Effects of neutron star dynamic tides on gravitational waveforms within the Effective One-Body approach

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Overview

• **Motivation**: potential to determine properties of ultra-dense matter using gravitational waves from NS-NS and NS-BH binaries
  - multimessenger studies (sGRBs, afterglows, neutrinos)
  - sources of r-process elements

• Requires robust models

• Recent improvements: **dynamical tides** during inspiral

• **Tidal Effective One-Body** model

• Conclusions
Neutron stars (NSs)

- strongest gravitational environment where matter can stably exist
- other extremes of physics:
  - spins up to 38000 rpm, huge magnetic fields, superfluidity, superconductivity, solid crust, …

- 1939: theoretical prediction [Oppenheimer & Volkoff]
- 1968: discovery of pulsars [Hewish, Bell, +]
- 1969: pulsars = neutron stars [Gold]
- > 2000 observed to date (~1/1000 stars)
- masses ≳ Msun, radii ~ 10km
- matter compressed to several times nuclear density

What is the nature of matter in such extreme conditions?
Phases of the strong force

H$_2$O

QCD (conjectured)

Neutron stars (NSs)

[credit: Garrido]

[Wambach+2011]
NS structure

- **crust**: $\sim km$
- **outer core**: $\sim$ few km
- uniform liquid?
- **deep core**: $\geq 2 \times \rho_{\text{nuclear}}$
- exotic states of matter?
- deconfined quarks? condensates?

- **Theoretical difficulties:**
  - many-body problem with strong interactions
  - unknown composition and equation of state (EoS)

- **Experiments**: properties of neutron-rich nuclei, phases of the strong force
  
  impossible to reproduce conditions in NSs
NS global properties from microphysics

- composition, multi-body forces, etc., reflected in the EoS
- EoS determines observables (mass, radius, …)

Einstein’s field equations

Pressure vs. density

Mass vs. radius

[ Özel & Freire 2016 ]
**NS radius measurements**

- **Masses**: to \( \sim 0.0001\% \) from pulsar timing
- **Radii**: difficult to determine

**Thermonuclear X-ray bursts**

![Thermonuclear X-ray bursts](image)

- Quiescent low-mass X-ray binaries, isolated cooling NS
- Millisecond pulsars: X-ray pulse shape of rotating hot-spot

![Millisecond pulsars](image)

- X-ray intensity vs. time relative to burst start

![X-ray intensity vs. time](image)

[Galloway+2006]
Results for NS radii

systematic uncertainties:
- distance
- atmosphere
- size of emitting region
- surface composition
- identification of spectral features
- magnetic field
- ....

Examples of results

potentially more robust EoS measurements with gravitational waves (GWs)
- asymmetric rotating NSs (crust physics)
- coalescing binaries

[Lattimer & Steiner 2014]
Gravitational waves (GWs) in brief

- Matter and energy curve space and warp time
- That **curvature** is responsible for **gravity**

- Accelerating masses generate **ripples in curvature**: GWs.

- Fractional deviation away from flat space:

  \[
  h \sim \frac{G}{c^4} \frac{\ddot{I}}{D} \sim \frac{G}{c^4} \frac{mv^2}{D} \sim 10^{-22}
  \]

  \[
  \approx 8 \times 10^{-45} \frac{s^2}{\text{kg m}}
  \]

- Carry **enormous power**: \(\approx 10^{51}\) Watts (c.f. sun radiates \(\approx 10^{26}\) Watts)

- Interact **very weakly** with matter.

- Also produced by processes in the early universe, supernova explosions, asymmetric pulsars …
Measuring GWs with interferometers

- change in intensity due to difference in phase:

\[ \Delta \phi = 2\pi f \frac{2\Delta L}{c} = \frac{4\pi f}{c} h(t) L \]

- laser frequency
- extra roundtrip travel time in the arm
Worldwide network of GW detectors

LIGO Hanford (WA)  
L = 4 km

LIGO Livingston (LA)  
L = 4 km

Advanced LIGO  
first observing run completed  
~ 2019 design sensitivity

Advanced Virgo major hardware upgrade almost completed

GEO 600

KAGRA  
~ 2020 +

LIGO India  
~ 2020 +
GW signal from black hole (BH) binaries

- BHs: regions of extreme spacetime curvature, characterized completely by only mass & spin

![Diagram showing the process of GW signal generation from black hole binaries](image)

- Inspiral: the orbit shrinks...
- Merger/ringdown: ...until they collide, ...and merge into a single BH...
- Velocity: ~0.6 c, orbital period ~10 ms...

![Waveform graph](image)
GW signal from BH binaries

- details of the waveform depend on the parameters (masses, spins, …)

<table>
<thead>
<tr>
<th>Type</th>
<th>Waveform</th>
</tr>
</thead>
<tbody>
<tr>
<td>equal mass, no spin</td>
<td></td>
</tr>
<tr>
<td>unequal mass, no spin</td>
<td></td>
</tr>
<tr>
<td>equal mass, with spins</td>
<td></td>
</tr>
</tbody>
</table>

courtesy A. Taracchini

- extracting the information from the signal requires highly accurate models as templates for data analysis
Approaches to the two-body problem

Newtonian dynamics

post-Newtonian theory

black hole perturbation theory

Numerical relativity

timescales:
\[ T_{\text{orbit}} \sim M \left( \frac{r}{M} \right)^{3/2} \]
\[ T_{\text{inspiral}} \sim M \left( \frac{M}{\mu} \right) \left( \frac{r}{M} \right)^{4} \]
Approaches to the two-body problem

- **Newtonian dynamics**
- **Newtonian theory**
- **Black hole perturbation theory**

**Orbital separation** $r/M$

**LIGO band**

**Test particle limit**

**Path to merger**

**Mass ratio** $M/\mu$

**Timescales:**

$T_{\text{orbit}} \sim M \left(\frac{r}{M}\right)^{3/2}$

$T_{\text{inspiral}} \sim M \left(\frac{M}{\mu}\right) \left(\frac{r}{M}\right)^{4}$
Approaches to the two-body problem

Newtonian dynamics

Effective One-Body (EOB) model:

combines all information into a complete waveform model for LIGO searches

[Buonanno, Damour 1999, 2000]
Effective-One-Body (EOB) approach

Binary problem

- Hamiltonian for the dynamics:
  \[ H_{EOB}(r, p_r, p_\phi; M, \nu) = M \sqrt{1 + 2\nu \left( \frac{H_{eff}}{\mu} - 1 \right)} \]

- radiation reaction forces
- factorized waveforms

Effective description

\[ ds^2_{eff} = -A(M, \nu, r)dt^2 + B(M, \nu, r)dr^2 + r^2d\phi^2 \]

\[ A = 1 - \frac{2M}{r} + \nu \delta A^{PN}(r; M, \nu) \]

effective Hamiltonian \( H_{eff} \)

lengthy PN description

\( \nu = \mu / M \)

MAP
Evolve the two-body dynamics up to the light ring (spherical photon orbit)

Smooth transition

Ringdown: quasinormal modes (QNM) of final BH
Performance of EOB waveforms

numerical relativity

EOB

m_1 = m_2, S_1 = S_2 = 0.98 S_{max}

no tuning

tuned

GW cycles

[courtesy A. Taracchini]

- recent extension to precessing spins  [Taracchini+ 2016]
GW150914 detected by LIGO

[LSC 2016]
The importance of models for GW150914

- establish >5σ detection significance
- measure source properties
- perform tests of general relativity

![Graph showing detection statistic and number of events](image)

**Detection statistic** $\hat{\rho}_c$

- $2\sigma$, $3\sigma$, $4\sigma$, $5\sigma$, >5.1σ

**Number of events**

- GW150914 search result
- Search Background
- Background excluding GW150914

**Tests of general relativity**

[LSC 2016]
Experimental progress

- LIGO’s visible volume of the universe for GWs from double neutron stars:

  ![Diagram showing the visible volume of the universe for GWs from double neutron stars](credit: atlasoftheuniverse)
GW signal from NS-NS binaries

NS-NS \(\approx\) point-masses

last \(~ 20\) cycles

rich characteristic frequency spectrum \(> kHz\)

BH-BH

merger

post-merger

collapse to BH

\(>10^3\) GW cycles

[data from T. Dietrich]
GW signal from NS-BH binaries

- NS-BH
- BH - BH

≈ point-masses

larger modeling uncertainty in point-mass GWs than for NS-NS

tidal effects

small \sim \frac{1}{(1 + q)^5}

q = \frac{m_{\text{BH}}}{m_{\text{NS}}}

tidal disruption or plunge

GW shutoff can be in aLIGO band

[data from F. Foucart]
Tidal effects during inspiral

- dominant effect:

\[ Q_{\text{NS}} = -\lambda \varepsilon_{\text{tidal}} \]

*induced quadrupole*  
*tidal deformability*  
*companion’s tidal field*

\[ \lambda = \frac{2}{3} \frac{k_2 R^5}{M} \]

\( \text{Love number} \)

\( \text{NS radius} \)

*Einstein’s Eqs: linear perturbations to equilibrium sol.*

\[ \text{[One 2nd order ODE]} \]

*pressure - density*

*\( \lambda \)- mass*

credit: B. Lackey
• **Energy** goes into deforming the NS

\[ E \sim E_{\text{orbit}} - \frac{1}{4} Q_{\text{NS}} e_{\text{tidal}} \]

• moving tidal bulges contribute to gravitational radiation

\[ \dot{E}_{\text{GW}} \sim \left[ \frac{d^3}{dt^3} (Q_{\text{orbit}} + Q_{\text{NS}}) \right]^2 \]

• **GW phase** from energy balance:

\[ \frac{d \phi_{\text{GW}}}{dt} = 2\Omega, \quad \frac{d\Omega}{dt} = \frac{\dot{E}_{\text{GW}}}{dE/d\Omega} \]

**tidal contribution:**

\[ \Delta \phi_{\text{GW}}^{\text{tidal}} \sim \lambda \frac{(v/c)^{10}}{M^5} \]

Influence on the GW phase

- **Tidal phase** contribution in the stationary phase approx.:

\[
\psi_{\text{tidal}} = \frac{3}{128 \nu x^{5/2}} \left[ -\frac{39}{2} \tilde{\Lambda} x^5 + \left( -\frac{3115}{64} \tilde{\Lambda} + \frac{6595}{364} \sqrt{1 - 4 \nu} \delta \tilde{\Lambda} \right) x^6 \right]
\]

- most sensitive to the weighted average:

\[
\tilde{\Lambda} = \frac{8}{13} \left[ (1 + 7 \nu - 31 \nu^2) \left( \frac{\lambda_1}{m_1^5} + \frac{\lambda_2}{m_2^5} \right) + \sqrt{1 - 4 \nu} (1 + 9 \nu - 11 \nu^2) \left( \frac{\lambda_1}{m_1^5} - \frac{\lambda_2}{m_2^5} \right) \right]
\]

- for identical NSs:

\[
\tilde{\Lambda} = \frac{\lambda}{m_{\text{NS}}^5} \quad \delta \tilde{\Lambda} = 0
\]
Approximate universality

- weak EoS-dependence between many NS quantities, e.g.:
  - “I - Love - Q“ [moment of inertia, tidal Love number, rotational quadrupole]
  - NS binaries: merger frequency \( f_{\text{peak}} \), post-merger spectrum

[Read+2013] [Rezzolla&Takami 2016] [Yagi & Yunes 2013]
What to expect from aLIGO+Virgo

• “standard” NS-NS event rate (40/yr), ~1 yr of data [some caveats with the analysis]:
  • $\lambda$ to ~10-50 %, radius to ~1-2 km, pressure to ~ factor of 2  [Lackey+2014]

• similar conclusions with hybrid NR waveforms [Shibata+2016]

• NS-BH systems: $\lambda/m^5$ to ~ 10-100 %  [Lackey+ 2013]
Recent model improvement: dynamic tides

- $Q_{NS}$ corresponds to the NS’s fundamental oscillation modes
  - eigenfrequency: $\omega_f \sim \sqrt{m_{NS}/R^3}$ (internal structure-dependent)
- **tidal forcing** frequency: $m\Omega \sim m\sqrt{M/r^3}$

**NS’s response to the tidal field**

$$\lambda_\ell = \frac{2(\ell-2)}{(2\ell-1)!!} k_\ell R^{2\ell+1}$$

Love number

$H4$ EoS, $m_{NS}=1.35M_\odot$
EOB Hamiltonian with tidal effects

- **adiabatic tides (AT):** \( A = A^{pp}(M, \nu, r) + \lambda_\ell A^{AT}(M, \nu, r) \) [Damour, Nagar, Bini+2009-2014]

- **dynamic tides:** effective description of from approximate solutions for \( Q_{NS} \):
  \[
  A = A^{pp}(M, \nu, r) + \lambda_\ell^{\text{eff}}(M, \nu, r, \lambda_\ell, \omega_f) A^{AT}(M, \nu, r)
  \]

- good agreement with full evolution:
  \[ H_{\text{EOB}}(r, p_r, p_\phi, Q_{\ell m}, P_{\ell m}; M, \nu, \lambda_\ell, \omega_f) \]

\[
\text{ds}_{\text{eff}}^2 = -Adt^2 + Bdr^2 + r^2d\phi^2
\]
Performance of the tidal EOB model

Nonspinning NS-BH mass ratio 2:1
$\Gamma=2$ polytropic
$C=0.1444$

GW cycles

$\Delta \phi_{22} \, [\text{rad}]$

$t - r^*/M$

Re($D_{h2}/M$)

NR error

Adiabatic tides

BH-BH

Dynamic tides
Performance of the tidal EOB model

- Nonspinning NS-BH mass ratio 2:1
- $\Gamma = 2$ polytropic
- $C = 0.1444$

**Graph Details:**
- **Re($D_l h_{22}/M$)**
- **(t - r*)/M**
- **Δϕ22 [rad]**
- **GW cycles**

**Tides:**
- Dynamic tides
- Adiabatic tides
- Self-force

**Models:**
- EOB
- NR

**Enhancements:**
- Enhanced tidal EOB

[Bernuzzi+]
Conclusions

- Main imprint of NS microphysics in the GWs from **inspirals**: tidal effects
- **Dynamic** f-mode tides can be significant, now included in **EOB**
- Also included: NS-BH **tidal disruption signal** (nonspinning case)

Outlet:

- Further **improve models** and measurement potential, **reduce systematics** (inspiral, NS-BH tidal disruption, NS-NS merger/post-merger)
- Include **more realistic physics**
- **Accurate NR** simulations are crucial to inform model developments
- data analysis strategies (e.g. parameterization)
- connection with multimessenger signals
Thank you