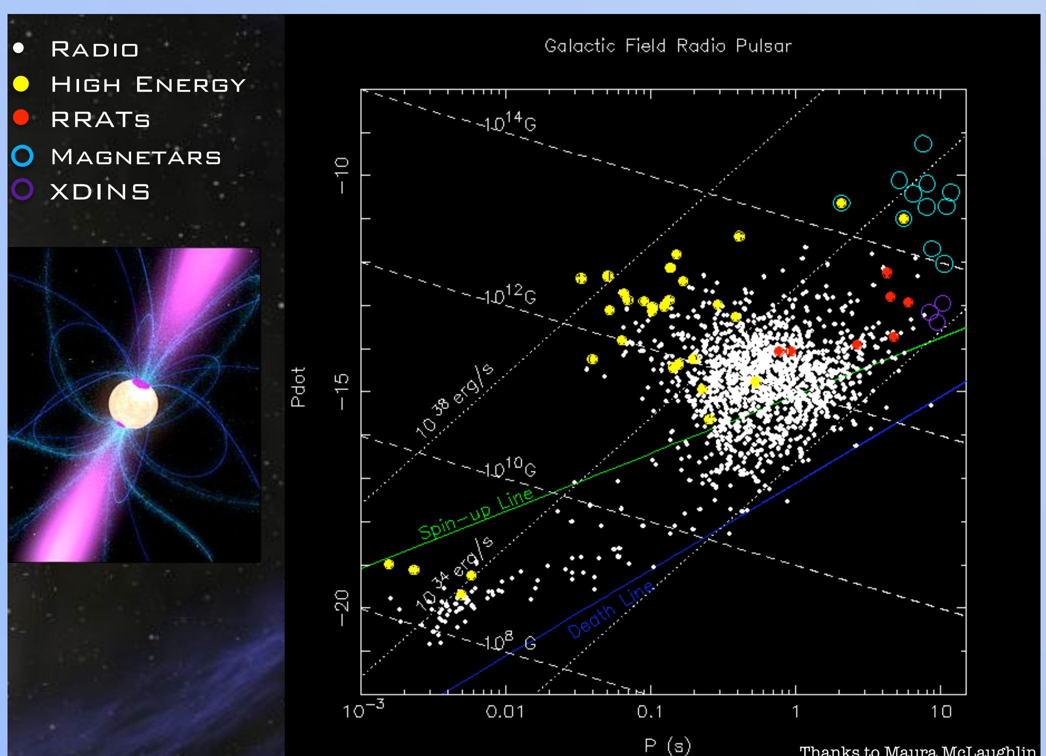
Consistent models for the structure of strongly magnetized neutron stars

Debarati Chatterjee
LUTH, Observatoire de Paris, Meudon

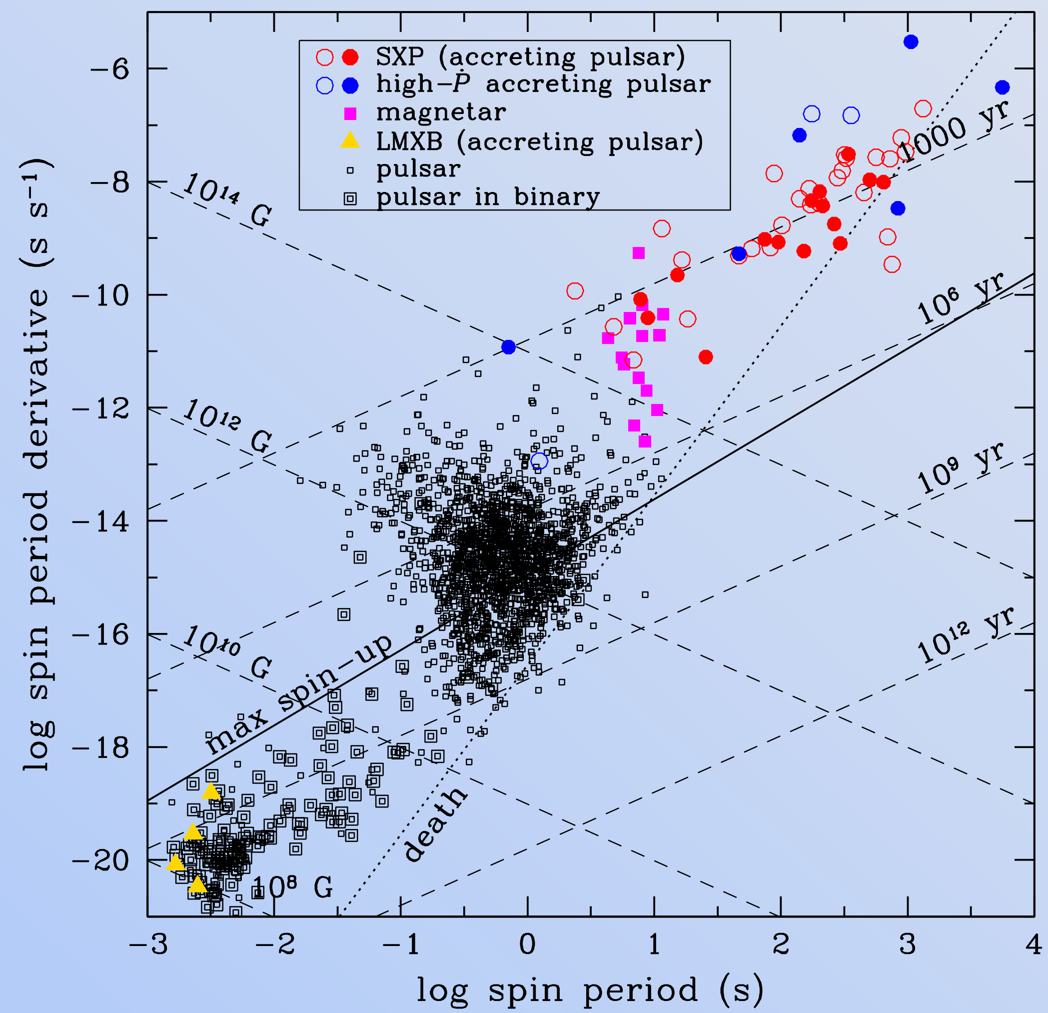
Collaborators:
Micaela Oertel
Jerome Novak

MOTIVATION

- *high B neutron stars: XDINSs and RRATs, SXPs, ESPs, having super critical magnetic fields*
- *Soft Gamma-ray Repeaters (SGRs), Anomalous X-ray Pulsars (AXPs)*
- *Observations indicate common features ⇒ SGRs/AXPs belong to a unified class of objects, i.e. magnetars*



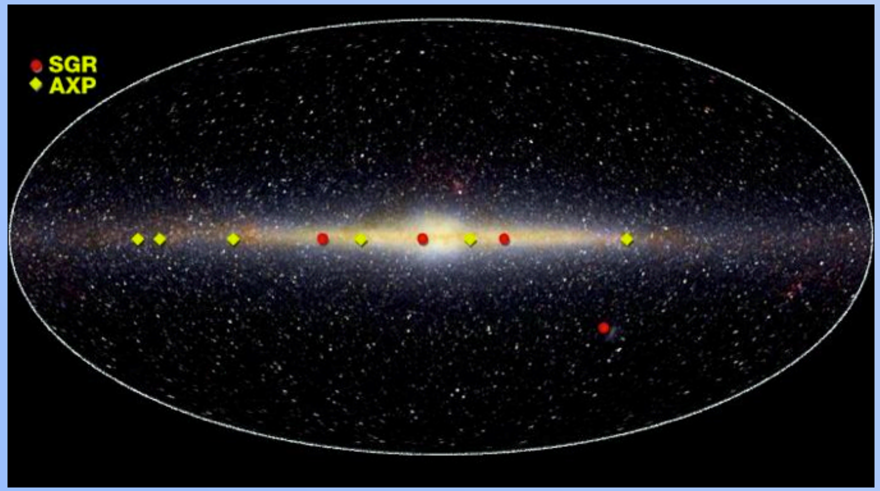
M. Burgay (2009)

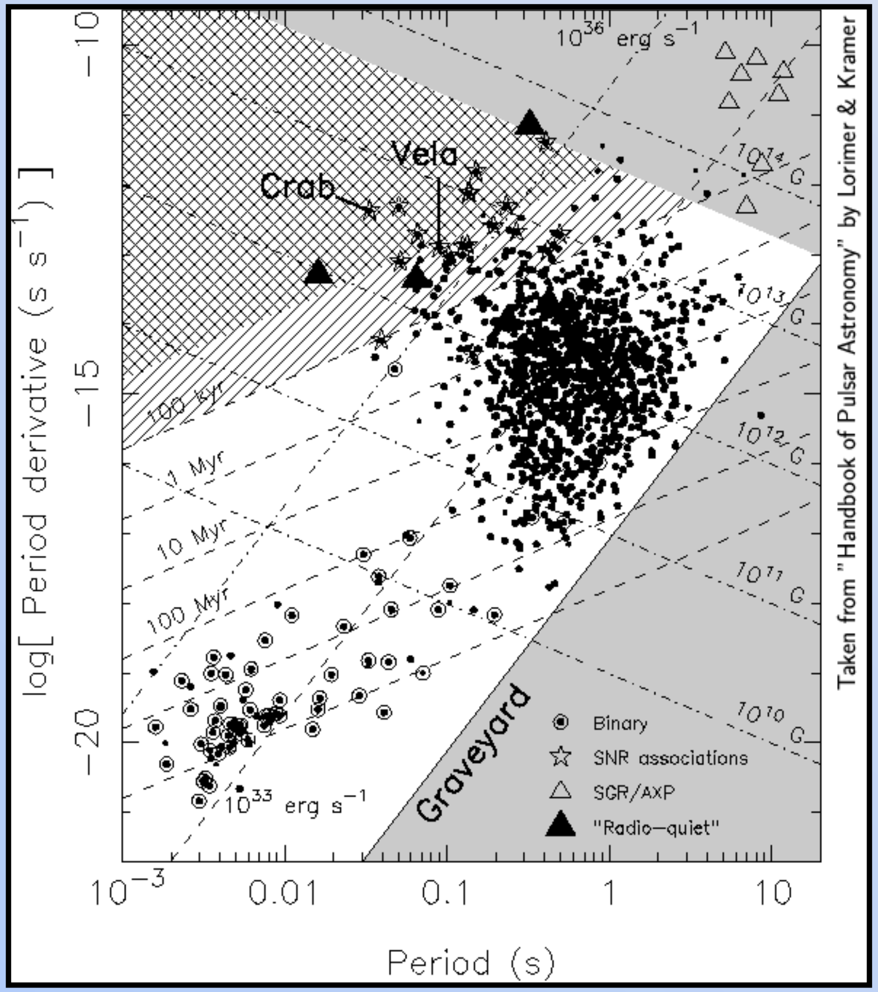


Ho, Klus, Coe, Andersson (2013)

MAGNETARS

- *quiescence: all SGRs/AXPs display a steady luminous X-ray emission with emission $L_x \sim 10^{35}$ erg/s*
- *characterized by X-ray bursting, flaring and outbursts, Luminosities super-Eddington ~ 10^{46} erg s^{-1}*
- *Large luminosities at quiescence and during outbursts can be explained in terms of magnetic dissipation*
- *long periods (P ~ 5-12 s), large spin-down rates (Pdot ~ $5 \times (10^{-13} - 10^{-10})$ s/s)*
- *$dE/dt > dE_{rot}/dt \Rightarrow$ powered by field decay*
- *P-Pdot, magnetic braking $\Rightarrow B \sim 10^{14}$ - 10^{15} G*
  (Duncan & Thomson 1992; Thomson & Duncan 1993)
- *Direct measurements of the field (Ibrahim et al.)*





MAXIMUM ALLOWED MAGNETIC FIELD

- *Inside the star, the magnetic field may be even higher*
- *The limiting interior field strength for a star can be estimated using the Virial theorem,*

  $2T + W + 3\,\Omega + M = 0$

  T = total rotational kinetic energy

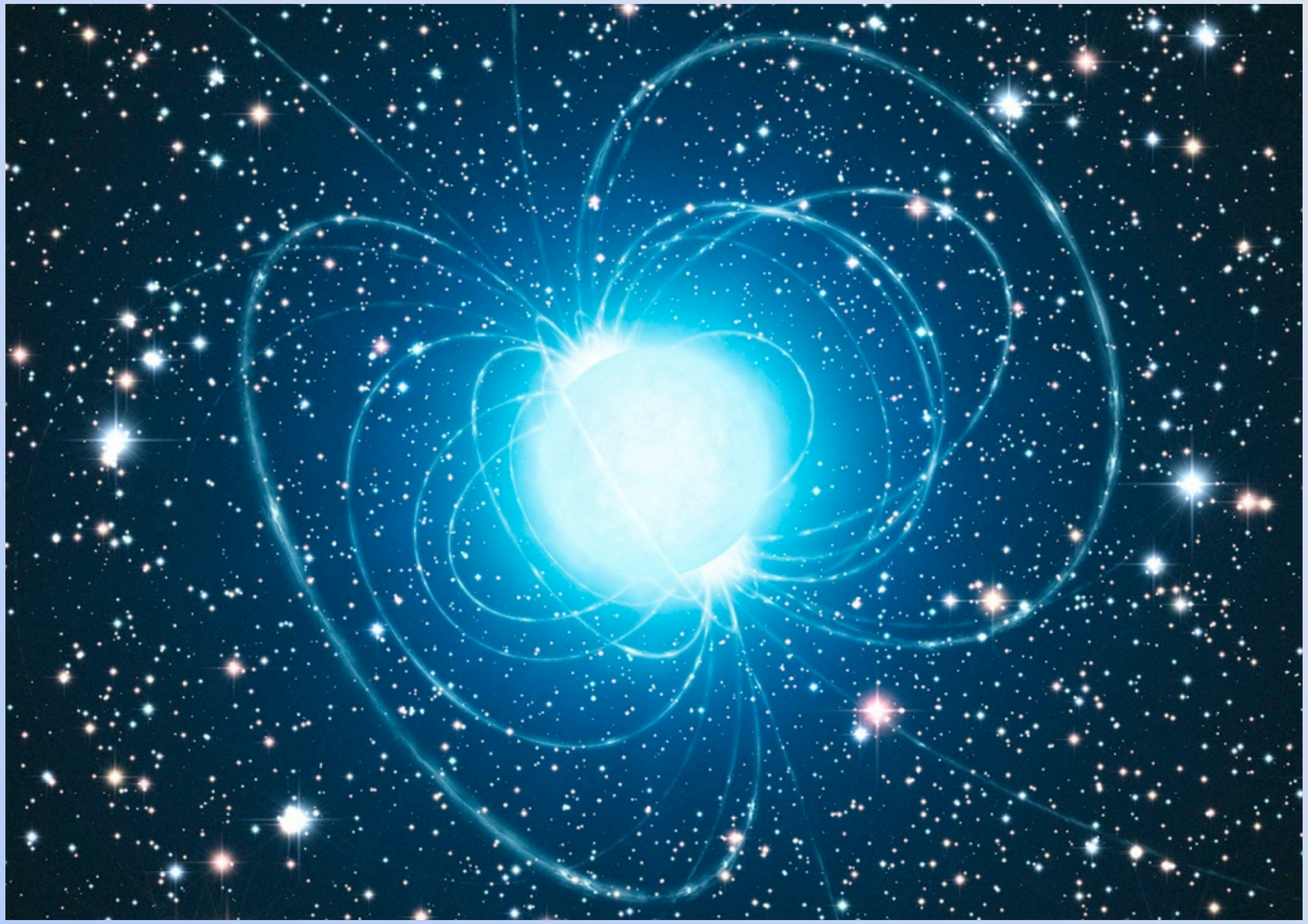
  W= gravitational potential energy

  Ω = internal energy

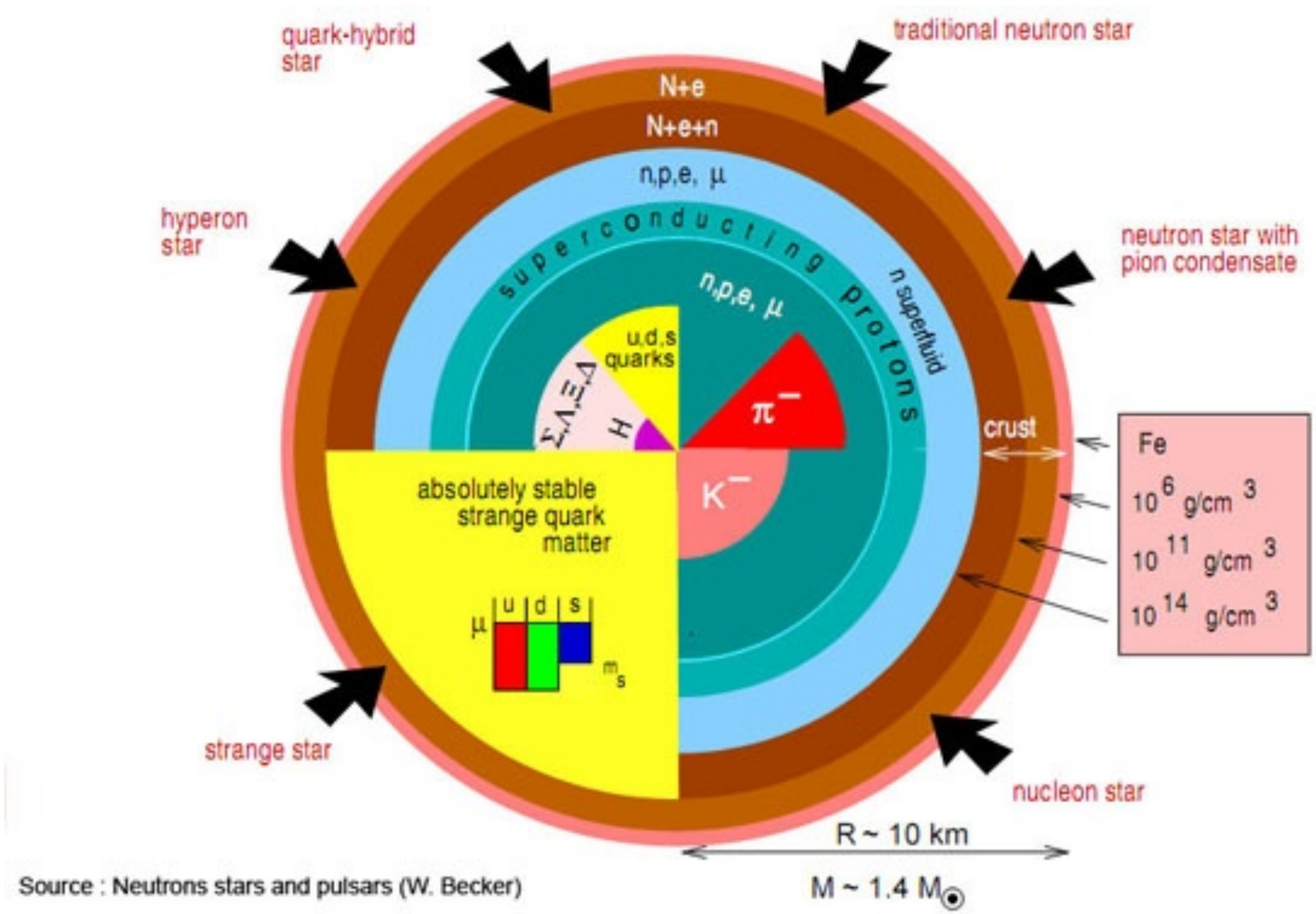
  M = magnetic energy

Since T, $\Omega > 0$, maximum magnetic energy can be comparable to, but cannot exceed gravitational energy in equilibrium

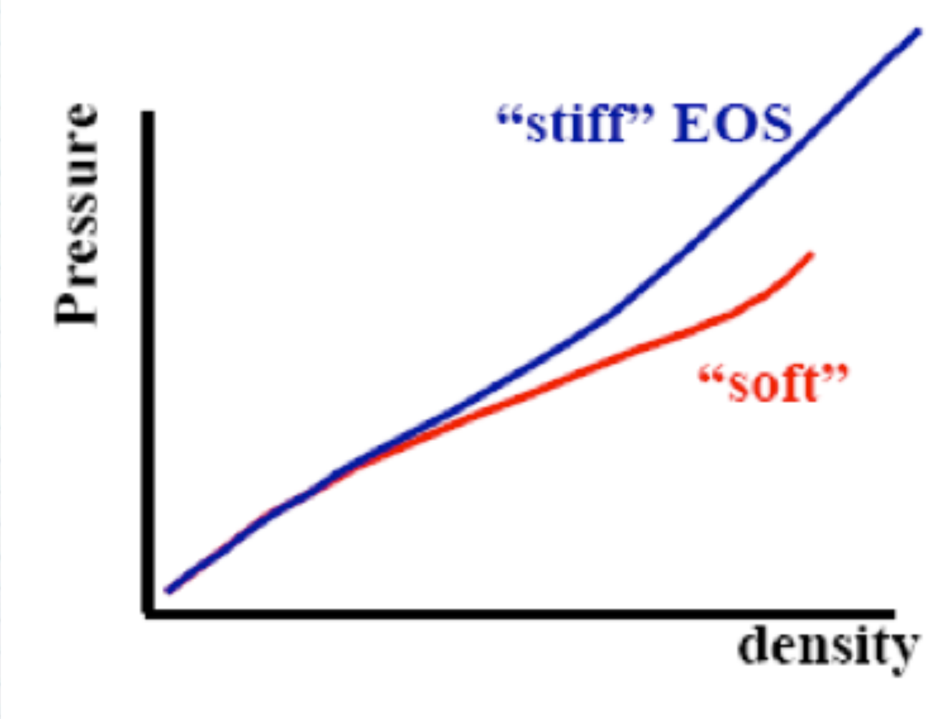
- *For a typical neutron star, $B_{max} \sim 10^{18}$ G*

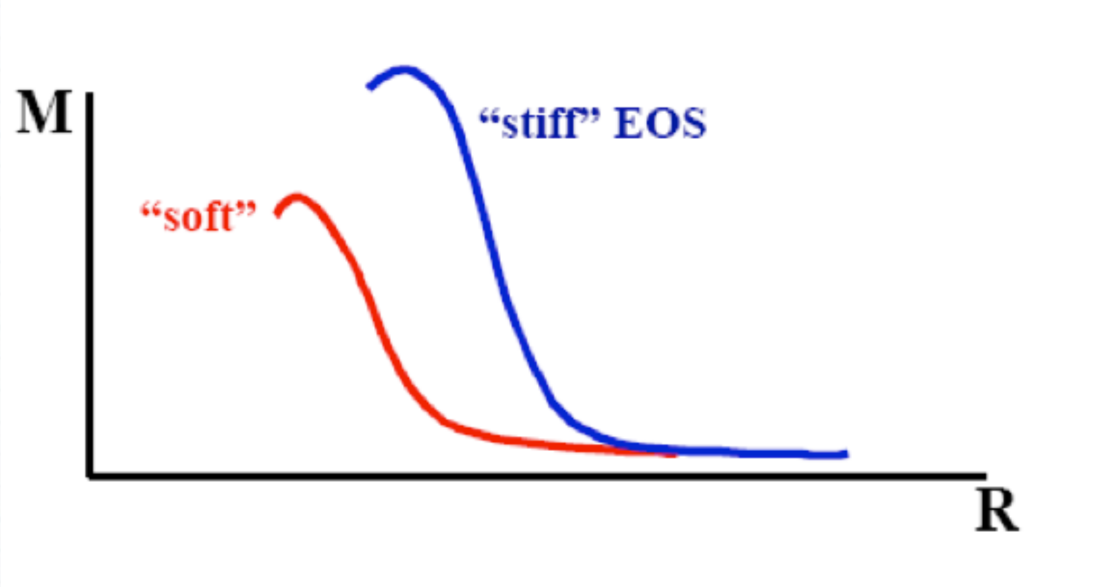


NEUTRON STAR STRUCTURE



Equation of state (EoS)





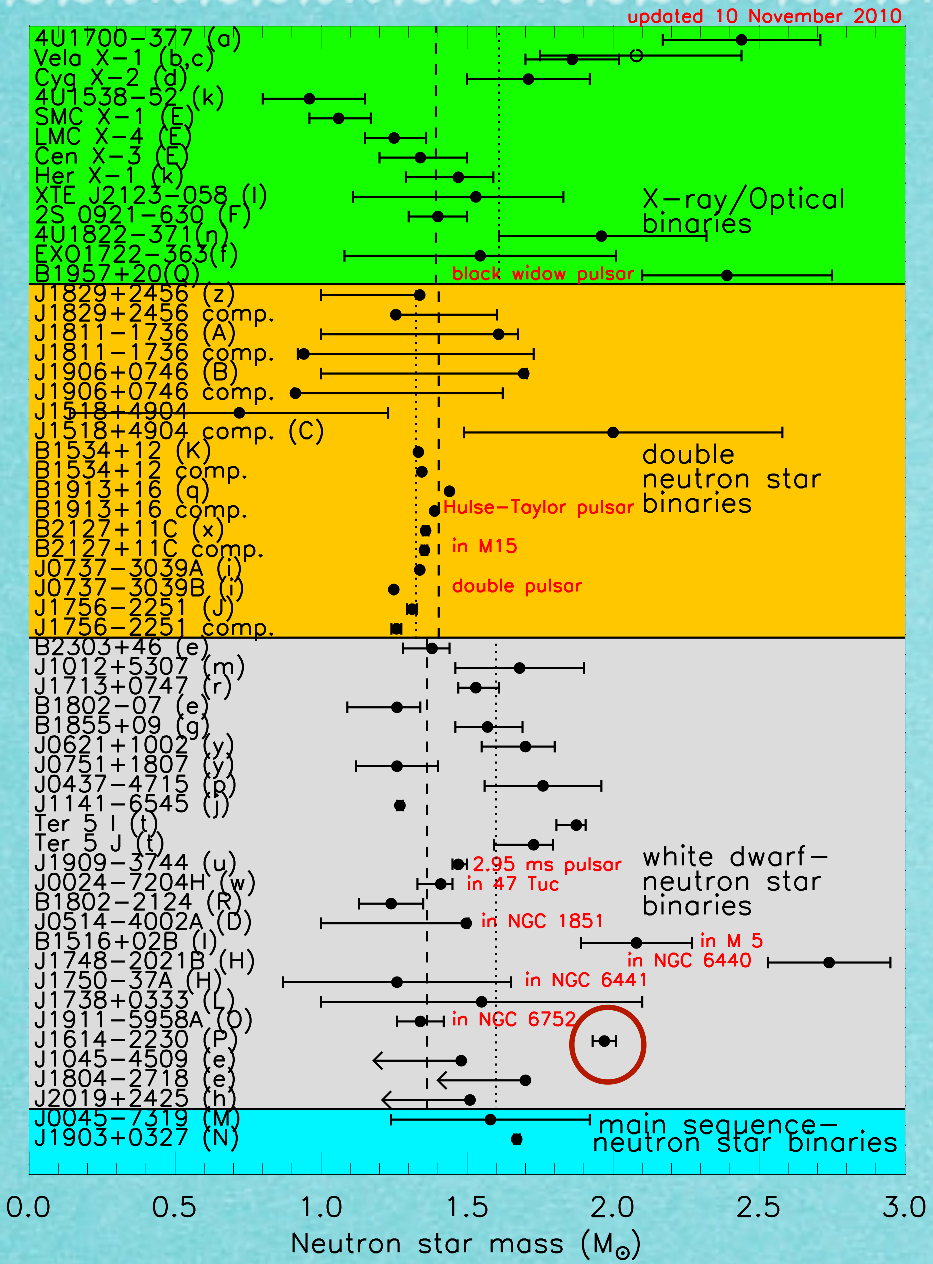
Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$

Constraining the EoS

$M^{max}(theo) > M^{max}(obs)$

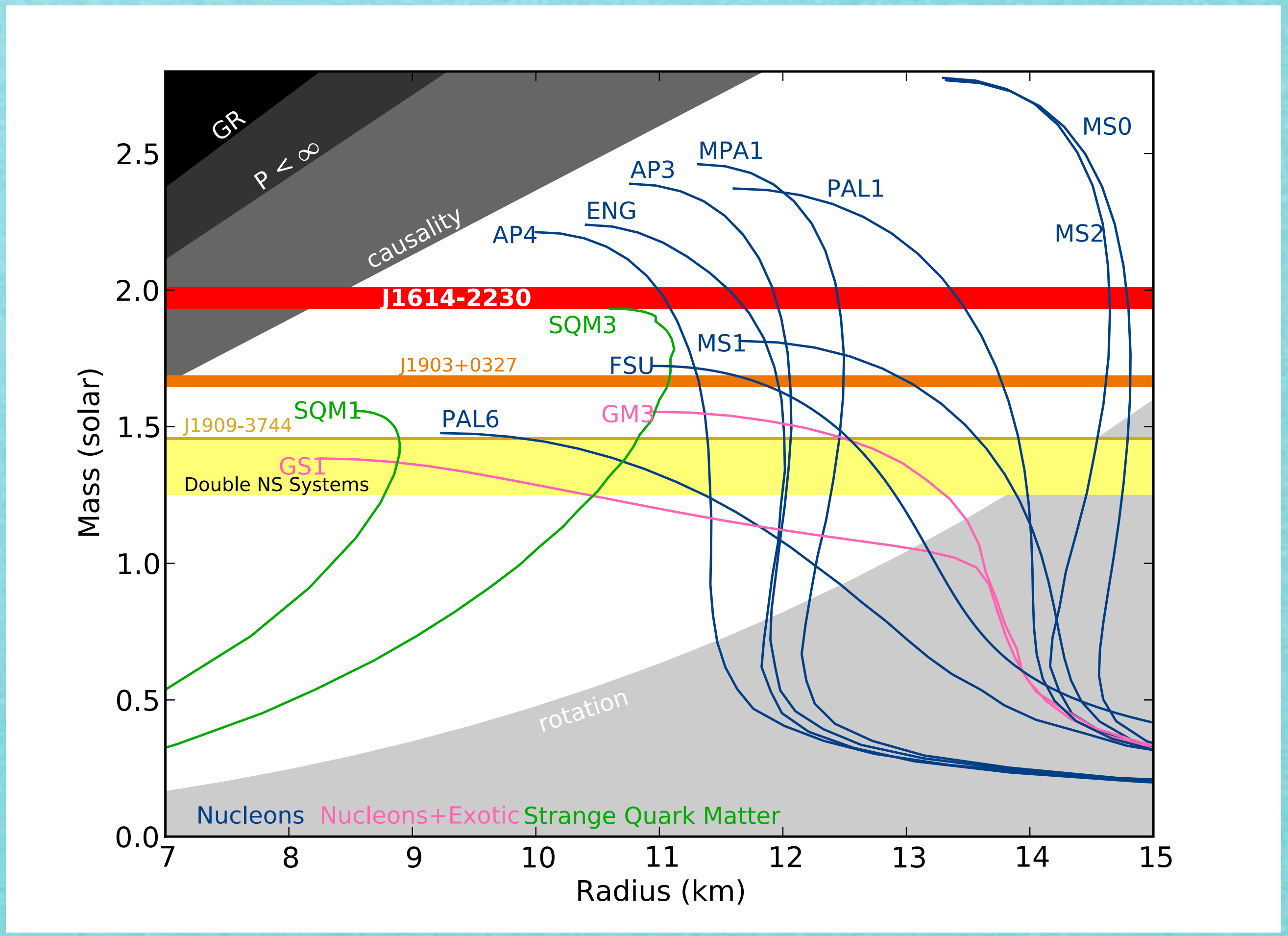


Demorest et al (Nature 2010)

$\mathbf{1.97 \pm 0.04\ M_\odot}$

Antoniadis et al (Science 2013)

$2.01 \pm 0.04\ M_\odot$

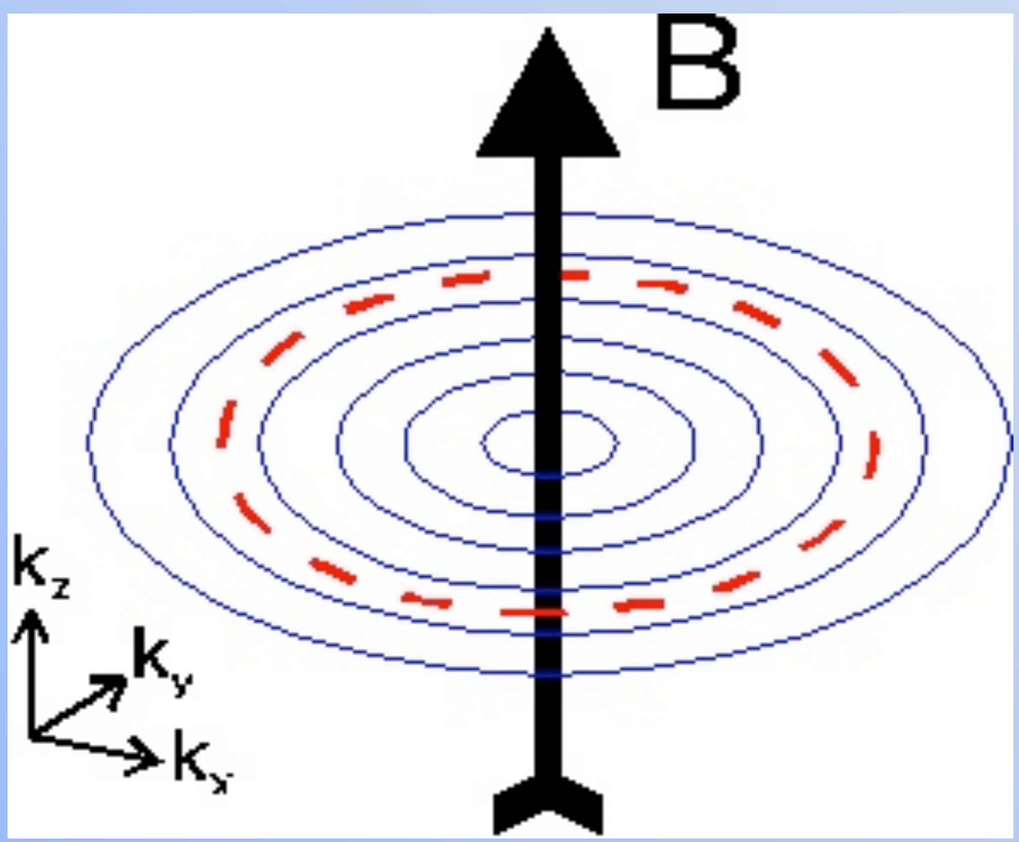


Lattimer and Prakash, arXiv:1012.3208

PARTICLES IN MAGNETIC FIELD: LANDAU QUANTIZATION

- *In the presence of the magnetic field, the motion of the charged particles is Landau quantized in the direction perpendicular to the magnetic field.*
- *The critical field (the value where the cyclotron quantum is equal to the rest energy of the charged particle) for electrons is $B_m^{(e)(c)} = 4.4 \times 10^{13}$ G, and for protons it is $B_m^{(p)(c)} \sim 10^{20}$ G*
- *Choosing the coordinate axes such that B is along z-axis, the single particle energy of any charged particle at n-th Landau level is given by*

$$E_n = \sqrt{p_z^2 + m^2 + 2ne|Q|B}.$$



Magnetic field effects in Neutron Stars

Noronha and Shovkovy (2007), Ferrer et al. (2010), Paulucci et al. (2010), Dexheimer, Menezes, Strickland (2012)

Microphysics

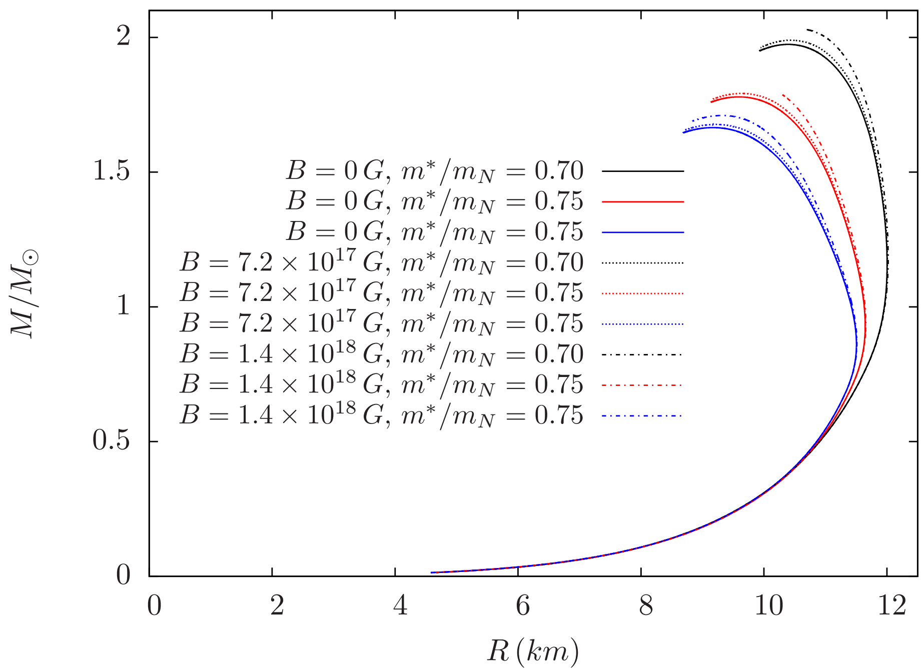
effects of magnetic field on the dense matter Equation of State

interaction of the electromagnetic field with matter (magnetisation)

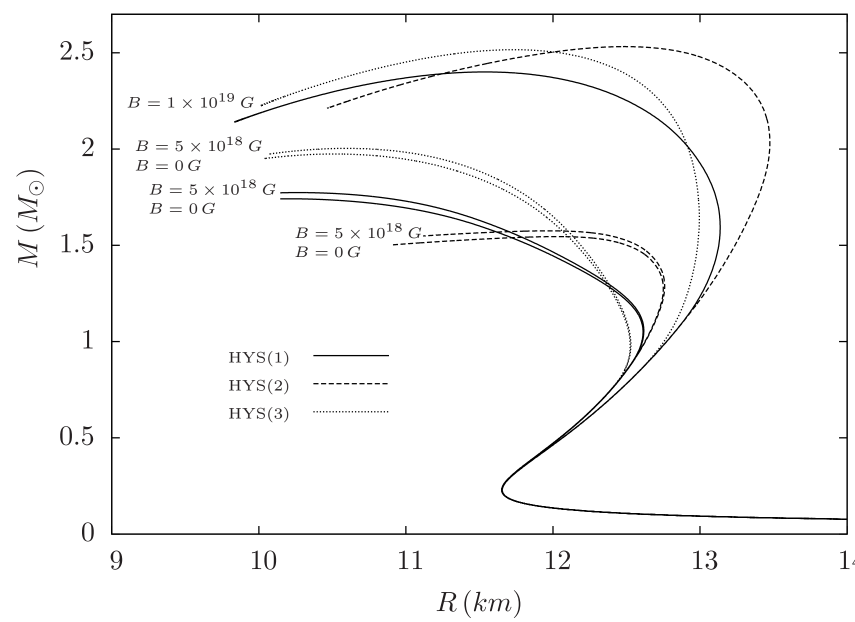
Macrophysics

anisotropy of the energy momentum tensor caused by breaking of the spherical symmetry by the electromagnetic field

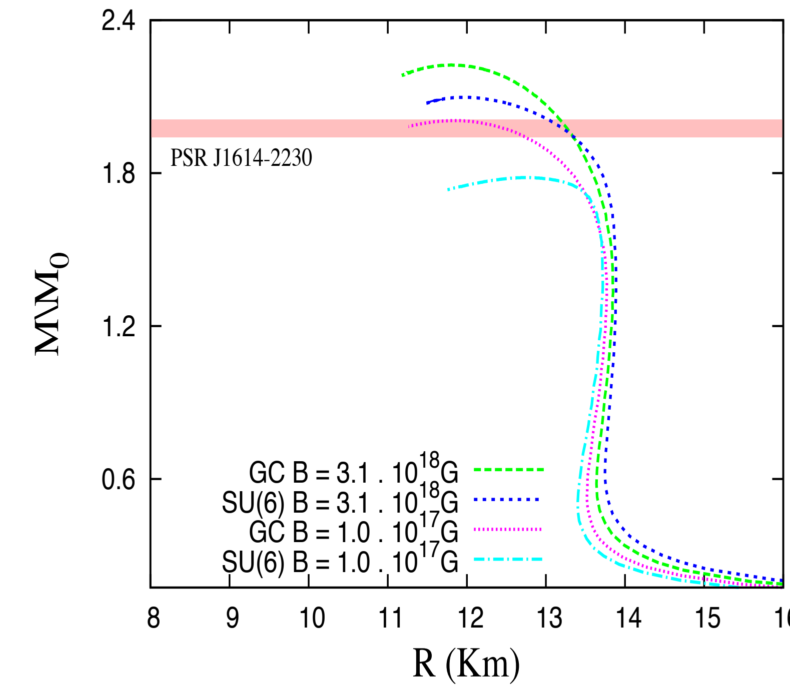
to calculate the structure and observable properties of the neutron star within General Relativistic framework



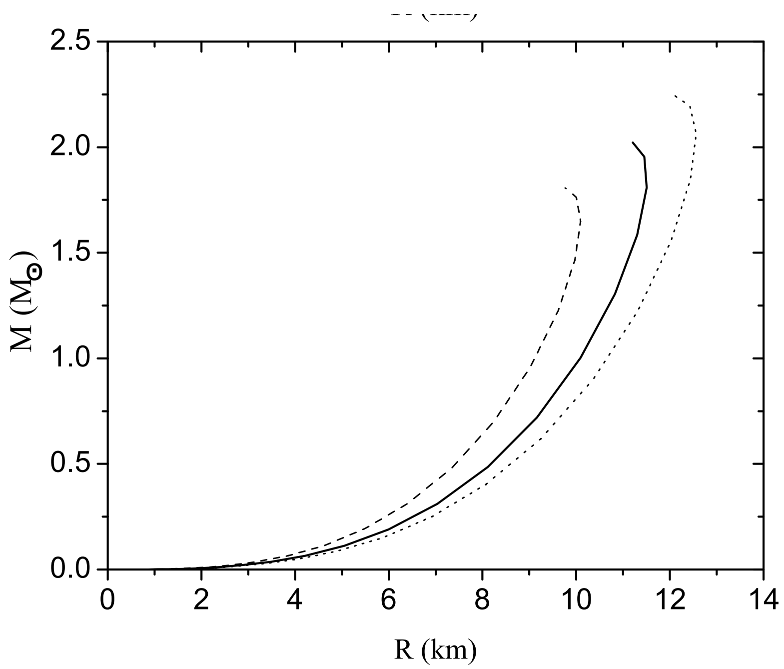
Gomes et al. (2013)



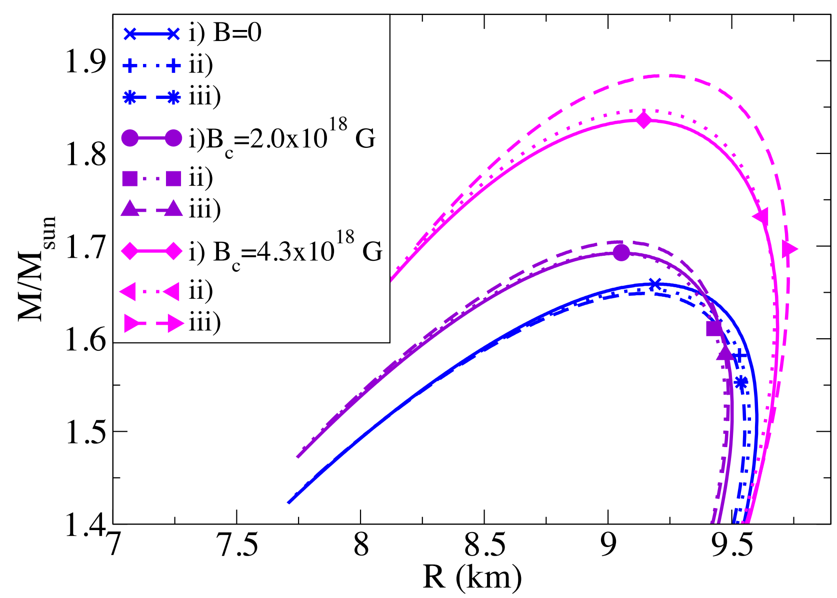
Gomes et al. (2014)



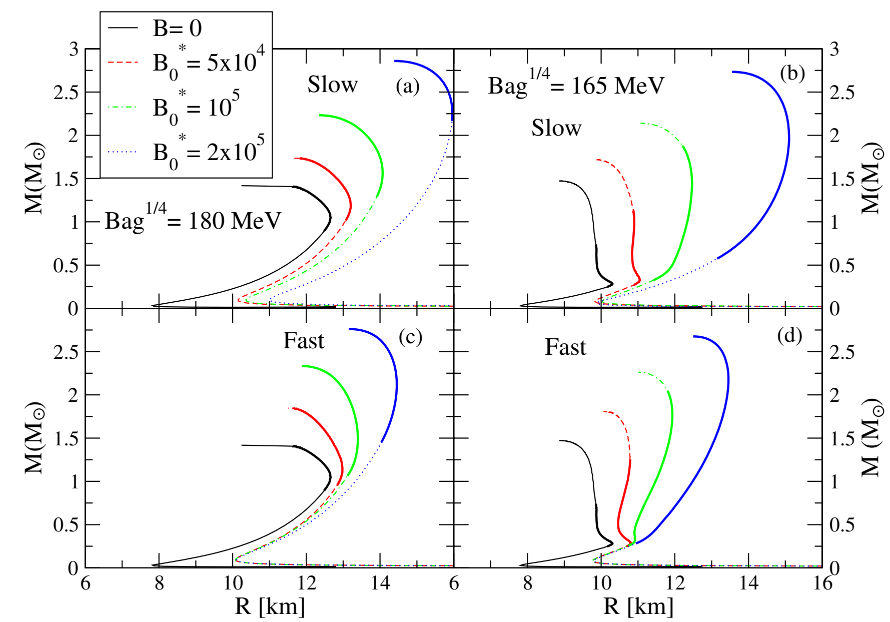
Lopes & Menezes (2013)



Paulucci et al. (2010)



Dexheimer et al. (2013)



Rabhi et al. (2009)

Magnetic field effects in Neutron Stars

Microphysics

effects of magnetic field on the dense matter
Equation of State

interaction of the electromagnetic
field with matter (magnetisation)

Macrophysics

anisotropy of the energy momentum tensor caused
by breaking of the spherical symmetry by the electromagnetic field

to calculate the structure and observable properties
of the neutron star within General Relativistic framework

Bonazzolla, Gourgoulhon, Salgado, Marck (1993)
Bocquet, Bonazzola, Gourgoulhon, Novak (1995)
Cardall, Prakash, Lattimer (2001)

AIM OF THE STUDY

effects of magnetic field on the dense matter Equation of State

interaction of the electromagnetic field with matter (magnetisation)

Microphysics

Consistent neutron star models in a strong magnetic field

Macrophysics

anisotropy of the energy momentum tensor caused by breaking of the spherical symmetry by the electromagnetic field

to calculate the structure and observable properties of the neutron star within General Relativistic framework

MICROSCOPIC ENERGY MOMENTUM TENSOR

- *Lagrangian density of a fermion system in the presence of a magnetic field*

$$\mathcal{L} = \underbrace{\boxed{\bar{\psi}(x)(D_\mu \gamma^\mu - m)\psi(x)}}_{matter} - \underbrace{\boxed{\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}}}_{field}$$

where $D_\mu = i\partial_\mu - eA_\mu$
and the field strength tensor of the electromagnetic field is $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

- *In flat space, the Canonical energy momentum tensor* $$\Theta^{\mu\nu} = \sum_\varphi \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi - g^{\mu\nu} \mathcal{L} \, ,$$

is the conserved Noether current associated with the symmetry of space-time translations.

- *this is neither symmetric nor gauge-invariant, hence unsuitable as source of Einstein equations*
- *The energy-momentum tensor can be written in a symmetrized and gauge invariant form (Belinfante-Rosenfeld tensor)*

L. Rosenfeld, Acad. Roy. Belg. , Memoires de classes de Science 18 (1940), F.J. Belinfante, Physica 7 (1940), 449.

$$T^{\mu\nu} = \underbrace{\boxed{\frac{1}{4\pi}(F^{\mu\alpha} F^\nu_\alpha + g^{\mu\nu} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta})}}_{field} + \underbrace{\boxed{\frac{1}{2} \bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi}}_{matter}$$

THERMODYNAMIC AVERAGE OF ENERGY MOMENTUM TENSOR

- *thermodynamic average of the microscopic energy-momentum tensor in the statistical ensemble*

$$\langle \tau^{\mu\nu} \rangle = \frac{1}{\beta V}\frac{1}{Z}\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_0^\beta d\lambda \int d^3x \tau^{\mu\nu} \exp(\tilde{S})$$

where the partition function is and S is the action.

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(\tilde{S})$$

- *thermal average of the energy-momentum tensor is given by*

$$\boxed{T_m^{\mu\nu} = \underbrace{\varepsilon_m u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)}_{\text{pure fermionic}} + \underbrace{\frac{1}{2}(M^{\mu\lambda}F_\lambda^\nu + M^{\nu\lambda}F_\lambda^\mu)}_{\text{magnetisation}}}$$

matter

$$\boxed{T_f^{\mu\nu} = -\frac{1}{4\pi}F^{\mu\lambda}F_\lambda^\nu + \frac{1}{16\pi}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}.}$$

field

FLUID REST FRAME

- *In the rest frame of the hadronic fluid, with B field along the z axis, the matter and field parts of the energy-stress tensor are given by*

$$T_m^{\mu\nu} = \begin{pmatrix} \varepsilon_m & 0 & 0 & 0 \\ 0 & P - MB & 0 & 0 \\ 0 & 0 & P - MB & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

$$T_f^{\mu\nu} = \frac{B^2}{8\pi} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- *In the literature, components of the energy-momentum tensor are defined as*

total energy density $\quad \varepsilon = \varepsilon_m + \dfrac{B^2}{8\pi}.$

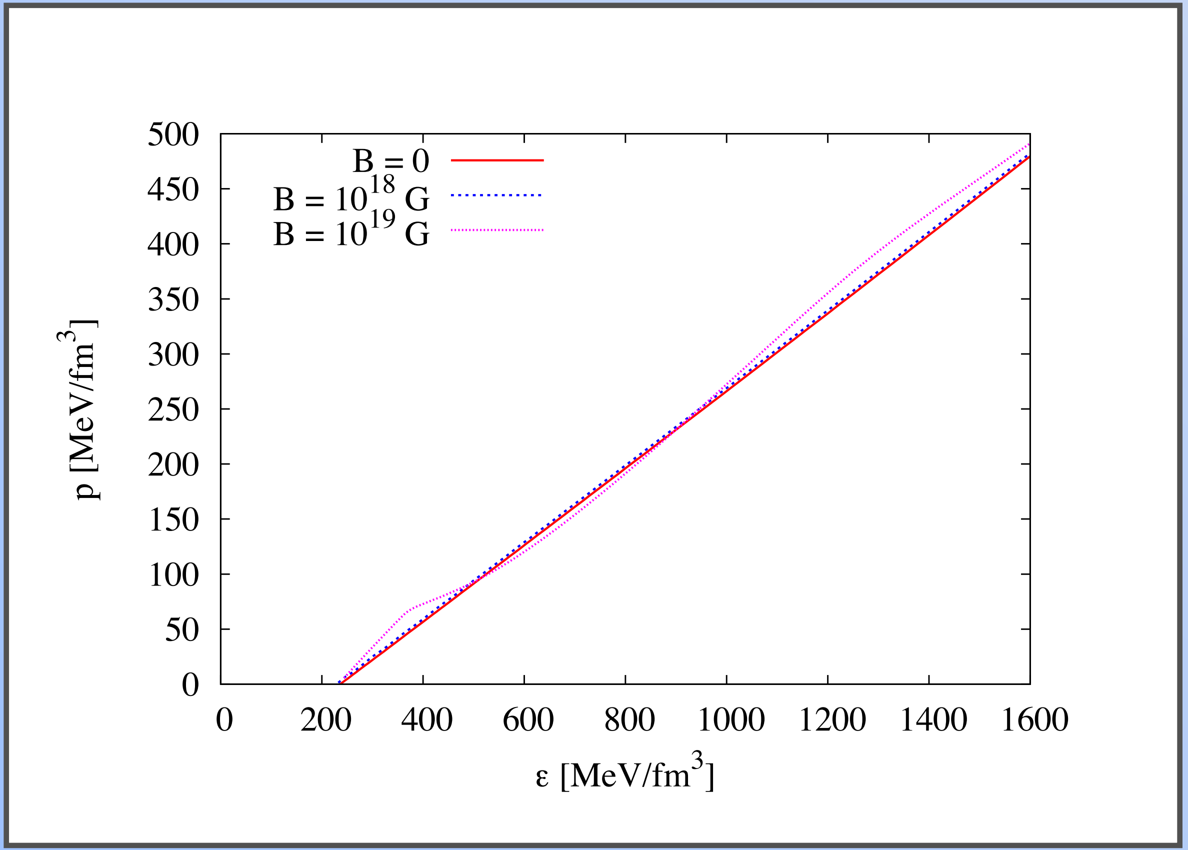
total "longitudinal" and "transverse" pressure

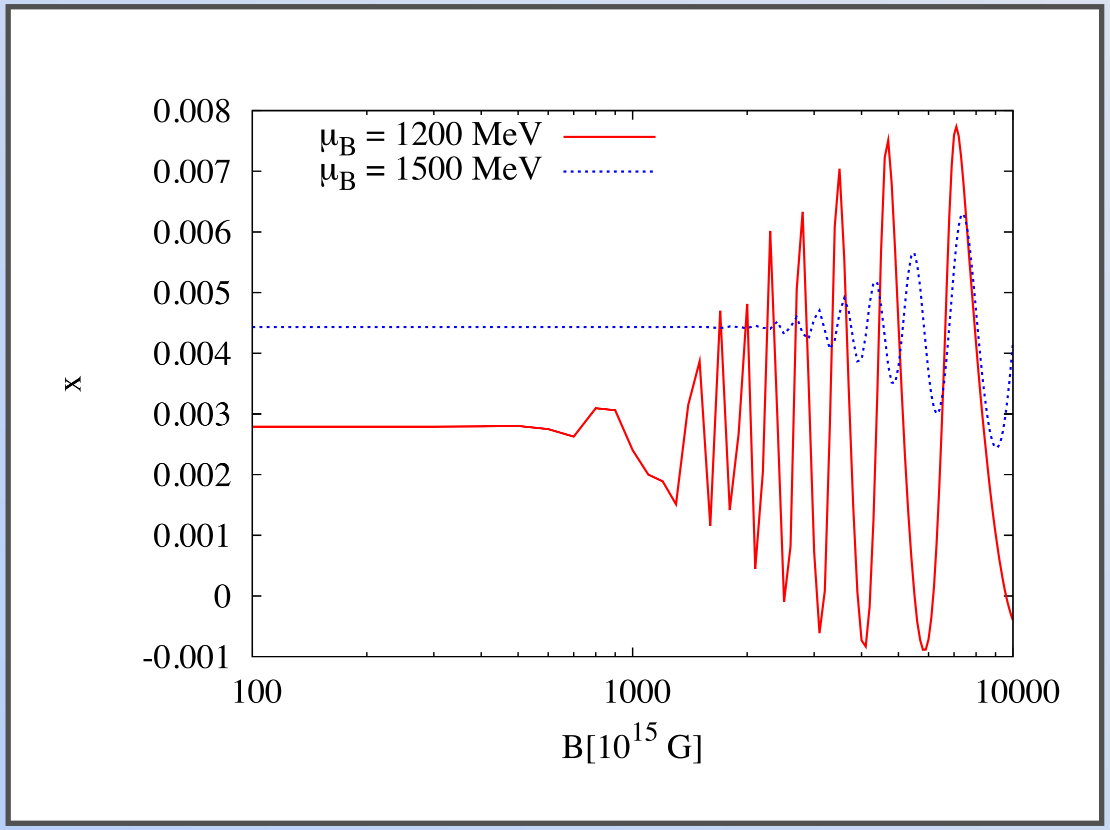
$$P_{\parallel} = P - \frac{B^2}{8\pi}.$$

$$P_{\perp} = P - MB + \frac{B^2}{8\pi},$$

MAGNETIC FIELD DEPENDENT EQUATION OF STATE

- *Example: Quark Matter in MCFL phase* *(Noronha and Shovkovy 2007, Paulucci et al. 2011)*
- *massless 3-flavor MIT Bag model (with B = 60 MeV/fm³) + Pairing interaction of NJL-type*





- *effects of Landau quantization become noticeable only for fields of ~ 10^{19}G.*

- *Magnetisation negligible*
- *de Haas-van Alphen oscillations*

D. C., T. Elghozi, M. Oertel, J. Novak, arXiv:1410.6332

NUMERICAL RESOLUTION

- *The structure equations of neutron stars are obtained by solving Einstein's field equations*
- *In the 3+1 Formalism, solving the Einstein's equations (system of 2nd order PDEs) are reduced to integration of a system of coupled 1st order PDEs subject to certain conditions:*

Bonazzolla, Gourgoulhon, Salgado, Marck (1993)

- *6 evolution equations for the extrinsic curvature*
- *1 Hamiltonian constraint equation*
- *3 momentum constraint equations*

- *The formulation has been employed to construct a numerical code (LORENE) using spectral methods* **L**angage **O**bjet pour la **RE**lativité **N**umériqu**E**
- *The code has been extended to include coupled Einstein-Maxwell equations describing rapidly rotating neutron stars with a magnetic field*

Bocquet, Bonazzola, Gourgoulhon, Novak (1995)

MAGNETOSTATIC EQUILIBRIUM
(WITHOUT MAGNETISATION)

- *Equations for magnetostatic equilibrium (from the conservation of energy and momentum):*

$$\nabla_\mu \mathcal{T}^{\mu\nu} = 0$$

- *Inhomogeneous Maxwell equations:*

$$\frac{1}{\mu_0} \nabla_\mu F^{\nu\mu} = j^{\nu}{}_{free}$$

Bonazzolla, Gourgoulhon, Salgado, Marck (1993)

- *Einstein-Maxwell equations*

$$\nabla_\alpha \mathcal{T}^{\alpha\beta} = \nabla_\alpha \mathcal{T}_f^{\alpha\beta} - F^{\beta\nu} j_{free\nu}$$

Bocquet, Bonazzola, Gourgoulhon, Novak (1995)

- *first integral of fluid stationary motion (momentum conservation) :*

$$\boxed{\ln h(r,\theta) + \nu(r,\theta) - \ln \Gamma(r,\theta) + M(r,\theta) = \text{const.}}$$

- *In terms of enthalpy per baryon for neutron star* $h := \frac{\varepsilon + p}{n_\mathrm{b}}$

and $$M(r,\theta) = -\int_0^{A_\varphi(r,\theta)} f(x) \mathrm{d}x.$$

the electromagnetic term associated with the Lorentz force

MAGNETOSTATIC EQUILIBRIUM
(WITH MAGNETISATION)

- *In the Fluid Rest Frame, assuming perfect conductor, E = 0* $F_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu} u^\beta b^\alpha$
- *assuming isotropic medium, the magnetisation is aligned with the magnetic field*

$$M_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu} u^\beta m^\alpha \qquad m_\mu = \frac{x}{\mu_0} b_\mu$$

- *Modified inhomogeneous Maxwell equations:*

$$\nabla_\mu F^{\sigma\mu} = \frac{1}{1-x}\left(\mu_0 j^\sigma{}_{free} + \boxed{F^{\sigma\mu}\nabla_\mu x}\right)$$

magnetisation

- *first integral of fluid motion :*

$$(\varepsilon + p)\left(\frac{1}{\varepsilon+p}\frac{\partial p}{\partial x^i} + \frac{\partial \nu}{\partial x^i} - \frac{\partial \ln \Gamma}{\partial x^i}\right) - F_{i\rho} j^\rho{}_{free} \boxed{-\frac{x}{2\mu_0} F_{\mu\nu}\nabla_i F^{\mu\nu}} = 0$$

- *it can be shown that*

$$\frac{x}{2\mu_0} F_{\mu\nu}\nabla_i F^{\mu\nu} = \frac{x}{\mu_0}\left(b_\mu \nabla_i b^\mu - b_\mu b^\mu u_\nu \nabla_i u^\nu\right) = b\nabla_i b = m\cancel{\frac{\partial b}{\partial x^i}},$$

- *enthalpy per baryon for neutron star with magnetic field* $h = h(n_b, b) = \dfrac{\varepsilon + p}{n_b}$
- *derivative of logarithm of enthalpy*

$$\frac{\partial \ln h}{\partial x^i} = \frac{1}{\varepsilon + p}\left(\frac{\partial p}{\partial x^i} - m\cancel{\frac{\partial b}{\partial x^i}}\right)$$

Blandford and Hernquist (1982), Potekhin and Yakovlev (2012)

NUMERICAL RESOLUTION

- *The structure equations of neutron stars are obtained by solving Einstein's field equations*
- *In the 3+1 Formalism, solving the Einstein's equations (system of 2nd order PDEs) are reduced to integration of a system of coupled 1st order PDEs subject to certain conditions:*

Bonazzolla, Gourgoulhon, Salgado, Marck (1993)

- *6 evolution equations for the extrinsic curvature*
- *1 Hamiltonian constraint equation*
- *3 momentum constraint equations*

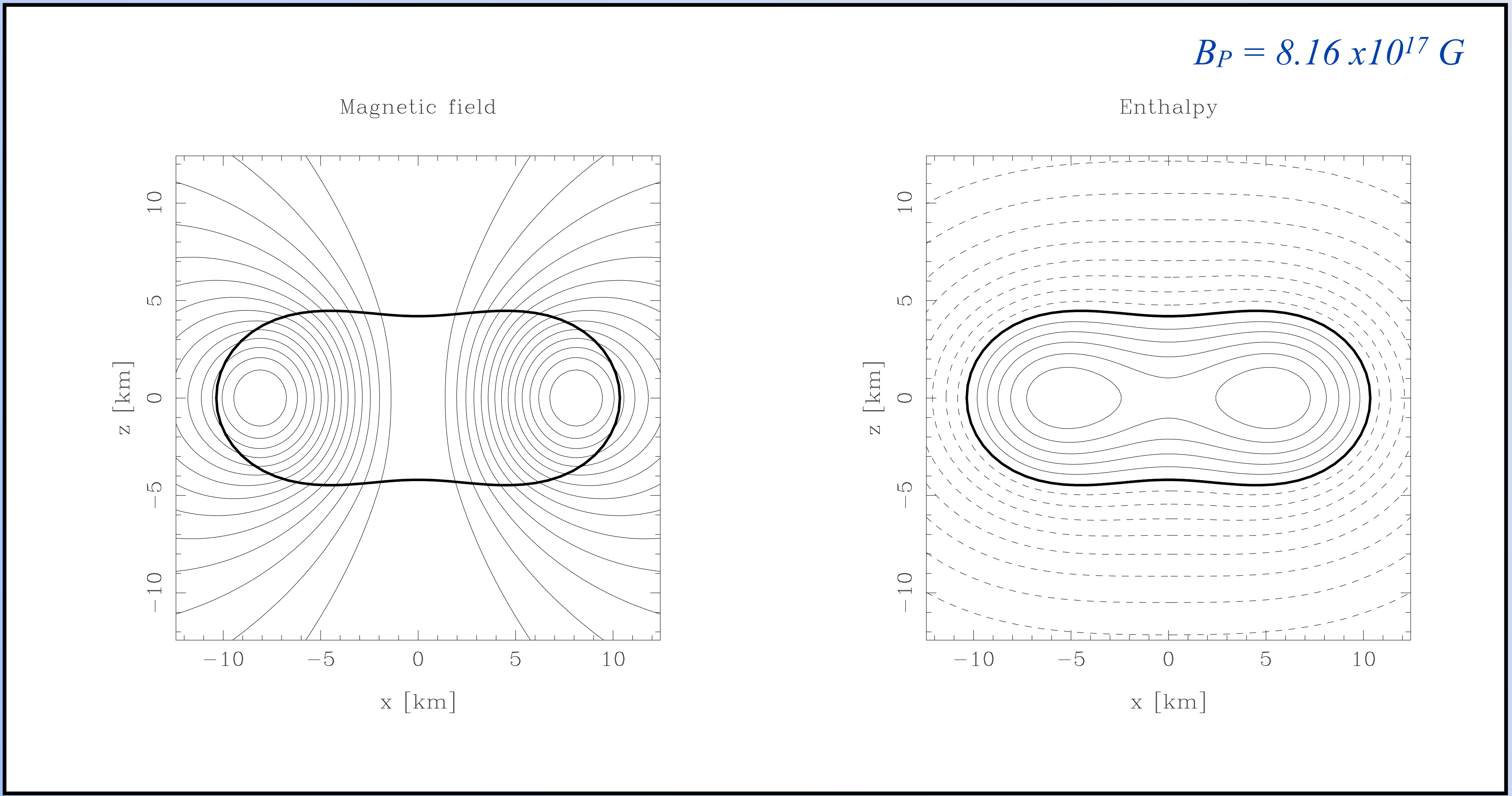
- *The formulation has been employed to construct a numerical code (LORENE) using spectral methods* **Langage Objet pour la RElativité NumériquE**
- *The code has been extended to include coupled Einstein-Maxwell equations describing rapidly rotating neutron stars with a magnetic field*

Bocquet, Bonazzola, Gourgoulhon, Novak (1995)

- *Incorporate magnetic field dependent EoS*
- *Incorporate magnetisation by modifying the inhomogeneous Maxwell equations*

D. C., T. Elghozi, M. Oertel, J. Novak, MNRAS 447 (2015) 3785

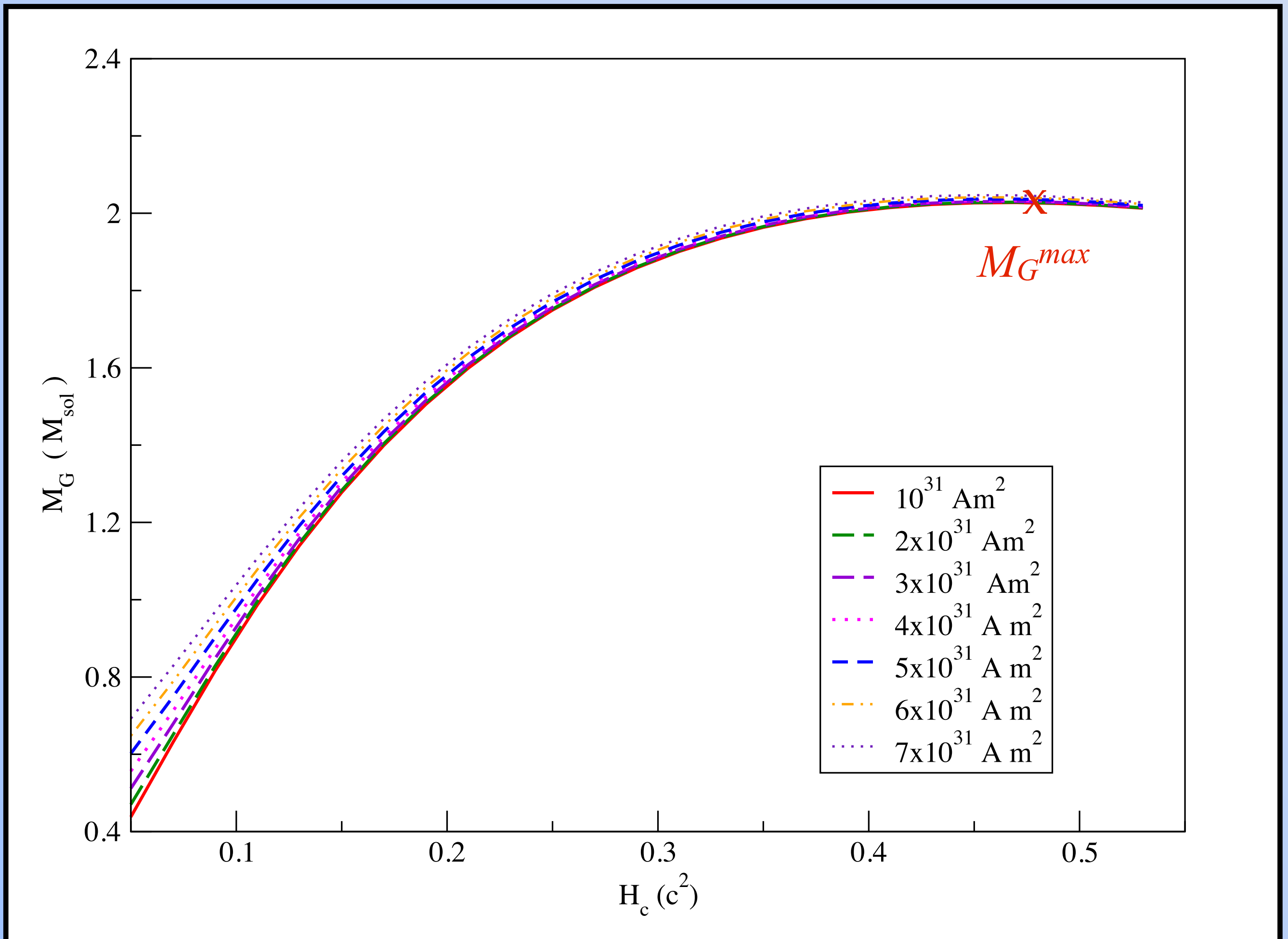
MAXIMAL DEFORMATION DUE TO MAGNETIC FIELD

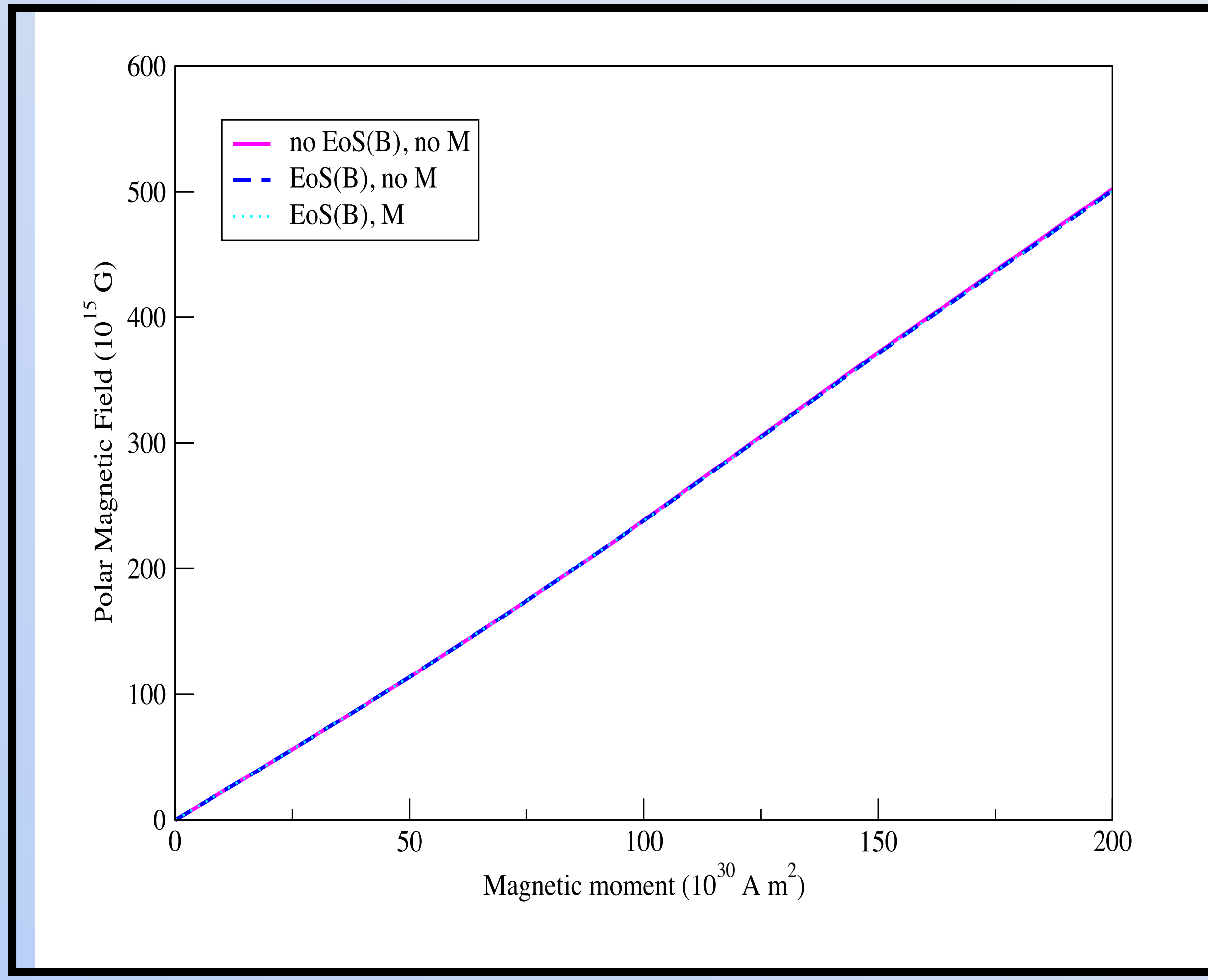


Magnetic field lines and enthalpy isocontours in the meridional (x, z) plane for static configuration for B_{polar}=8.16x10^{17}G, Mag moment = 3.25x10^{32} Am^2, M_G =2.22 M_{sol}

- *Stellar configurations strongly deviate from spherical symmetry*
- *Upon increasing magnetic field strength, the shape of the star becomes more and more elongated, finally reaching toroidal shape*

MAXIMUM GRAVITATIONAL MASS FOR A GIVEN MAGNETIC MOMENT

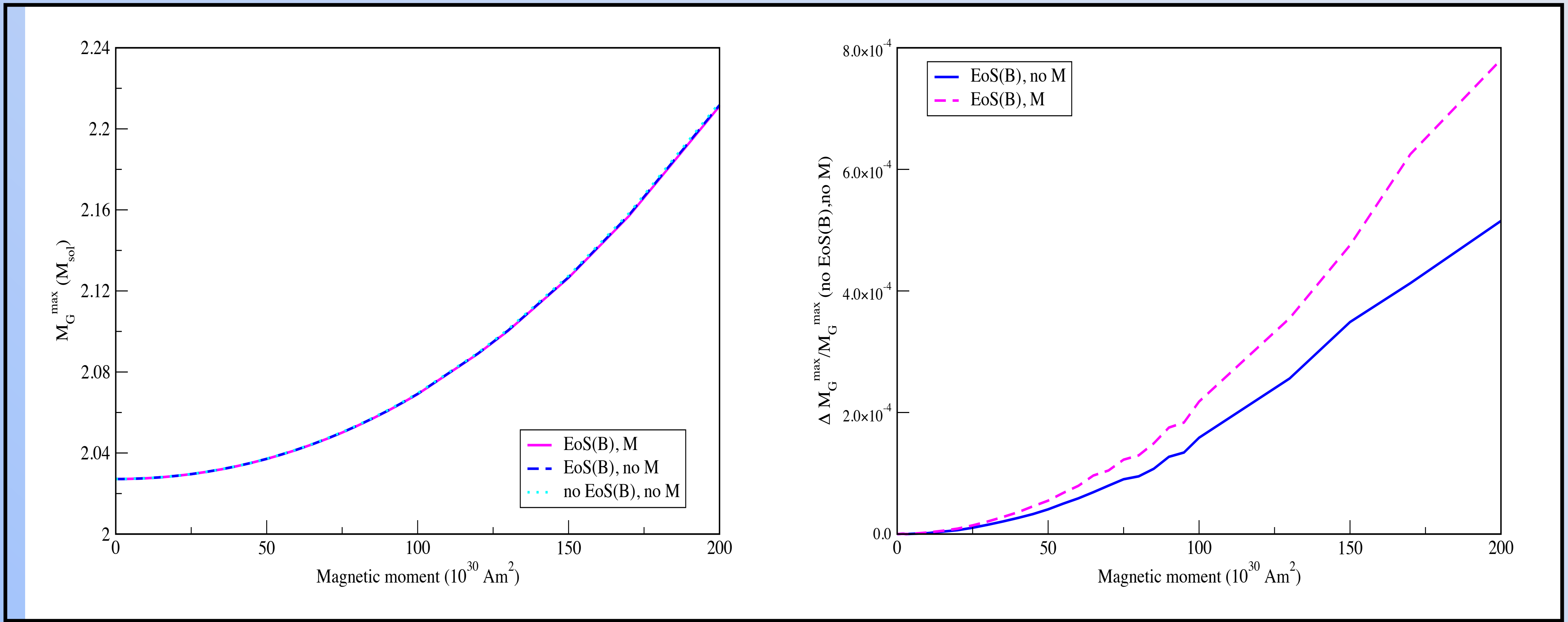




- *Gravitational mass varies with central log enthalpy and magnetic field*
- *Static configurations determined by different values of central log-enthalpy along constant sequences of magnetic dipole moment*
- *Plot of polar magnetic field corresponding to the values of magnetic moment for a neutron star of $M_B = 1.6\ M_{sol}$*
- *Maximum gravitational mass M_G^{max} was determined by parabolic interpolation*

D. C., T. Elghozi, M. Oertel, J. Novak, MNRAS 447 (2015) 3785

effect of EoS(B) and M



- *The 3 cases:*

(i) without magnetic field dependence in EoS, without magnetisation : no EoS(B), no M

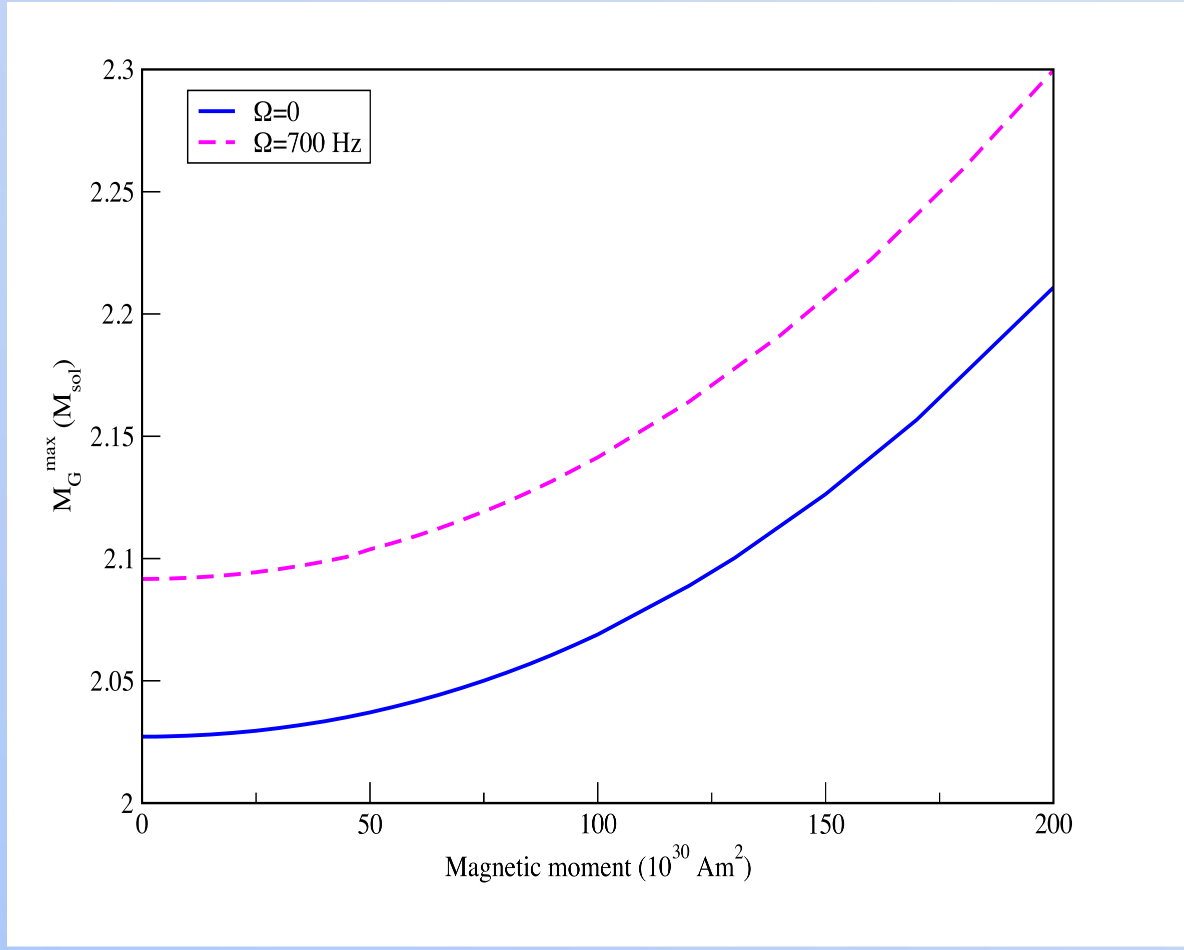
(ii) with magnetic field dependence in EoS, without magnetisation: EoS(B), no M

(iii) with magnetic field dependence in EoS, with magnetisation: EoS(B), M

- *Maximal gravitational mass is an increasing function of magnetic moment*
- *The effects of inclusion of magnetic field dependence of the EoS and the magnetisation are negligible, contrary to the claims of several previous works*

D. C., T. Elghozi, M. Oertel, J. Novak, MNRAS 447 (2015) 3785

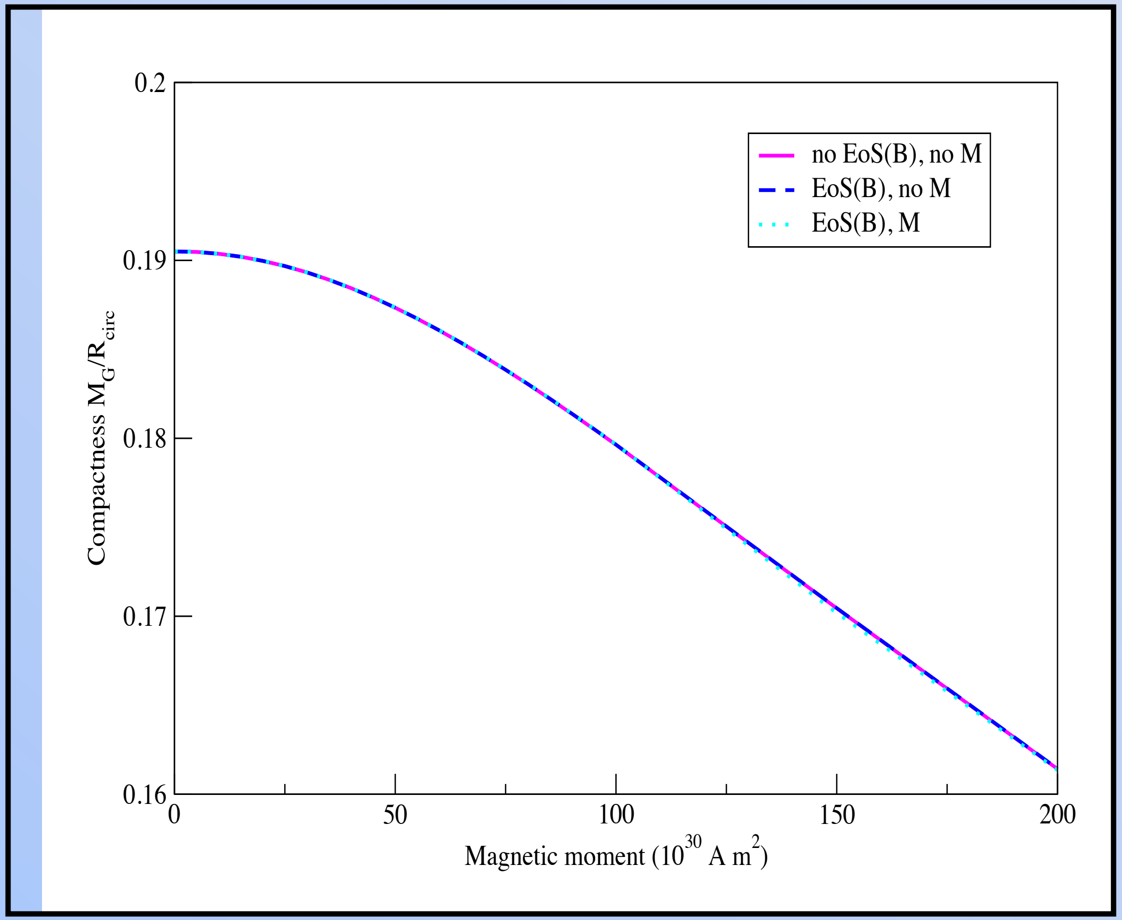
ROTATING CONFIGURATIONS



- *Observed magnetars are slowly rotating (P ~ s)*
- *We chose a sequence of neutron stars rotating at 700 Hz, close to fastest known rotating pulsar (716 Hz)*
- *Maximum mass increases with magnetic moment*
- *Effect of magnetisation and magnetic field dependence of EoS again found to be negligible*

D. C., T. Elghozi, M. Oertel, J. Novak, MNRAS 447 (2015) 3785

COMPACTNESS



$$\mathcal{C} = \frac{GM_G}{R_{\rm circ}c^2}$$

- *We studied the behaviour of compactness of a neutron star with baryon mass 1.6 with magnetic moment*
- *The compactness was found to decrease with increase in magnetic moment*
- *Centrifugal forces exerted by the Lorentz force on matter increases with increasing magnetic moment*
- *The influence of magnetic field dependence of EoS and magnetisation are negligible*
- *The main effect arises from the purely electromagnetic part*

D. C., T. Elghozi, M. Oertel, J. Novak, MNRAS 447 (2015) 3785

SUMMARY

- *In this work, we developed a self-consistent approach to determine the structure of neutron stars in strong magnetic fields, relevant for the study of magnetars*

- *Taking as an example the EoS of quark matter in MCFL phase, we investigated the effect of inclusion of magnetic field dependence of the EoS and magnetisation*

- *In particular, it was found that the equilibrium only depends on the thermodynamic EoS and magnetisation explicitly only enters Einstein-Maxwell equations*

- *In contrast to previous studies, we found that these effects do not significantly influence the stellar structure, even for the strongest magnetic fields considered*

- *The difference arises due to the fact that in previous works isotropic TOV equations were used to solve for stellar structure, whereas magnetic field causes the star to deviate from spherical symmetry considerably*