Current driving mechanism and role of the negative energies in Blandford-Znajek process

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Relativistic Jets

Cygnus A

Active Galactic Nucleus

\[ M_{\text{BH}} = 10^6 - 10^9 \, M_\odot \]

\[ L_j \lesssim L_{\text{Edd}} \sim 10^{46} \, M_8 \, \text{ergs}^{-1} \]

\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = 10 - 100 \]

- Energy source?
- Mass source?
- Acceleration
- Collimation
- Stability
Promising Scenario

- Energy injection into dilute region above BH → Relativistic speed
- Steady extraction of BH rotational energy (Blandford & Znajek 1977) → Poynting-dom jet
- Origin of jet matter debated (see KT & Takahara 2012)
- Lorentz force acceleration or magnetic dissipation
- Collimation by external pressure (many literatures; see Lyubarsky 2009)

\[ L_j = L_{EM} + L_{th} + \gamma \dot{M}_j c^2 \]
The high-resolution radio imaging is being improved, revealing the structure and composition of M87 jet.
Blandford & Znajek (1977)

- Slowly rotating Kerr BH
  \[ a = \frac{J}{M r_g c} \ll 1 \]
- Steady, axisymmetric
- Split-monopole B field
- Force-free approximation (Electromagnetically dominated)

\[ \rho \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{J}_p \parallel \mathbf{B}_p \quad \mathbf{E} \perp \mathbf{B} \]

(see also Beskin & Zheltoukhov 2013)
BZ process with large BH spin $\alpha$

(Komissarov 2004)

- Many other FF/MHD numerical studies show BZ process works with large $\alpha$. (e.g. Komissarov 01; Koide+ 02; McKinney 06; Barkov & Komissarov 08; Tchekhovskoy+ 11; Ruiz+ 12; Contopoulos+ 13; Nathanail & Contopoulos 14)
- It is proved analytically that $E = 0$ cannot be maintained for open field lines (KT & Takahara 14)

But the detailed mechanism of flux production is still debated
Vacuum Solution

(Wald 1974; Punsly & Coroniti 1989)

- Space-time rotation produces $E$, but not $B_\phi$
- $B_\phi$ requires $J_p$. What drives $J_p$?
Unipolar induction

\[ E = -V \times B \]

Pulsar winds (Goldreich & Julian 1969)

\[ \nabla \cdot S_p = -E \cdot J_p \]

Matter rotational energy reduced  

Energy source!
There is no matter-dominated region in BZ process

\[ \nabla \cdot \mathbf{S}_p = 0 \]

What drives \( J_p \) (across field lines) ??

(Comissarov 04; 09)

(Blandford & Znajek 1977)
Membrane Paradigm?

- Horizon is assumed as a rotating conductor (Thorne et al. 1986; Penna et al. 2013)
- But the horizon is causally disconnected (Punsly & Coroniti 1989)
- Current driving mechanism is unclear
- -> Mechanism producing the flux must work outside the horizon
Negative electromagnetic energy?

\[ S_p = e \nu_p > 0 \quad \text{for } e < 0 \text{ and } \nu_p < 0 \]

(Lasota et al. 2014; Kiode & Baba 2014)

- Electromagnetic energy density \( e \) in the Boyer-Lindquist coordinates can be negative for \( \Omega < \Omega_F \)

\[ -\alpha T^t_t = e = \frac{1}{8\pi\alpha} \left[ \alpha^2 B^2 + \gamma \varphi \varphi (\Omega^2_F - \Omega^2)(B^\theta B_\theta + B^r B_r) \right]. \]

- But \( \nu_p \) is not defined. The concept of advection of steady field is ambiguous.

- We showed \( e > 0 \) in the Kerr-Schild coordinates

\[ S_p = \frac{\mathbf{E} \times \mathbf{H}_\varphi}{4\pi} \]

(KT & Takahara 2016)
Current driven in a pair creation gap?

(Okamoto 2006)

This case could be relevant for the upcoming high-resolution radio observations and the observed high-variability gamma-rays.

M87 radio jet (Hada et al. 2016)

IC310 TeV gamma-rays (Aleksic et al. 2015, Science)
\[ ds^2 = g_{\mu \nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(\beta^i dt + dx^i)(\beta^j dt + dx^j), \]

\[ E^\mu = \gamma^{\mu \nu} F_{\nu \alpha} \xi^\alpha, \quad H^\mu = -\gamma^{\mu \nu *} F_{\nu \alpha} \xi^\alpha \]

Fields in the coordinate basis

\[ D^\mu = F^{\mu \nu} n_\nu, \quad B^\mu = -* F^{\mu \nu} n_\nu \]

Fields as measured by FIDOs/ZAMOs

\[ \nabla \cdot B = 0, \quad \partial_t B + \nabla \times E = 0, \]

\[ \nabla \cdot D = 4\pi \rho, \quad -\partial_t D + \nabla \times H = 4\pi J, \]

Electromagnetic energy equation

\[ \partial_t \left[ \frac{1}{8\pi} (E \cdot D + B \cdot H) \right] + \nabla \cdot \left( \frac{1}{4\pi} E \times H \right) = -E \cdot J, \]

Energy density

\[ E = \alpha D + \beta \times B, \]

\[ H = \alpha B - \beta \times D, \]

Poynting flux

(Landau & Lifshitz 1975; Komissarov 2004)
General conditions of magnetosphere

- Kerr spacetime with arbitrary spin $a$ (fixed)
- Axisymmetric
- Poloidal $B$ field (with arbitrary shape) threading the ergosphere
- Plasma with sufficient number density
  
  \[ \mathbf{D} \cdot \mathbf{B} = 0 \]

\[ (\mathbf{E} \cdot \mathbf{B} = 0) \]
Origin of Electromotive Force

\[ E = \alpha D + \beta \times B, \]

If \( E=0, H_\phi=\alpha B_\phi=0 \) (No ang. mom. or Poynting flux) along a field line,

\[ D = -\frac{1}{\alpha} \beta \times B_p \quad \Rightarrow \quad D^2 > B^2 \text{ for } \alpha^2 < \beta^2 \]

(in the ergosphere)

Then the force-free is violated, and the strong \( D \) field drives \( J_p \) across \( B_p \) (\( H_\phi \neq 0 \)), weakening \( D \) (\( E \neq 0 \)).

The origin of the electromotive force is ascribed to the **ergosphere**.

(KT & Takahara 2014, MNRAS; see also Komissarov 2004; 2009)
- From the regularity condition
  \[ H_\varphi \neq 0, \quad D^2 < B^2 \]
- Force-free approx. valid
- No \( J_p \) crosses field lines
- No AM/energy transferred from particles to electromagnetic field (unlike mechanical Penrose process)

Origin of Poynting flux? What drives \( J_p \)?
Origin of Schwarzschild spacetime

The source of the Schwarzschild gravitational field is the mass inside the horizon, but the outside of horizon cannot know it.

The spacetime metric is determined by the mass distribution at prior times.

Collapsing material

Event horizon

Light cones

Information of mass distribution

$t$

$r_H$

$r$
Process toward steady state: toy model

First consider a vacuum, and then begin injecting force-free plasma continuously between the two light surfaces.
Causal production of the flux

We derived junction conditions from Maxwell equations, and found current must cross field lines.

\[ \eta^r = \frac{-D^r_{\text{vac}}}{4\pi} \left| \begin{array}{c} V, \end{array} \right|_{R=0} \]

\[ V = \frac{1}{\sqrt{\gamma}} \left( H^\text{ff}_\varphi + 4\pi \sqrt{\gamma} \eta^\theta \right) \left| \begin{array}{c} D^\theta_{\text{ff}} - D^\theta_{\text{vac}} \end{array} \right|_{R=0} \]

\[ V = \frac{1}{\sqrt{\gamma}} \left( E^\text{ff}_\theta - E^\text{vac}_\theta \right) \left| \begin{array}{c} B^\varphi_{\text{ff}} \end{array} \right|_{R=0} \]

\[ \nabla \cdot S_p = -\partial_t e - E \cdot J_p \]

Fluxes are produced at the ingoing boundary.

Assumed to have steady-state structures.
Steady State

\[ \nabla \cdot L_p = -\partial_t l - (J_p \times B_p) \cdot m \]
\[ \nabla \cdot S_p = -\partial_t e - E \cdot J_p \]

\[ \nabla \cdot L_p = 0 \]
\[ \nabla \cdot S_p = 0 \]

The boundary (AM/energy source) does not affect the exterior

- No electromagnetic sources are required in the steady state
  *(partly because of no resistivity)*

- BH decelerates directly by Poynting flux (different from mechanical Penrose process)

**Diagram:**
- BL coordinates
- Light cones
- The boundary (AM/energy source) does not affect the exterior
Conclusion

• The current driving ($S_p$ production) mechanism in BZ process can be discussed only in the time-dependent state towards steady state

• In the steady state, $S_p$ needs no electromagnetic source. The steady currents can keep flowing in the ideal MHD condition. No gap is needed. The BH rotational energy is reduced directly by $S_p$ without being mediated by the negative energies.

• Our argument is based on some assumptions. Detailed plasma simulations are needed to validate it
Back-up slides
We consider that a small field-aligned electric field may appear in numerical simulations and in reality with small resistivity.
These 2-fluid analyses show the global violation of \( \mathbf{E} \cdot \mathbf{B} = 0 \)
Blandford & Znajek (1977)

- Kerr space-time
- Steady, axisymmetric
- Slowly rotating BH

\[ a = \frac{J}{Mr_g c} \ll 1 \]

- Split-monopole B field

\[ B^r \sqrt{\gamma} = \text{const.} \]

- Force-free approximation (Electromagnetically dom.)

\[ H_\varphi = \text{const.} \]

\[ E = -\Omega_F e_\varphi \times B \]

“Field line angular velocity”

\[ H_\varphi = -2\pi \Omega_F B^r \sqrt{\gamma} \sin \theta \]

Condition at infinity

At event horizon

\[ H_\varphi = 2\pi(\Omega_F - \Omega_H)B^r \sqrt{\gamma} \sin \theta \]

\[ \Omega_F = \Omega_H / 2 + O(a^3) \]
MHD model

Energy flux density

\[ S_p = 4\pi \rho c^2 \Gamma v_p \mathcal{E} > 0 \quad \text{for} \quad v_p < 0, \quad \mathcal{E} < 0 \]

Separation surface may be located outside the ergosphere.

- Cross-field (inertial drift) currents cannot produce all of \( S_p \)
- MHD simulations show the steady state without negative particle energy (Komissarov 2005)

Fig. 1.—Positions of the light surfaces \( r = r_{L}^{in}, r_{L}^{out} \) (solid lines) and the separation point \( r = r_S \) (broken lines) for a monopole geometry in the equatorial plane with \( a = 0.8m \). These points are determined by \( \Omega_F \).
Field lines threading equatorial plane

- $D^2 > B^2$ possible, creating AM flux $(H_\phi)$ & Poynting flux

\[
\nabla \cdot L_p = -(J_p \times B_p) \cdot m
\]

\[
\nabla \cdot S_p = -E \cdot J_p
\]

- For $D^2 \sim B^2$, particles are strongly accelerated in direction of $-\phi$, obtaining negative energies

- Analogous to the mechanical Penrose process

(KT & Takahara 2014, 2016)
Inflow of negative-energy particles

\[ S_p \]

\[ -U_t < 0, \quad U^r < 0 \]

\[ \partial_r \sqrt{\gamma} \left( -\alpha \rho_m U_t U^r \right) = \mathbf{E} \cdot \mathbf{J}_p < 0 \]
Znajek condition

\[ H_\varphi = -\alpha \sqrt{\frac{\gamma_\varphi \varphi}{\gamma_{\theta \theta}}} D_\theta \]  

BL coordinates

- Ohm’s law for the current flowing on the membrane (Thorne et al. 1986 “Membrane Paradigm”)

- Rather, it should be interpreted as displacement current (see also Punsly 2008)

\[ H_{\varphi}^{\text{ff}} = \sqrt{\gamma}(D_{\varphi}^{\text{ff}} - D_{\varphi}^{\text{vac}})V - 4\pi \sqrt{\gamma} \eta^\theta \]

\[ V = \frac{\pm \alpha}{\sqrt{\gamma_{\varphi \varphi}}} \sqrt{1 + \frac{4\pi \sqrt{\gamma} \eta^\theta}{H_{\varphi}^{\text{ff}}}}. \]

\[ \eta^\theta \to 0 \]

\[ \alpha D_{\text{vac}}^{\theta} \to 0 \]