Critical behaviour: Head-on collision of rotating relativistic neutron stars

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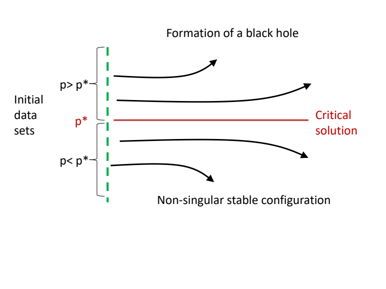


- Universality
- Scale-invariance or time independence

Type I

Lifetime of the metastable solution:

$$t_p = -\gamma \ln |p - p^*| + C$$

γ: Universal critical index

Type II

Black hole mass scaling relation:
$M_{Bh} = C|p - p^*|^\gamma$

Scale-invariance:
$\Phi^*(r,t) = \Phi^*(r \cdot e^\Delta, t \cdot e^\Delta)$

- Implosion of massless scalar fields in spherical symmetry (1993)
- Collapse of axisymmetric gravitational waves in pure vacuum spacetimes (1993)
- Collapse of a massive scalar field
- Collision of non-rotating neutron stars (2007)
- Collapse of non-rotating and rotating radiation fluids (1994, 2016)

Kerr black holes

axially-symmetric solution; characterized by its mass and spin

Event horizon at $r_+ = M + \sqrt{M^2 - (J/M)^2}$

Cosmic Censorship Hypothesis

Gravitational collapse from physically reasonable, generic set of initial conditions never gives rise to a 'naked' singularity which is not clothed by an event horizon.

Spin restriction

$$J/M^2 \leq 1$$

Numerical set-Up

Einstein Toolkit: Finite differencing, HRSC, CCZ4, AMR, yz-reflection symmetry

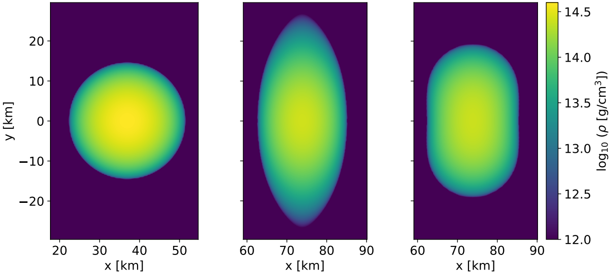
Initial data

Standard superposition method:

$$g_{\mu\nu} \approx g_{\mu\nu}^{(Star\ \ A)} + g_{\mu\nu}^{(Star\ \ B)} - \eta_{\mu\nu}$$

Equation of state

$$\mathrm{p} = \mathrm{K}\rho^{\Gamma}$$



TOV

$J/M^2 = 0$

ID: pizzaTOV-Solver

Uniform rotation

$\Omega = constant$

$J/M^2:\ 0.6 - 0.9$

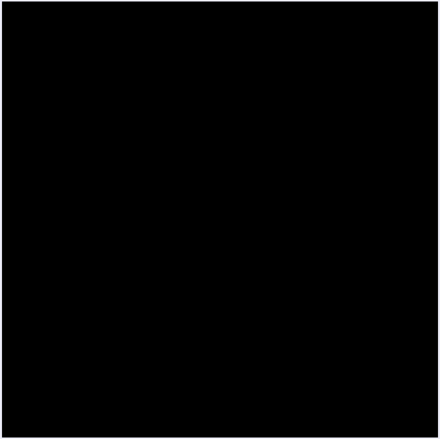
ID: RNSID

Differential rotation

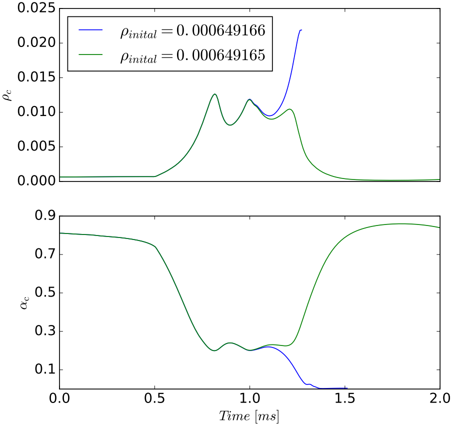
$\Omega - \Omega_C = \frac{F(\Omega)}{A^2}$

$J/M^2:\ 1 - 2$

ID: RNSID

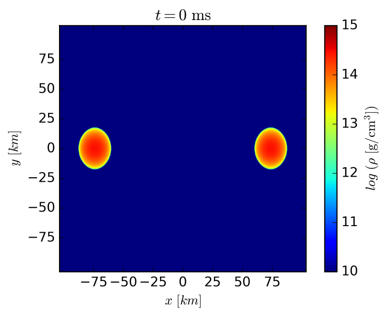


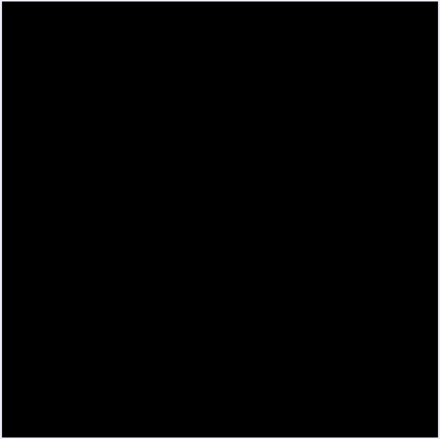
Critical central rest mass density:
$p^* = \rho_c^* = 0.0006491655 \mathrel{\widehat{=}} 4.01 \cdot 10^{14}\ [g/cm^3]$



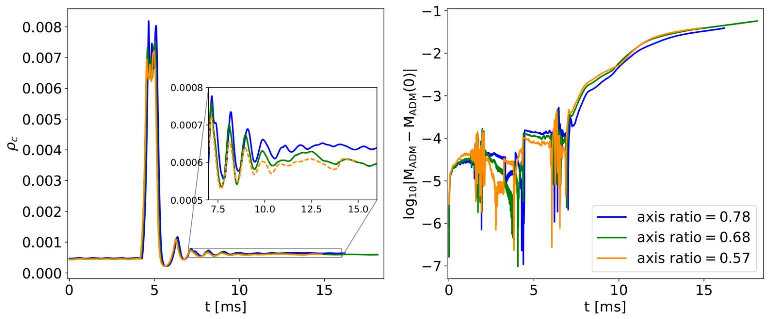
Initial velocity	0.15
Initial distance	73.5 km

Initial velocity	0
Initial distance	150 km
$M_{initally\ \ per\ \ NS}$	0.9 - 1.0 $M_{\odot}$
$J_{initally\ \ per\ \ NS}$	0.5 - 0.9
J/M^2	0.6 - 0.9



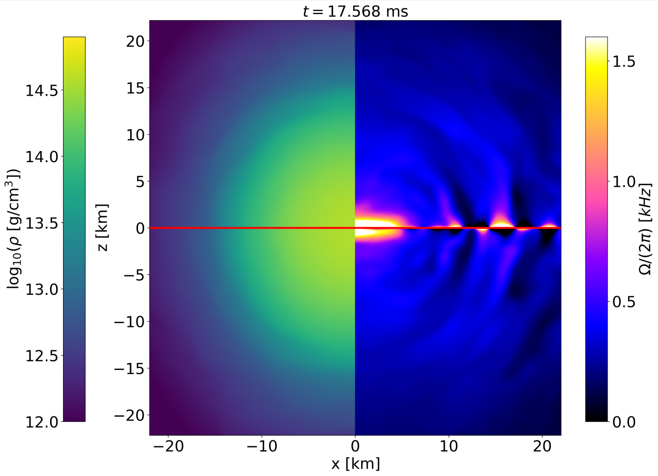


Subcritical long-run simulations



(r_p/r_e)	0.57		0.68		0.78	
	sub	super	sub	super	sub	super
$\mathrm{M_{ADM}}$	1.0011020	1.0011024	0.973460	0.973461	0.9328829	0.9328830
$\mathrm{J_{ADM}}$	0.899865	0.899866	0.753060	0.753062	0.5581656	0.5581657
$\left(\mathrm{J/M^2}\right)$	0.8978850	0.8978853	0.794681	0.794682	0.64137023	0.64137021
M_{max}	1.70		1.68		1.66	

Head-on collision of uniformly rotating stars: rest-mass density and angular velocity



Calculation of the critical index

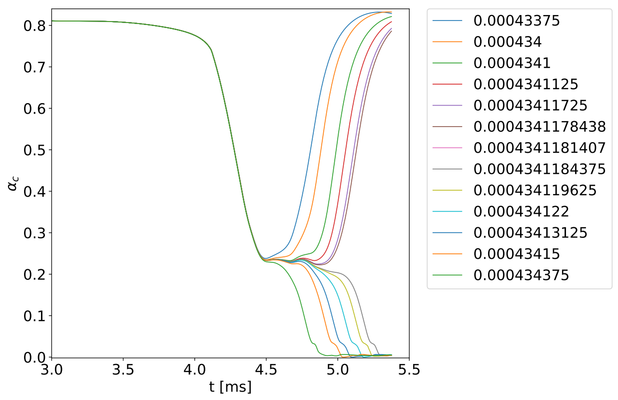


Figure: Evolution of the central lapse function for collisions of stars with $r_p/r_e = 0.57$. The legend on the right side terms the initial central rest-mass densities.

Calculation of the critical index

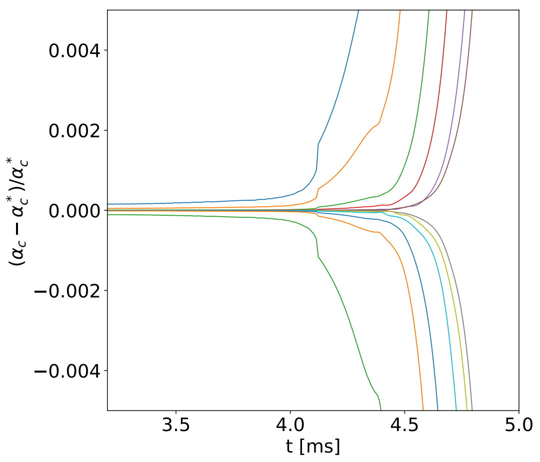


Figure: Evaluation of the lifetimes of the metastable phases

Calculation of the critical index

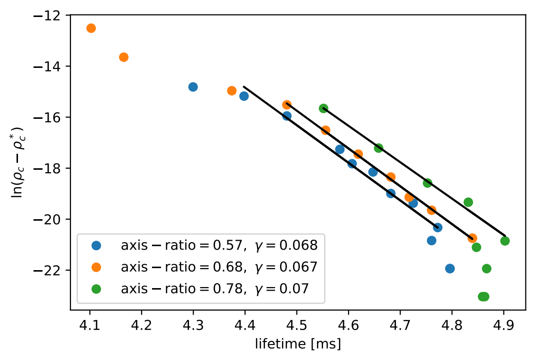


Figure: $\ln(\rho_c - \rho_c^*)$ vs. the lifetime of the metastable state with $r_p/r_e = 0.57,\ 0.68$ and 0.78. The slope of the linear fit gives the critical index.

Calculation of the critical index

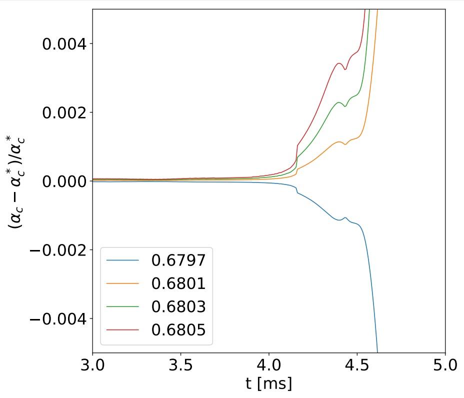


Figure: Evolution of $(\alpha_c - \alpha_c^*)/\alpha_c^*$ for collisions of stars with a fixed rest-mass density of 0.000448683595 but varying angular momentum which is adjusted by the axis ratio parameter as indicated in the

Calculation of the critical index

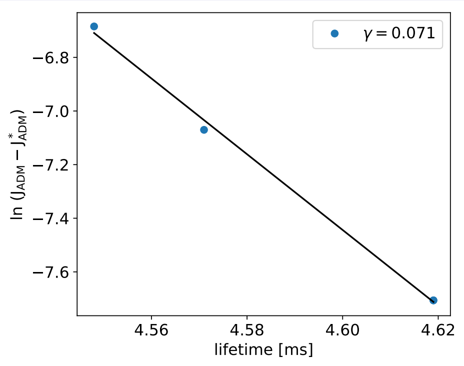


Figure: $\ln(J_{ADM} - J^*_{ADM})$ plotted against the lifetime of the metastable state determined for collisions of stars with $r_c = 0.000448683595$. The slope of the linear fit gives the critical index.

ρ_c	r_p/r_e	A_{diff}	Initial $\lvert J_{ADM}\rvert$	Initial M_{ADM}	Initial J/M^2
0.0004	0.6	0.9	1.15	1.10	0.96
0.00043	0.46	2.0	1.26	1.10	1.04
0.00042	0.37	1.3	2.03	1.29	1.21

Table: Properties of the investigated three differentially rotating stellar models.

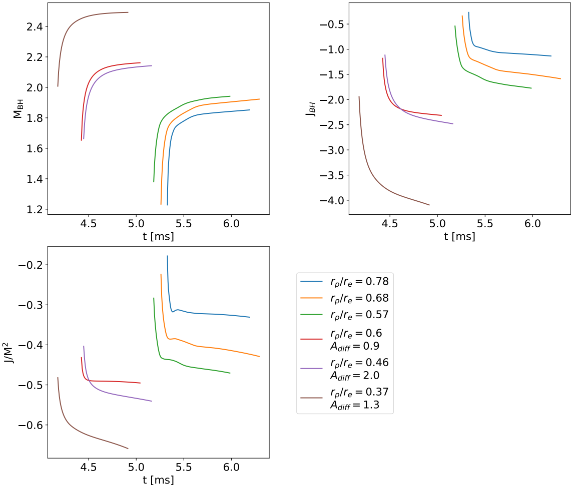
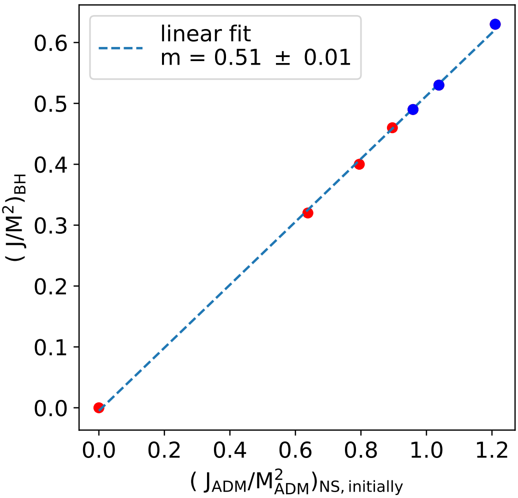


Figure: Supercritical head-on collision of uniformly and differentially rotating neutron stars.


linear fit
m = 0.51 ± 0.01
(J/M²)BH
(JADM/M²ADM)NS, initially
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Conclusions and Outlook

- Validation: Critical behaviour of colliding non-rotating stars
- Evaluated: Critical behaviour of colliding uniformly-rotating stars.
  Observed: universal critical index.
- Collision of differentially rotating stars: Still too far away from the $J/M^2 \approx 2$ models.

THANK YOU FOR YOUR ATTENTION

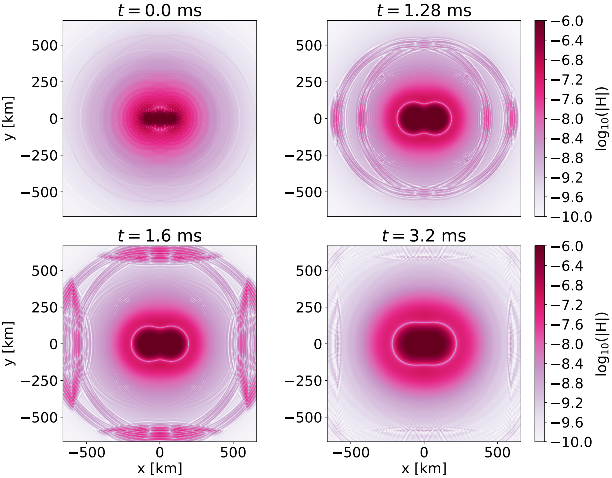


Figure: Hamiltonian constraint violation of a subcritical collision in the xy-plane - inner region.

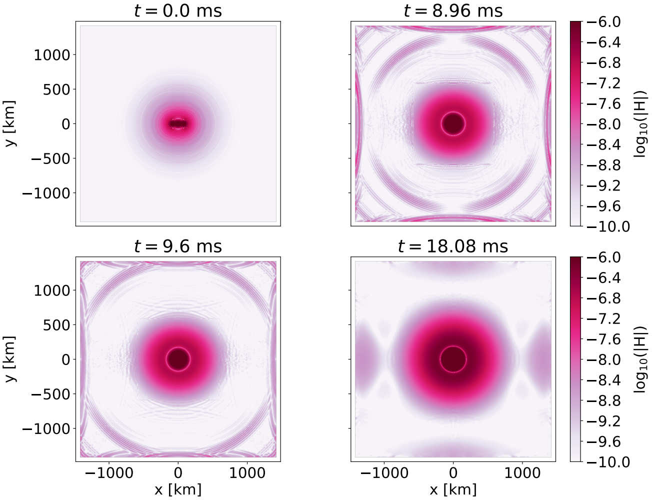
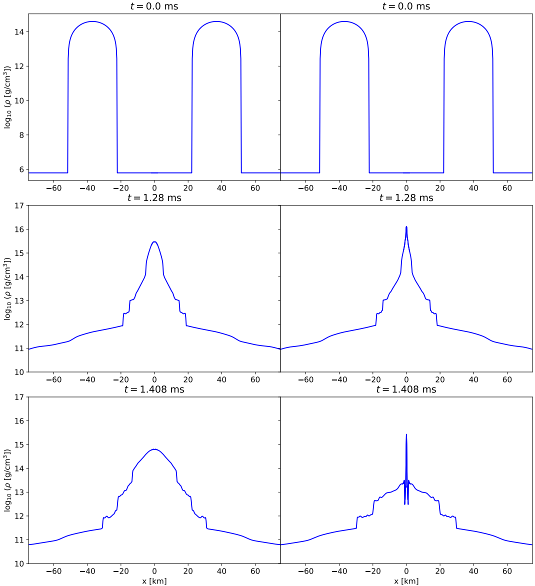
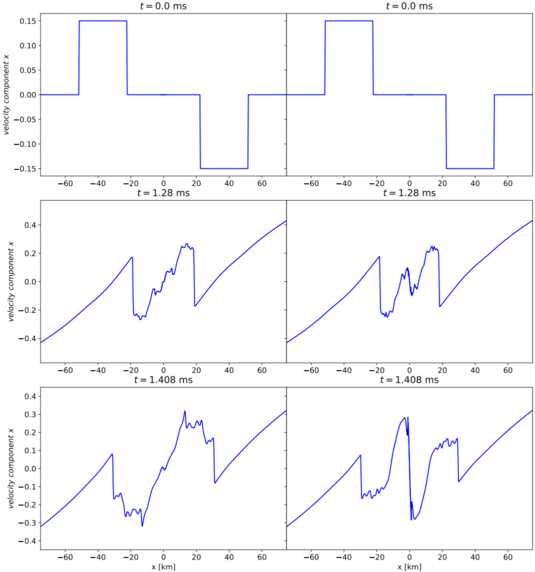


Figure: Hamiltonian constraint violation of a subcritical collision in the xy-plane - outer region.


$t = 0.0$ ms
$t = 0.0$ ms
$t = 1.28$ ms
$t = 1.28$ ms
$t = 1.408$ ms
$t = 1.408$ ms
$\log_{10}(\rho\ [g/cm^3])$
x [km]


$t = 0.0$ ms
$t = 0.0$ ms
$t = 1.28$ ms
$t = 1.28$ ms
$t = 1.408$ ms
$t = 1.408$ ms
velocity component x
x [km]