Radial oscillations of compact stars and collapse to black holes

Alessandro Brillante, Prof. Igor N. Mishustin, FIAS
Outline

1 Motivation
2 Derivation of oscillation equation
3 Results on neutral hybrid stars
4 Results on charged strange and hybrid stars
5 Non-linear effects, critical phenomena
How much charge allowed?

\[ \frac{F_C}{F_G} = \frac{e^2}{G m_p^2} \approx 10^{36} \]

Number of baryons in one neutron star: \( N_B \approx 3 \cdot 10^{57} \)

Number of net unit charges allowed to build “reasonable” charged compact stars: \( N_c < 10^{-18} N_B \)

assumption: EOS for charged compact stars calculated at charge neutrality (only 1 independent chemical potential)
prescription to find oscillation equation

- time-dependent spherically symmetric metric

- equations: \( G_{\mu\nu} = 8\pi T_{\mu\nu} \), \( T^{\mu\nu}_{\cdot\nu} = 0 \), \( (nu^\mu)_{;\mu} = 0 \)

\[
\partial_\mu \left[ \sqrt{-g} F^{\nu\mu} \right] = 4\pi \sqrt{-g} j^\nu
\]

- decompose variables: \( A(r, t) = A_0(r) + \delta A(r, t) \)

- linearize nonlinear equations

- subtract equilibrium equations from time-dependent equations and get perturbations: \( \delta A(r, t) \)

- substitute perturbations in \( T^{\mu\nu}_{\cdot\mu} = 0 \) and get pulsation equation
Derivation of oscillation equation 1

\[
ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\]

\[
G_0^0 = -e^{-2\Lambda} \left[ 2r^{-1} \Lambda' - \left( 1 - e^{2\Lambda} \right) r^{-2} \right]
\]

\[
G_1^1 = e^{-2\Lambda} \left[ 2r^{-1} \Phi' + r^{-2} \right] - r^{-2}
\]

\[
G_2^2 = e^{-2\Lambda} \left[ \Phi'' - \Phi' \Lambda' + \Phi'^2 + r^{-1} (\Phi' - \Lambda') \right]
+ e^{-2\Phi} \left[ \Phi \dot{\Lambda} - \ddot{\Lambda} - \dot{\Lambda}^2 \right]
\]

\[
G_0^1 = 2r^{-1} e^{-2\Lambda} \dot{\Lambda}
\]

\[
T_{\mu}^{\gamma} = (\rho + P) u_{\mu} u^{\gamma} + P g_{\mu}^{\gamma} + \frac{1}{4\pi} \left[ F_{\mu\alpha} F^{\alpha\gamma} - \frac{1}{4} g_{\mu}^{\gamma} F_{\beta\gamma} F_{\beta\gamma} \right]
\]
Derivation of oscillation equation 2

\[ \delta \Lambda = - (\Phi_0' + \Lambda_0') \xi \]

\[ \delta \rho = - \xi \rho_0' - (\rho_0 + P_0) \frac{e^{\Phi_0}}{r^2} \left( r^2 e^{-\Phi_0} \xi \right)' \]

\[ \delta \Phi' = 4\pi r e^{2\Lambda_0} \delta P + 2\Phi_0' \delta \Lambda + r^{-1} \delta \Lambda - \frac{Q_0 \delta Q e^{2\Lambda_0}}{r^3} \]

\[ \delta P = \frac{dP_0}{d\rho_0} \delta \rho = - \xi P_0' - \frac{\gamma P_0 e^{\Phi_0}}{r^2} \left( r^2 e^{-\Phi_0} \xi \right)' \]

Energy-momentum conservation:

\[ e^{2\Lambda_0 - 2\Phi_0} (\rho_0 + P_0) \dot{\nu} + \delta P' + \frac{Q_0 Q_0' \xi'}{4\pi r^4} + \frac{Q_0 Q_0'' \xi}{4\pi r^4} \]

\[ + \frac{Q_0' \xi}{4\pi r^4} + \Phi_0' (\delta \rho + \delta P) + (\rho_0 + P_0) \delta \Phi' = 0 \]
The oscillation equation

\[ \sigma^2 e^{\lambda_0 - \nu_0} (\frac{1}{\rho_0 + \epsilon_0}) \xi \frac{4}{r} \frac{d}{dr} \xi - e^{-(\lambda_0 + 2\nu_0)/2} \frac{d}{dr} \left[ e^{(\lambda_0 + 3\nu_0)/2} \frac{\gamma \rho_0}{r^2} \frac{d}{dr} \left( r^2 e^{-\nu_0/2} \xi \right) \right] \]

\[ + \frac{8\pi G}{c^4} e^{\lambda_0} \rho_0 (\rho_0 + \epsilon_0) \xi - \frac{1}{\rho_0 + \epsilon_0} \left( \frac{d}{dr} \rho_0 \right)^2 \xi. \]

\[ \omega^2 e^{2\Lambda_0 - 2\Phi_0} (\rho_0 + P_0) \xi = -e^{-\Lambda_0 - 2\Phi_0} \left[ e^{\Lambda_0 + 3\Phi_0} \frac{\gamma P_0}{r^2} \left( r^2 e^{-\Phi_0} \xi \right)' \right]' \]

\[ - (\rho_0 + P_0) \frac{\Phi_0'}{2} \xi + 4r^{-1} \xi P_0' + 8\pi (\rho_0 + P_0) \xi e^{2\Lambda_0} P_0 \]

\[ + (\rho_0 + P_0) r^{-4} \xi e^{2\Lambda_0} Q_0^2 \leftarrow \text{CHARGE TERM} \]
Neutral hybrid stars – Gibbs constr.

\[ \Omega_{QM} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2} (1 - a_4) + B_{\text{eff}} \]

\[ m_s = 100 \text{ MeV}, a_4 = 0.8 \]

Charged hybrid stars – Gibbs constr.

Hadronic phase: relativistic mean-field model, TM1 parameter set
Quark phase: MIT bag model, \( m_s = 100 \text{ MeV} \), \( a_4 = 0.8 \), \( B^{1/4} = 200 \text{ MeV} \)

\[ x = 10^{19} \frac{N_c}{N_b} \]

Charged strange stars

Quark phase: MIT bag model, $m_s = 100 \text{ MeV}$, $a_4 = 1.0$, $B^{1/4} = 140 \text{ MeV}$

$$x = 10^{19} \frac{N_c}{N_b}$$

Schwarzschild-like coordinates

\[ G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T^{\mu\nu};_{\gamma} = 0 \]

constrain equations:

\[ \Lambda' = 4\pi r (\rho + P) e^{4\Lambda} u^2 + 4\pi r \rho e^{2\Lambda} + \frac{1 - e^{2\Lambda}}{2r} + \frac{Q^2 e^{2\Lambda}}{2r^3} \]

\[ \Phi' = 4\pi r (\rho + P) e^{4\Lambda} u^2 + 4\pi r P e^{2\Lambda} + \frac{e^{2\Lambda} - 1}{2r} - \frac{Q^2 e^{2\Lambda}}{2r^3} \]

evolution equations:

\[ \frac{\partial P}{\partial t} + c_{11} \frac{\partial P}{\partial r} + c_{12} \frac{\partial u^1}{\partial r} + c_{13} = 0 \]

\[ \frac{\partial u^1}{\partial t} + c_{21} \frac{\partial P}{\partial r} + c_{22} \frac{\partial u^1}{\partial r} + c_{23} = 0 \]

\[ \xi = \frac{\partial \rho}{\partial P} - e^{2\Lambda} u^2 + \frac{\partial \rho}{\partial P} e^{2\Lambda} u^2 \]

\[ c_{11} = \frac{1}{\xi} \left( \frac{\partial \rho}{\partial P} - 1 \right) e^{2\Phi} u_0 u^1 \]

\[ c_{12} = \frac{\rho + P}{\xi u^0} \]

\[ c_{13} = \frac{e^{\Phi} u^1}{2r^3 (1 + e^{2\Lambda} u^2)} \xi \left[ 2e^{\Lambda} Q r P_{ch} + 2e^{3\Lambda} Q r P_{ch} u^2 + 5e^{\Phi} r^2 (\rho + P) u^0 \right. \]

\[ + \left. e^{2\Lambda + \Phi} (\rho + P) \left( Q^2 + r^2 \left( -1 - 8\pi P r^2 + 4u^2 \right) \right) u^0 \right] \]

\[ c_{21} = \frac{1}{(\rho + P) \xi} \frac{\partial \rho}{\partial P} e^{2\Phi} u_0 u^0 \]

\[ c_{22} = c_{11} \]

\[ c_{23} = -\frac{e^{\Phi - 2\Lambda}}{2r^3 (\rho + P) \xi} \left[ \frac{\partial \rho}{\partial P} \left( 2e^{\Lambda} Q r P_{ch} + 2e^{3\Lambda} Q r P_{ch} u^2 + e^{\Phi} r^2 (\rho + P) u^0 \right. \right. \]

\[ + \left. \left. e^{2\Lambda + \Phi} (Q^2 - r^2 - 8\pi P r^4) (\rho + P) u^0 \right) + 4e^{2\Lambda + \Phi} r^2 (\rho + P) u^1 u^0 \right] \]
1D code

- 2nd order time, 4th order space accuracy

- constrained evolution

- implicit discretization for evolution equations (iterated Crank-Nicolson)

- bicgstab algorithm for sparse linear system

- strengths: good tracking of stellar surface (Lagrangian coordinates), CFL condition always fulfilled, for spheres faster than multidimensional codes, better resolution possible

- weaknesses: unable to penetrate Schwarzschild radius (coordinate singularity), limited to barotropic fluids, limited to spheres
Generalized Zeldovich EoS

\[ P = a (\rho - \rho_0) \]

Compactness \( x = \frac{2M}{R} \) of maximum mass and maximum radius configuration depends only on \( a \).

Different choices for \( \rho_0 \) preserve compactness (for max m. and max r. configurations).
Critical phenomena

Maximum mass configuration (km): 2.984016494
Selected masses (km): 2.962963, 2.963477514, 2.9635508309, 2.96356153, 2.96358311021
Gravitational collapse to black hole, initially outgoing velocity
Gravitational collapse to black hole, initially outgoing velocity
Gravitational collapse to black hole, initially outgoing velocity
Parametrization for barotropic EoS

\[ f = \sum_{k=1}^{3} \frac{a_{k1} + a_{k2}x + a_{k3}x^2}{\left(1 + e^{a_{k5}(a_{k6}+x)}\right)\left(1 + a_{k4}x\right)} \]

\[ f = \log(\text{density}) \]

\[ x = \log(\text{pressure}) \]
Results

- generalization of Chandrasekhar's equation to charge

- enhancement of masses of hybrid and strange stars due to Coulomb repulsion

- both Coulomb interaction and deconfinement lead to lower frequencies of radial eigenmodes at given central density

- A naïve application to stars with sharp density discontinuity contradicts with the static stability criterion.

Outlook

- Are there hollow charged spheres?

- introduce viscosity

- Critical phenomena with realistic microphysics
Radial oscillations in neutral and charged compact stars

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EMMI Workshop "Quark Matter in Compact Stars", FIAS, Frankfurt am Main, October 7-10, 2013
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Why care about charge in compact stars?

- even if the global charge is zero, there can be separation of charges inside / freedom to arrange charge within

- charge might prevent gravitational collapse and support supermassive stars

- charged balls might be natural candidates to form extremal black holes
Radial oscillations

- spherical symmetry preserved
- type of oscillation described by number of nodes
- Sturm-Liouville equation as in Newtonian gravity
- discrete set of frequencies given by boundary conditions: \( \xi(r=0) = 0, \Delta P(r=R) = 0 \)
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