SPATIAL GROWTH OF THE CURRENT-DRIVEN INSTABILITY IN ROTATING, RELATIVISTIC JETS AND THE ROLE OF MAGNETIC RECONNECTION

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Motivation for our work

• GRMHD simulations with a spinning black hole indicate jet production with a magnetically dominated high Lorentz factor spine with $v \sim c$, and a matter-dominated sheath with $v > c/2$ possibly embedded in a lower speed, $v << c$, disk/coronal wind.

• Numerical simulations indicate that the jet spine and sheath are accelerated and, in part, collimated by strong magnetic fields twisted in the rotating black hole ergosphere and in the accretion disk, respectively.
Relativistic jets on the largest scales are kinetically dominated. The large scale magnetic field may dissipate if the regular magnetic structure is destroyed as a result of a global MHD instability. The kink instability is one of the possible candidates.

The current carrying plasma columns containing strong toroidal fields are unstable to non-axisymmetric perturbations. Among these, the kink instability is the most dangerous one.
• In configurations with strong toroidal magnetic field, the current driven (CD) kink mode is unstable. This instability excites large scale helical motions that can strongly distort or even disrupt the system. Non-linear development of the kink instability might trigger the violent magnetic dissipation.

• In the absence of CD kink instability and resistive relaxation, helical structures may be attributed to the Kelvin–Helmholtz (KH) helical instability driven by velocity shear at the boundary between the jet and the surrounding medium (e.g., Hardee 2004, 2007) or triggered by precession of the jet ejection axis (Begelman et al. 1980).
• It is still not clear whether current driven, velocity shear driven, or jet precession is responsible for the observed structures, or whether these different processes are responsible for the observed twisted helical structures at different spatial scales in AGN jets.

Mizuno et al. (2009) studied the kink instability in relativistic systems. By relativistic they mean not only relativistically moving systems but any with magnetic energy density comparable to or greater than the plasma energy density, including the rest mass energy. In that paper, they presented three-dimensional results of the CD kink instability of a static plasma column.
• Basically, the instability of helically magnetized static plasma columns (or more generally, rigidly moving flows considered in the proper reference frame).

• Simulation results showed that the initial configuration was strongly distorted but not disrupted by the CD kink instability.

• The growth rate depended on the radial profile of the magnetic pitch and density. The plasma column was more unstable when the helicity of the magnetic field lines increased with radius (magnetic pitch parameter, \( P \equiv R B_z / B_\phi \), decreased with radius \( R \)) and when the density decreased with radius.
• The simulation grid is periodic along the axial direction.
• The grid is a Cartesian (x, y, z) box of size $4L \times 4L \times L_z$ with grid resolution of $L/40$. For case A, $L_z = 2L$, and for case B, $L_z = 3L$.
• Outflow boundary conditions on the transverse boundaries at $x = y = \pm 2L$. 
• In Mizuno et al. (2011), they investigated the influence of velocity shear on the CD kink instability and found that the kink propagates along the jet with speed and flow structure dependent on the radius of the velocity shear relative to the characteristic radius of the helically twisted force free magnetic field.

• two different jet velocities: slow, $v_j = 0.2c$ and fast, $v_j = 0.3c$. In order to study the effect of the velocity, simulations with four different velocity shear surface radii: $R_j = 1/8L$, $1/4L$, $1/2L$, $1L$ and characteristic radius, $a = (1/4)L$. 
• In Mizuno et al. (2012), they studied the influence of jet rotation and differential motion on the CD kink instability.

• In accordance with linear stability theory, development of the instability depends on the lateral distribution of the poloidal magnetic field.

• When the profile of the poloidal field is shallow, the instability develops slowly and eventually saturates. When the profile of the poloidal field was steep and the magnetic pitch in the inner portion of the jet was sufficiently small, i.e., the magnetic field was sufficiently tightly twisted, multiple growing helical wavelengths disrupted the initial configuration.
• In order to investigate the effect of jet rotation, simulations with four different angular velocities, $\Omega_0 = 1, 2, 4,$ and 6 were performed.

• fixed boundary conditions on the transverse boundaries at $x = y = \pm 3L$ to maintain jet rotation. The simulation grid is periodic along the axial $z$-direction. The grid is a Cartesian ($x, y, z$) box of size $6L \times 6L \times L_z$. $L_z$ is the axial grid length. In most cases, we choose the axial grid length, $L_z = 3L$. The grid resolution is the same in all directions with $L/40$.

• O’Neill et al. (2012) have also investigated the CD kink instability in a local, commoving frame using the three-dimensional RMHD module within the Athena code.
• Mizuno et al. (2014) : a systematic study of the spatial development of the CD kink instability in a relativistic jet via three-dimensional RMHD simulations using a nonperiodic computational box.

• A non-periodic simulation box that is longer in the jet propagation direction. The computational domain is $6L \times 6L \times 20L$ in a Cartesian ($x$, $y$, $z$) coordinate system with a grid resolution of $\Delta L = L/40$ for the transverse $x$- and $y$-directions and $\Delta L = L/20$ for the axial $z$-direction.

• Outflow boundary conditions on all surfaces except the inflow plane at $z = 0$. In the simulations, a pre-existing jet flow is established across the computational domain. This set up represents the case in which the jet is in equilibrium with an external medium far behind the leading-edge.
• the jet to be mildly relativistic, \( v_j = 0.2c \),

• simulations with velocity shear radii of \( R_j = L/8 \) and \( L \) at which a sharp transition from jet with speed \( v = v_j \) to stationary ambient medium with \( v = 0 \) occurs.
Our attempt

• To extend the works of Mizuno et al. (2014), coupling with that of Mizuno et al. (2012).
• The code setup for spatial development of the CD kink instability is similar to the setup as mentioned in Mizuno et al. (2014).
• The computational domain is $8L \times 8L \times 30L$ in a cartesian $(x, y, z)$ coordinate system with grid resolution of $L/30$ for the transverse $x$- and $y$-directions and $L/6$ for the axial $z$-direction.
• The outflow boundary conditions have been imposed on all surfaces except the inflow plane at $z=0$.
• A pre-existing jet is established across the computational domain.
\[ B_z = \frac{B_0}{[1 + (R/a)^2]^\alpha}, \]

\[ B_\phi = -\frac{B_0 (R/a)[1 + (\Omega R/a)^2]^{1/2}}{[1 + (R/a)^2]}. \]

*R is the radial position in cylindrical coordinates,*
*B₀* parameterizes the magnetic field,
*a is the characteristic radius of the magnetic field = L/4 (the toroidal field component is a maximum at a for constant magnetic pitch),
*a is a pitch profile parameter.*
*a = 1 which gives constant magnetic pitch and magnetic helicity.*
\[
\Omega = \begin{cases} 
\Omega_0 & \text{if } R \leq R_0 \\
\Omega_0 \left(\frac{R_0}{R}\right)^\beta & \text{if } R > R_0,
\end{cases}
\]

R is the radial position in cylindrical coordinates normalized by a simulation scale unit \( L \equiv 1 \),
\( \Omega_0 \) is the angular velocity amplitude,
\( R_0 \) is the radius of the core,
\( \beta \) is an angular velocity profile parameter.
In simulations, we choose \( R_0 = (1/4)L \), and \( \beta = 1 \).
a low gas pressure medium with pressure decreasing radially, similar to the angular velocity decrease with radius, and with $p_0 = 0.02$ in units of $\rho_0 c^2$.

The equation of state is that of an ideal gas with $p = (\Gamma - 1)\rho e$, where $e$ is the specific internal energy density and the adiabatic index $\Gamma = 5/3$. 
The decreasing density profile,

$$\rho = \rho_1 \sqrt{B^2 / B_0^2}$$

The increasing density profile,

$$\rho = \rho_1 \sqrt{B_0^2 / B^2}$$

Where, $$\rho_1 = 0.8 \rho_0$$

The magnetic field amplitude, $$B_0 = 0.7 \text{ in units of } \sqrt{4\pi \rho_0 c^2}.$$
• Jet velocity, $v_j = 0.9c$
• Velocity shear radius = $1L$
• To break the equilibrium state, a precessional perturbation is applied at the inflow using a transverse velocity component with a magnitude of $v_\perp = 0.01v_j$ and a characteristic wavelength $\lambda_j = 3L$
The time evolution of the volume-averaged relativistic energies within a cylinder of radius $R/L < 2.0$ is an indicator of the growth of the CD kink instability. $E_{\text{kin,xy}}$ is a volume-averaged kinetic energy transverse to the $z$-axis

$$E_{\text{kin,xy}} = \frac{1}{V_b} \int_{V_b} \frac{\rho v_x^2 + \rho v_y^2}{2} \, dx \, dy \, dz$$
The volume averaged relativistic electromagnetic energy,

\[ E_{EM} = \frac{1}{V_b} \int_{V_b} \frac{B^2 + [v^2 B^2 - (v \cdot B)^2]}{2} dx dy dz. \]
In order to study the effect of jet rotation, simulations are performed with three different angular velocity amplitudes: 1, 2, and 4.
Heavy jet, angular velocity amplitude is 1. Images at $t = 50$ and 100.
Heavy jet, angular velocity amplitude is 4. Images at t = 50 and 70.
Light jet, angular velocity amplitude is 1. Images at $t = 80$ and $150$. 
Light jet, angular velocity amplitude is 4. Images at $t = 80$ and $150$
Heavy jet with angular velocity amplitude 2.
Images at $t = 0, 50, 100$ and $136$
Light jet with angular velocity amplitude 2.
Images at $t = 0, 50, 100$ and $150$
Heavy jet with angular velocity amplitude 1.
Images at $t = 100$. 

- (a) Density
- (b) $B_y$
- (c) $v_x$
- (d) $v_z$
Heavy jet with angular velocity amplitude 4. Images at $t = 70$. 
Light jet with angular velocity amplitude 1. Images at $t = 150$. 
Light jet with angular velocity amplitude 4. Image at $t = 150$. 
Kinetic energy and electromagnetic energy for heavy jet
Kinetic energy and electromagnetic energy for light jet
Possible sites of magnetic reconnection
Heavy jet, slow rotation
Heavy jet, fast rotation
Light jet, slow rotation
Light jet, fast rotation
Summary

• We studied the heavy as well as light jets with variation of angular velocity amplitude. Our work includes the regimes of linear as well as non-linear evolution of kink instability.

• The kink instability along the jet seem to cause the conversion of electromagnetic energy to kinetic energy. The light jet seems to be less distorted by the kink instability.
• The growth of kink instability seems to be weakly dependent on jet angular velocity. Beside the propagation speed of kink instability increases with increase in angular velocity of the jet.

• The location of large kink instability along the jet seems to be associated with the sites of magnetic reconnection and eventually rapid energy dissipation.

• This work is in progress and detailed analysis of the results remain to be done so comments are welcome.
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