Methods for simulating starquakes in neutron stars

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see C. Gundlach, I. Hawke, and SJE, CQG 29 015055 (2012)
Astrophysical motivation

- NS has crust
- Crust breaks
  - spindown
  - magnetic fields
  - tidal forces
- Starquakes observable
  - precursors to sGRB’s
  - pulsar glitches

Goal: Investigate dynamics of neutron star quakes
Technical aspects to address:

- Elasticity
- Interfaces
- Stellar surface
- Starquakes
Elasticity formulation
Elasticity formulation
Motivation Elasticity Interfaces Atmosphere Treatment Starquake mechanisms 2D Toy Model Future work

Elasticity formulation

- Map $\chi : \text{spacetime} \rightarrow \text{matter-space}$

- *Configuration gradient:*

  $$\psi^A{}_a := \frac{\partial \chi^A}{\partial x^a} \quad \text{with} \quad \psi^A{}_{[a,b]} = 0$$

- Particle labels dragged with particles ($u^a \psi^A{}_a = 0$) so

  $$\psi^A{}_t = -\hat{\nu}^i \psi^A{}_i$$

- So we get an *evolution equation* and a *constraint:*

  $$\psi^A{}_{i,t} + \left(\hat{\nu}^j \psi^A{}_j\right){}_i = 0 \quad \text{and} \quad \psi^A{}_{[i,j]} = 0.$$
Physical meaning of $\psi^A_i$:

- Integrate to find conserved quantity
- Count up matter-space lines crossed in a particular direction
- Represent crystal axes
Jump Conditions

\[
\psi^A_{[i,j]} = 0 \quad \text{and} \quad \psi^A_{i,t} + \left( \hat{\nu}^j \psi^A_j \right)_{,i} = 0
\]

- Covector normal to the shock \( n_i \)
- Shock velocity \( s^i \) and normal shock speed \( s = s^i n_i \)
- Projector into surface tangent: \( \|_{i,j} := \delta^i_j - n^i n_j \)
- Jump conditions become

\[
[\psi^A_{\|k}] = 0 \quad \text{and} \quad [\psi^A_n(\hat{\nu}^n - s)] + \psi^A_{\|i}[\hat{\nu}^{\|i}] = 0
\]
Jump Conditions: Constraint Examples

\[ [\psi^A_{||k}] = 0 \]

\[ [\psi^Y_y] \neq 0 \]

\[ [\psi^X_y] \neq 0 \]
Jump Conditions: Evolution Equation Examples

\[
[\psi^A_n(\hat{v}^n - s)] + \psi^A_{\parallel i}[\hat{v}^\parallel_i] = 0
\]

\[
\hat{v}^\parallel_i = 0, \quad s = 0, \quad [\psi^A_n \hat{v}^n] = 0
\]

\[
\hat{v}^n = 0, \quad \psi^A_{\parallel i}[\hat{v}^\parallel_i] - s[\psi^A_n] = 0
\]
Elasticity: Shear Stresses

Perfect Fluid

Initial contact stays stationary with evolution in time.
Elastic Solid

Initial discontinuity in velocity produces shear waves as time evolves.
How is the stress-energy tensor changed?

- More general stress-energy tensor

\[ T^{ab} = e u^a u^b + p^{ab} = e u^a u^b + ph^{ab} + \pi^{ab} \]

- Anisotropic stress \( \pi^{ab} \) comes from

\[ \pi_{ab} := \psi^A_a \psi^B_b \pi_{AB} \]

- On matter space, \( \pi_{AB} \) relates \( k_{AB} \) to \( g^{AB} \)

- Relaxed state at \( k_{AB} = n^{2/3} g_{AB} \)

- Have \( u^a h^{bc} T_{ab} = 0 \) \( \rightarrow \) heat-flow terms are zero
Matter evolution equations

<table>
<thead>
<tr>
<th>Conserved Quantity</th>
<th>Flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_i = \text{fluid} + \pi_{ij} v^j$</td>
<td>$\mathcal{F}(S_j)' = \text{fluid} + \pi_{ij}'$</td>
</tr>
<tr>
<td>$\tau = \text{fluid} + \pi$</td>
<td>$\mathcal{F}(\tau)' = \text{fluid} - \pi \alpha^{-1} \beta^i + \gamma^{ij} \pi_{jk} v^k$</td>
</tr>
<tr>
<td>$\psi^A_i$</td>
<td>$\mathcal{F}(\psi^A_j) = \hat{v}^k \psi^A_k \delta^i_j$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\mathcal{F}(D)$</td>
</tr>
</tbody>
</table>

Advection equation: $k_{AB,t} + \hat{v}^i k_{AB,i} = 0$
Elasticity results

- Newtonian limit of our relativistic code matches published Newtonian Riemann tests
- 2D tests match 1D tests and exact solutions where available
- 2D cylindrical coordinates demonstrates that we can use a general metric

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Outline

Technical aspects to address:
- Elasticity
- Interfaces
- Stellar surface
- Starquakes
Separate grid into regions governed by different physical models

- Track the moving boundary using a level-set function, $\phi$
- Level set is associated with particles, so $u^a \nabla_a \phi = u^a \phi, a = 0$
- In $3+1$ split: advection
Interfaces

Separate grid into regions governed by different physical models

Track the moving boundary using a level-set function, $\phi$

Level set is associated with particles, so $u^a \nabla_a \phi = u^a \phi, a = 0$

In $3+1$ split: advection
Interfaces: What happens at the interface?

- Apply appropriate boundary conditions
- Use approximate solution of multimaterial Riemann problem to determine behavior at the boundary
- Extension of ghost fluid method (GFM)
Example: Original GFM

Ghost fluid method:
- Continuous across contact: $p, \nu^{(n)}$
- Discontinuous across contact: $s, \nu^{(t)}$
- Calculate $n = n(s, p)$
Example: Original GFM

Ghost fluid method:
- Continuous across contact: $p, v^{(n)}$
- Discontinuous across contact: $s, v^{(t)}$
- Calculate $n = n(s, p)$
In general relativity

- Say interface has normal covector $s_a$ and $s_a u^a = 0$
- In 3 + 1 split $s_a = v_\perp n_a + k_a$ so
  $$[v^a k_a] = [v_\perp] = 0$$
- We have either of the following boundary conditions
  $$[T^{ab} s_a s_b] \leftrightarrow [p^{ab} s_a s_b] = 0 \quad \text{Slip}$$
  $$[T^{ab} s_a] \leftrightarrow [p^{ab} s_a] = 0 \quad \text{Stick}$$
In general relativity

- We split $p_{ab}$ into isotropic and anisotropic stress

$$p_{ab} = p h_{ab} + \pi_{ab}$$

- Interface conditions become

$$[p(1 - v_{\perp}^2) + \pi^{ab}(k_a - v_{\perp} v_a)(k_b - v_{\perp} v_b)] = 0 \quad \text{Slip}$$
$$[ps^b + \pi^{ab}(k_a - v_{\perp} v_a)] = 0 \quad \text{Stick}$$

- In the Newtonian Limit

$$[p + \hat{\pi}^{ij} k_i k_j] = 0 \quad \text{or} \quad [p k^i + \hat{\pi}^{ij} k_j] = 0$$
Interface results

- Newtonian and relativistic interfaces in 1D
- Moved to *multimodel* code for 2D infrastructure
- 2D Newtonian interfaces
Technical aspects to address:
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Why do we need an atmosphere?

- In terrestrial solids, when \( p \to 0 \), \( n \) and \( \epsilon \) do not
- In NS, shear and thermal terms both scale with \( n \):
  - \( n \to 0 \) and \( \epsilon \to 0 \) as \( p \to 0 \)
  - All zero at the surface
- Small fluctuations at the surface can cause these to go negative, which causes numerical problems
- Positivity preserving schemes have worked for fluids (Radice et al. 2014), but unclear how to extend to elasticity
What is the atmosphere?

- Treat the atmosphere as another *model* in our code.
- Model consists of only a pressure, $p_{\text{atm}}$.
- Don’t evolve.
- Just use to apply boundary conditions at the surface.
Atmosphere Results

Least-Squares Fit:

\[ E = p_{\text{atm}}^{1.00}/872.13 \]

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Outline:

- Fluid core
- Solid crust
- Shattered region
- Atmosphere
- Gravity
- Solid-wall
- Periodic
Starquake mechanisms

**Cracking**
- Material breaks and slips along a surface, handled using interfaces (previous slide)
- Suppressed by pressure in NS?

**Shattering**
- Instantaneous relaxation, matter-space metric proportional to spacetime metric
- Suggested by molecular dynamics simulations
2D toy model

Combine technical aspects:
- Elasticity
- Interfaces
- Stellar surface
- Starquakes
2D toy model
**Future Work**

- Assess effect of atmosphere
- Toy model in GR
- Elasticity and interfaces in a 3D fully relativistic code
- Magnetic fields