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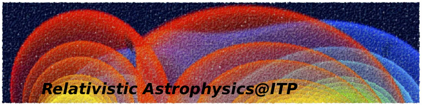

Stability of Differentially Rotating Neutron Stars

Lukas R. Weih

30. Oct 2017

AstroCoffee Seminar, Goethe-University, Frankfurt a.M.

Results available as arXiv preprint: L. R. Weih, E. R. Most, and L. Rezzolla (arXiv:1709.06058)

Motivation

Why differentially rotating neutron stars?

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NS-NS binary merger

$\downarrow$

differentially rotating NS

$\swarrow$ $\searrow$

prompt collapse to BH? stable NS?

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Why equilibrium solutions?

Full general-relativistic simulation	$\approx$ 200000 core hours
Sequence of equilibrium solutions	$\lesssim$ 1 core hour

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Why equilibrium solutions?

Full general-relativistic simulation	$\approx$ 200000 core hours
Sequence of equilibrium solutions	$\lesssim$ 1 core hour

$\Rightarrow$ Use equilibrium models of dif. rot. NS to determine stability of merger remnant

Overview

- Short review of equilibrium solutions of neutron stars (NS)
  - How to include differential rotation
  - Stability of non- and uniformly rotating models
- Stability of Differentially Rotating NSs
- Universal Relation for Determining the Turning Point
- Maximum Mass of Differentially Rotating Neutron Stars
- Validity of approximating the merger remnant
- Summary and Outlook

General Relativity and the Metric of NSs

Axisymmetric, stationary metric:

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}(d\Phi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2)$$

Solve Einstein eq. together with eq. of hydrostationary equilibrium

$$\ln h - \ln u^t + \int_{\Omega_{pole}}^{\Omega} F(\Omega')d\Omega' = \nu_{pole} \qquad \left(\text{with } h = \frac{e+P}{\rho}\right)$$

and zero-temperature EOS

$$P = P(\rho) \tag{1}$$

iteratively until convergence is reached.

$\Rightarrow$ Specify ρ_c, r_{ratio}, $F(\Omega)$ and `RNS`-code yields $\approx 10\,\mathrm{stars/min}$

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The Rotation Law

$F(\Omega) = A^2(\Omega_c - \Omega)$ (j-constant law) determines the rotation profile.

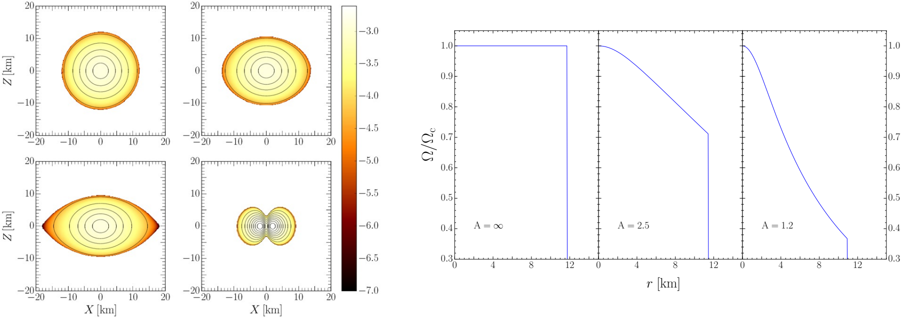

Figure: *Left:* Cross section of rest-mass density in x-z-plane.
Right: Rotation profiles depending on degree of differential rotation, A.

Equilibrium Solutions

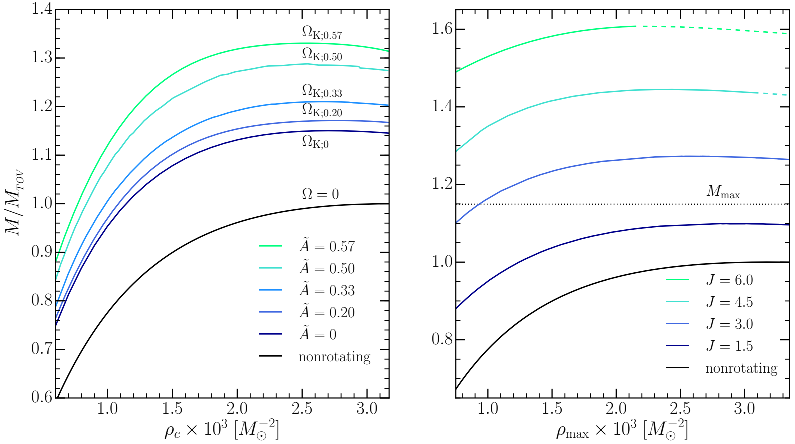

Figure: *Left:* Rest mass over central rest-mass density. Non-rotating sequence and mass-shedding limit for uniformly and dif. rotating models. *Right:* Sequences of constant angular momentum for high degree of differential rotation.

The Turning Point Criterium

- Along a sequence (parameterized ρ_0) of constant angular momentum, the point of maximum mass marks the onset of secular instability (collapse to BH).
- For rotating (uniformly) NSs neutral-stability line (F-mode=0) shifts to smaller ρ_0.
- What about differentially rotating NSs?

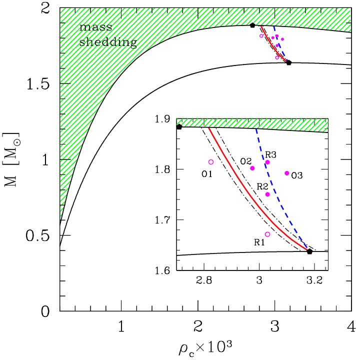

Figure: Turning-point line (blue) and neutral-stability line (red) for uniformly rot. models.

Stability of Differentially Rotating NSs

Evolve selected models numerically in time:

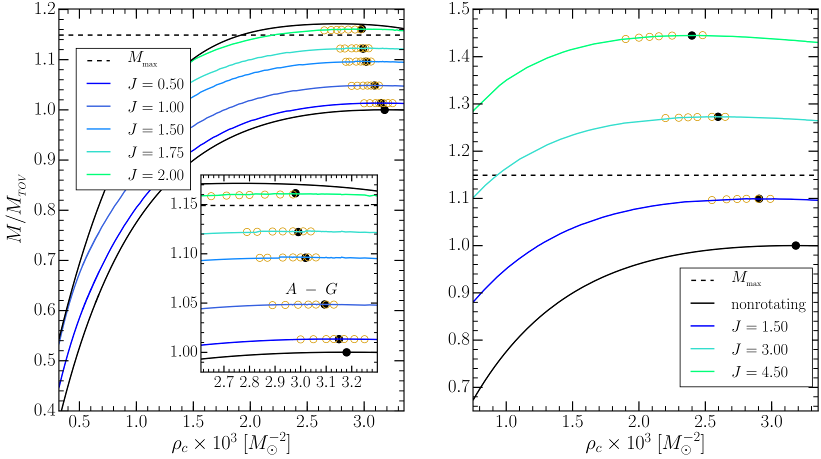

Figure: Lines of const. ang. mom. for moderate (*left*) and high (*right*) degree of dif. rot. Selected models are marked with open circles.

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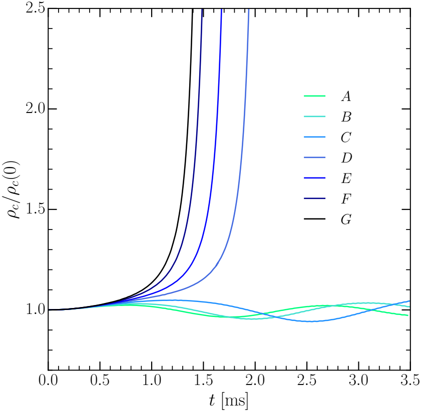


Figure: Time evolution of central rest-mass density for models *A*-*G*.

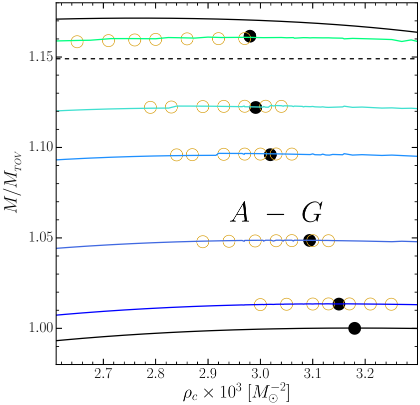
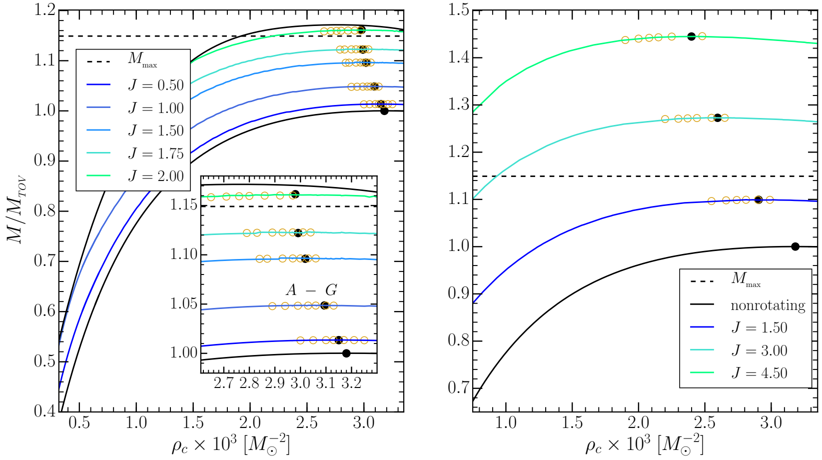


Figure: Models *A*-*G* in $M - \rho_0$ plane.

Stability of Differentially Rotating NSs

Evolve selected models numerically in time:



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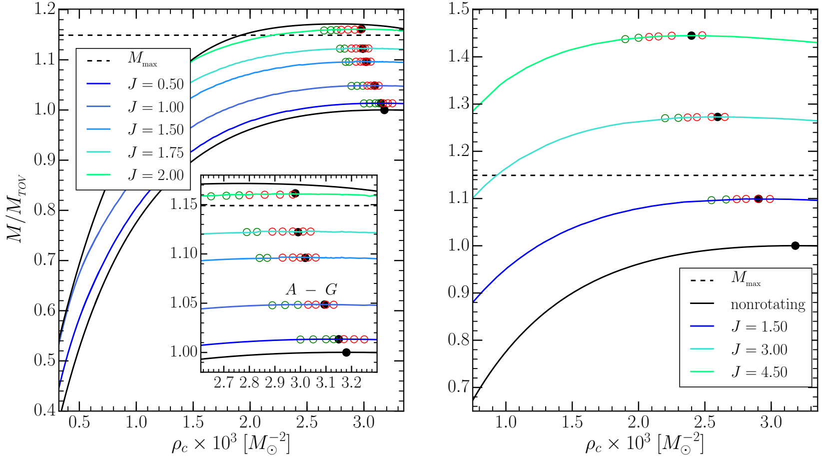
Stability of Differentially Rotating NSs

Evolve selected models numerically in time (○ stable, ○ unstable)



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Universal Relation for Determining the Turning Point

Determine turning-point mass, $M_{\mathrm{max,dr}}$, as function of dimensionless angular momentum $j = J/M^2$ for different EOSs:

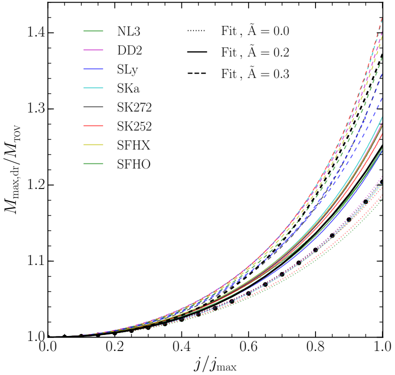

Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.

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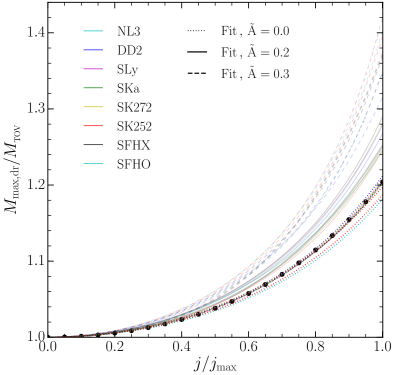

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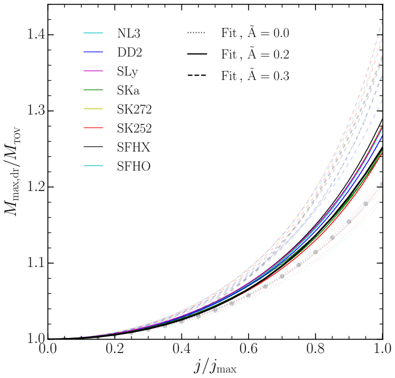

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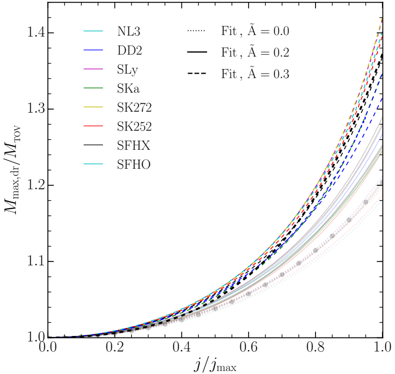


Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.

Universal Relation for Determining the Turning Point

For uniform rotation (lower bundle) from Breu & Rezzolla (2016):

$$M_{\max} = \left(1 + a_1 \left(\frac{j}{j_{\max}}\right)^2 + a_2 \left(\frac{j}{j_{\max}}\right)^4\right) M_{\mathrm{TOV}}$$

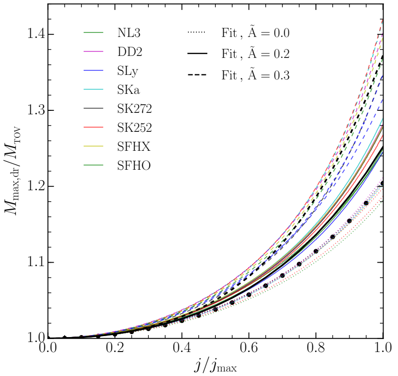
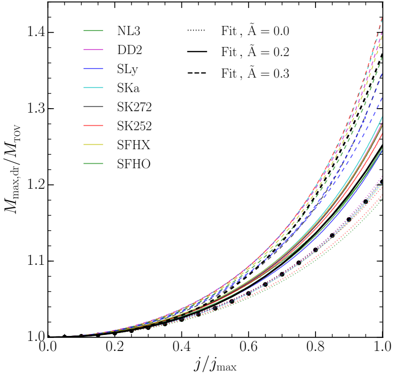


Figure: Turning-point mass over normalised dimensionless angular momentum. Three bundles for three degrees of dif. rot.

Universal Relation for Determining the Turning Point

For differential rotation (upper bundles):

$$M_{\mathrm{max,dr}} = \left(1 + a_1(A)\left(\frac{j}{j_{\mathrm{max}}}\right)^2 + a_2(A)\left(\frac{j}{j_{\mathrm{max}}}\right)^4\right) M_{\mathrm{TOV}}$$



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$$= \left(1 + a_1\left(\frac{j}{j_{\mathrm{max}}}\right)^2 + (b_0 + b_1\tilde{A}^2 + b_2\tilde{A}^4)\left(\frac{j}{j_{\mathrm{max}}}\right)^4\right) M_{\mathrm{TOV}}$$

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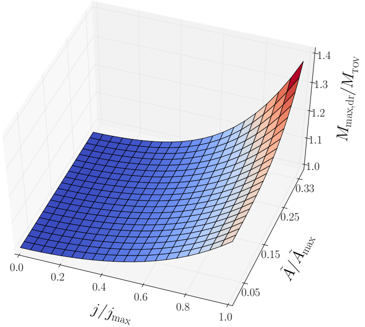

Figure: Normalised turning-point mass over dimensionless angular momentum and degree of differential rotation.

Maximum mass of differentially rotating neutron stars

For $j = j_{\max}$ the maximum mass for given $\tilde{A}$ is obtained:

$$M_{\max,\mathrm{dr}}\left(j_{\max},\tilde{A}\right)/M_{\mathrm{TOV}} = 1.2 + c_1\left(\frac{\tilde{A}}{\tilde{A}_{\max}}\right)^2 + c_2\left(\frac{\tilde{A}}{\tilde{A}_{\max}}\right)^4$$

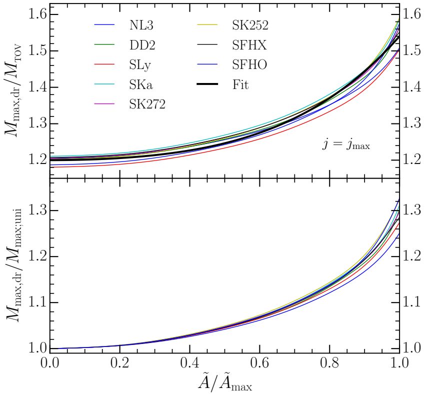

Figure: Maximum mass as function of degree of differential rotation.

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$$\Rightarrow \text{For } \tilde{A} = \tilde{A}_{\max} : M_{\max,\mathrm{dr}} = (1.54 \pm 0.05) M_{\mathrm{TOV}} .$$

Validity of approximating the merger remnant

- Actual rotation profile not monotonically decreasing

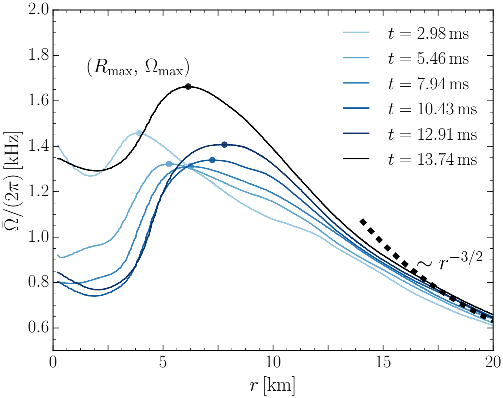

Figure: Rotation profile of BNS merger remnant (Hanauske et al. (2017)).

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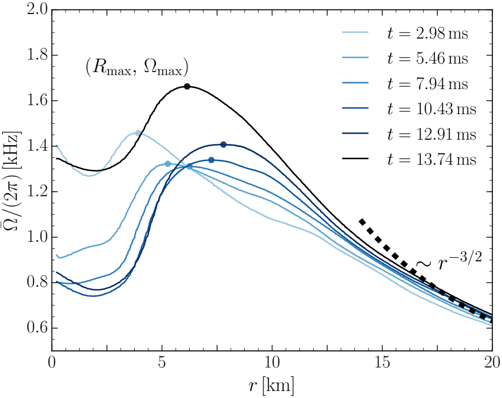
- But, which $F(\Omega)$ to chose?



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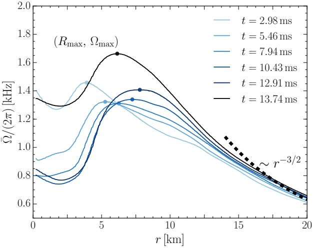

Figure: Rotation profile from BNS merger remnant *Hanauske et al (2016)*.

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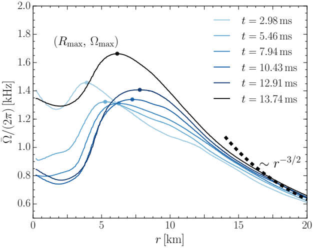

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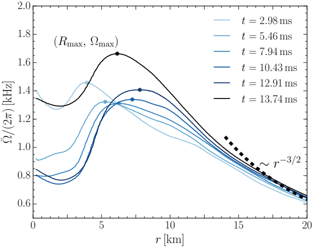
- Solve iteratively until convergence of $F(\Omega) = u^t u_\phi$ is reached



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Comparison of equilibrium model to merger remnant

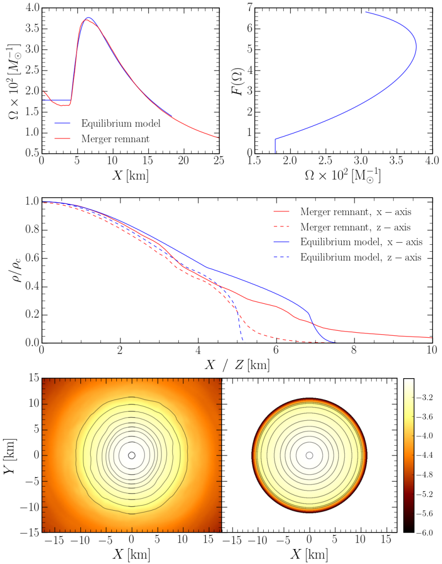

Figure: Equilibrium model with realistic rotation profile compared to actual data of a remnant from a BNS merger simulation.

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- Stability criterium for uniformly NS can be extended also to differential rotation.
- Turning point as valid approximation for stability limit. HMNS are not unconditionally unstable.
- Calculate turning-point mass for given j and A simply in terms of M_{TOV}.
  $\Rightarrow$ Maximum mass possible with dif. rot.: $(1.54 \pm 0.05) M_{\mathrm{TOV}}$.

Outlook

Recently, $F(\Omega)$ proposed to compute more realistic equilibrium solutions *Uryu et al (2017)*.

$\Rightarrow$ Sequences of const. ang. mom. have a turning point.

$\Rightarrow$ Stability criterion also true for realistic equilibrium models?