gamma_{th} - 1) \rho_d \epsilon_d = \rho_u v_u^2 + p_u \]
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Resolution Dependance

Explosion more difficult at higher resolution!

ULR 3.78 km
LR 1.89 km
MR 1.42 km
IR 1.24 km
HR 1.06 km

Resolutions
Why?

- Lower resolution favors the formation of larger, longer lived structures
- Secondary instabilities (Kelvin-Helmholtz) is suppressed by numerical viscosity
- When is the resolution good enough?
Turbulent Cascade I

\[ \partial_t E + \partial_k \Pi = -2\nu k^2 E + \epsilon \]

Energy flux

Energy injection

Specific kinetic energy

Energy dissipation
Kolmogorov 1941: $\Pi \sim \text{const} \implies E \sim k^{-5/3}$
The Water-Spill Analogy

Adapted from Boris 1992

Figure 8. LES water-spill analogy.

- Faster so the mass flow past any radius is constant.
- Increasing radius away from the center of the table is analogous to increasing wavenumber of the eddies in a turbulent cascade.
- The decreasing depth of the water is analogous to the decreasing energy content in each wavelength scale of the turbulence.
- The incompressibility of the water gives effectively the same influence at a distance that the nonlocal interaction of disparate scales does in considering turbulence.
- The inertial range of the turbulent cascade is represented by the region between the vertical dashed lines where the profile is smoothly decreasing in fig. 8a.
- The radius of the table and how the water eventually falls off the table is analogous to the viscous dissipation of turbulent energy at the Kolmogorov scale in very high Re flows. This dropoff clearly does not significantly affect the depth of the water near the center of the table.

In this hydrodynamic analogy, different possible contours at the edge of the table correspond to the different properties of various high Re Navier-Stokes models, conventional filtered LES models, and MILES models.

In MILES models based on monotone convection algorithms, the nonlinear flux limiter acts as a built-in subgrid model coupled intrinsically to the short wavelength errors in the solution. Turbulent energy reaching the grid scale is extracted from the calculation and converted to the correct conserved quantities. This has the effect of curving the table edge sharply downward, as illustrated in fig. 8b, so that the water can flow smoothly off at a finite radius without significant perturbations reaching the center of the table. The dissipation in MILES algorithms is physically matched to the grid-scale errors to minimize effects on long wavelengths which are accurately resolved.

With conventional, high-order CFD algorithms which are not monotone, dissipation is added through the physical viscosity. Thus a blocking or damming phenomenon occurs.

Bottleneck: water piles up

Numerical viscosity

Adapted from Boris 1992
The Bottleneck Effect

$E(k) k^{5/3}$ vs $k$

PPM_HLLC_N64
PPM_HLLC_N128
PPM_HLLC_N256
PPM_HLLC_N512

Bottleneck
Energy Cascade: PPM

Need very high resolution!!!
Turbulent Energy Spectrum

\[ E(\ell) \text{ [erg cm}^{-3}\text{]} \]

\[ t - t_{\text{bounce}} = 90 \text{ ms} \]
Semi-Global Convection Study

ULR
$\Delta r \approx 1.9 \text{ km}$

XLR
$\Delta r \approx 3.8 \text{ km}$

LR
$\Delta r \approx 960 \text{ m}$

MR
$\Delta r \approx 640 \text{ m}$

$r_s \approx 190 \text{ km}$

$t = 0.000 \text{ [ms]}$
Nuclear dissociation is included in a parametrized way using an approach similar to the one of Fernández & Thompson (2009b,a), but with some important differences discussed here.

Fernández & Thompson (2009b) suggested to parametrize the amount of specific internal energy lost to nuclear dissociation, $\bar{\varepsilon}_{\text{ND}}$, as a fraction, $\bar{\varepsilon}$, of the free-fall kinetic energy at the initial location of the shock:

$$\varepsilon_{\text{ND}} = \frac{1}{2} \bar{\varepsilon}_{\text{FF}}^2,$$

(A1)

where $\varepsilon_{\text{FF}}$ is the free-fall velocity at the initial location of the shock. In the relativistic case this translates to

$$\varepsilon_{\text{ND}} = \bar{\varepsilon} (W_{\text{FF}} - 1),$$

(A2)

where $W_{\text{FF}}$ is the free-fall Lorentz factor (see Appendix B). A typical range of values for $\bar{\varepsilon}$ is $0.2 - 0.4$ (Fernández & Thompson 2009a).
Convective Instability

![Graph showing convective instability](image)
Radial Reynolds Stresses

\[ R_r/c^2 [\times 10^{40} \text{g cm cm/s}^2] \]

\[ r/r_s \]

- XLR
- ULR
- LR
- MR
Not Quite There Yet

\[ E(k) \propto k^{-1} \]
A New Ingredient: Intermittency I

Turbulent energy density

Tangential Reynolds stress
A New Ingredient: Intermittency II

Shock radius evolution

![Graph showing shock radius evolution over time](image)
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Conclusions

• Turbulence: crucial role for supernova explosions

• Local simulations: very high resolution is needed

• Idealized global simulations: rich dynamics of turbulent convection
The Standing Shock Flow

From Janka 2001
Initial Data

Stationary initial data