“When the world was still flat…“ - On the stability of protoplanetary disks

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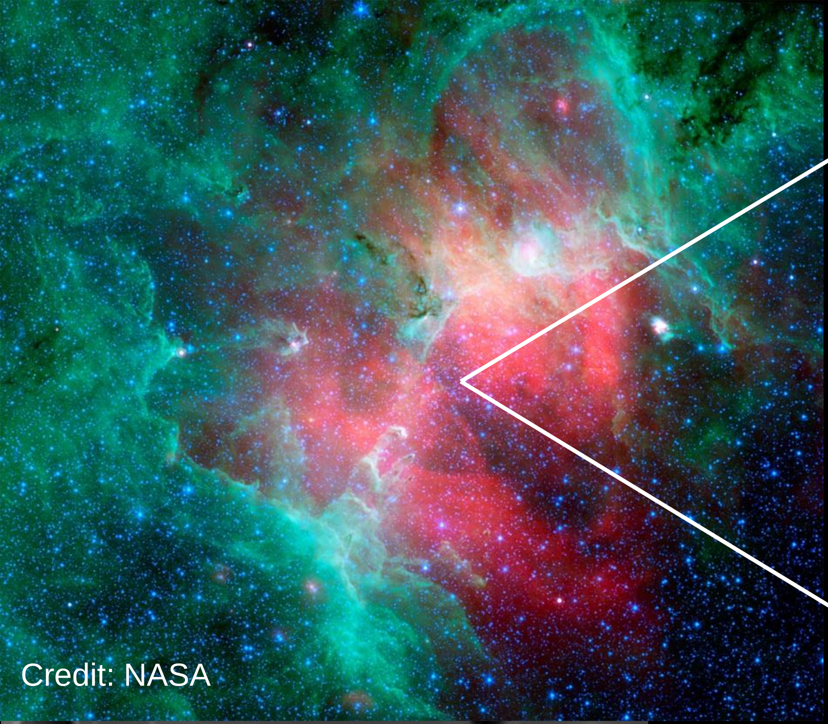
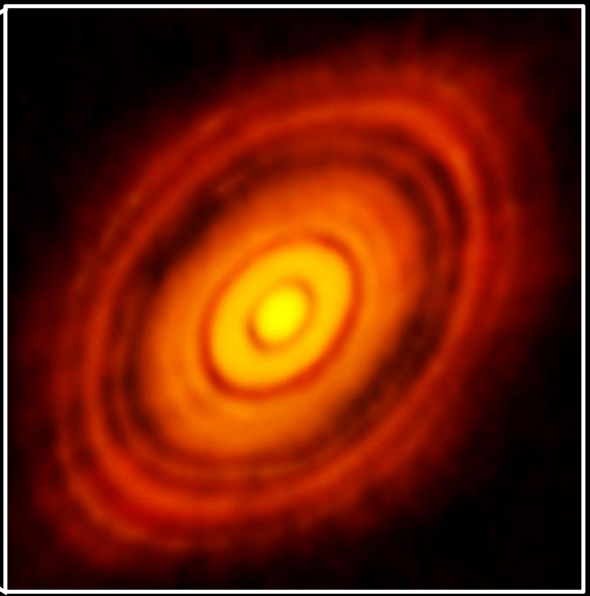
Astrocoffee FIAS, Frankfurt, January 23, 2018

Credit:ALMA (ESO/NAOJ/NRAO)

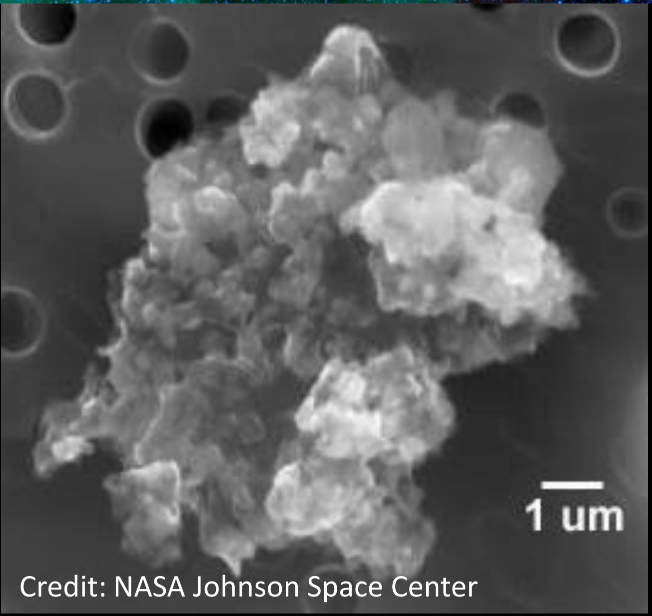
Source: https://www.youtube.com/watch?v=E4yirtvUurA (January 5, 2018, 22:20 h)



Credit: NASA



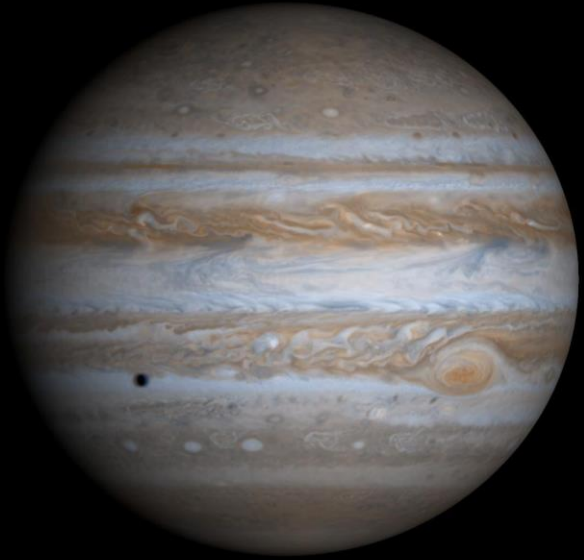
Credit: ALMA (ESO/NAOJ/NRAO)



Credit: NASA Johnson Space Center

size: $\sim \cdot 10^{13}$

mass: $\sim \cdot 10^{40}$



Credit: NASA/JPL/University of Arizona

Topics

Content-wise: Protoplanetary disks

- properties
- evolution

Method-wise: Linear Stability Analysis

Assumptions

- $M_{disk} \ll M_*$
- scale height h satisfies $\frac{h}{r} \ll 1$
- neglect self-gravity $\frac{M_{disk}}{M_*} < \frac{1}{2}\frac{h}{r}$ (Toomre criterion)
- "passive disks" → heated by central star
  → slow accretion rates ($\dot{M} \leq 2 \cdot 10^{-8} \frac{M_{sun}}{yr}$)
  → late times in disk evolution

Disk evolution

- angular momentum must be transported:
  → winds → viscosity ν
- Navier-Stokes equation yields:

$$\boxed{\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{\frac{1}{2}}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right]}$$

- Shakura & Sunyaev (1973)
- Lynden-Bell & Pringle (1974)

Disk evolution equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{\frac{1}{2}}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right]$$

$$X = 2r^{1/2}$$

$$f = \tfrac{3}{2}\Sigma X = 3\Sigma r^{1/2}$$

$$D = \frac{12\nu}{X^2} = \frac{3\nu}{r}$$

Diffusive form: $\boxed{\frac{\partial f}{\partial t} = D\,\frac{\partial^2 f}{\partial X^2}}$

Diffusion time scale: $\frac{(\Delta X)^2}{D}$

$\Rightarrow$ viscous time scale: $\tau_\nu \approx \frac{r^2}{\nu}$

Viscosity?

- α-prescription: $\boxed{\nu = \alpha c_s h}$
- Molecular collisions: $\nu_m \sim \lambda c_s = \frac{c_s}{n\sigma}$

$$\Rightarrow \tau_\nu = \frac{r^2}{\nu_m} \approx 3 \cdot 10^{13} yr$$

- Reynolds number: $Re = \frac{UL}{\nu_m} = \frac{c_s h}{\nu_m} \sim 10^{10} \gg 1$

  $\Rightarrow$ highly turbulent ***IF*** there is an instability

Rayleigh criterion

A non-self-gravitating, non-magnetized disk flow is linearly stable to axisymmetric perturbations if

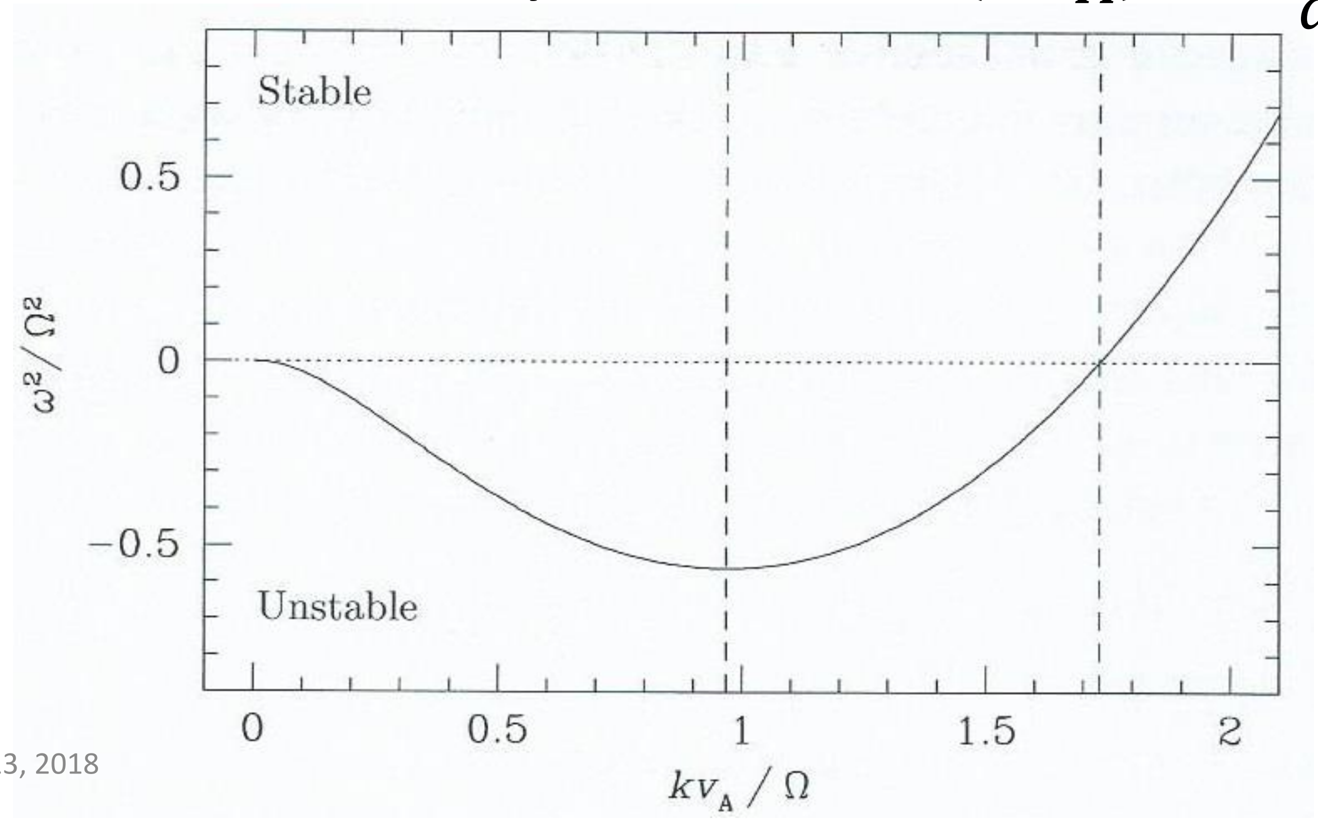
$$\frac{dl}{dr} = \frac{d}{dr}(r^2\Omega) > 0$$

$\Omega \propto r^{-3/2}$

$\Rightarrow$ no turbulence?!

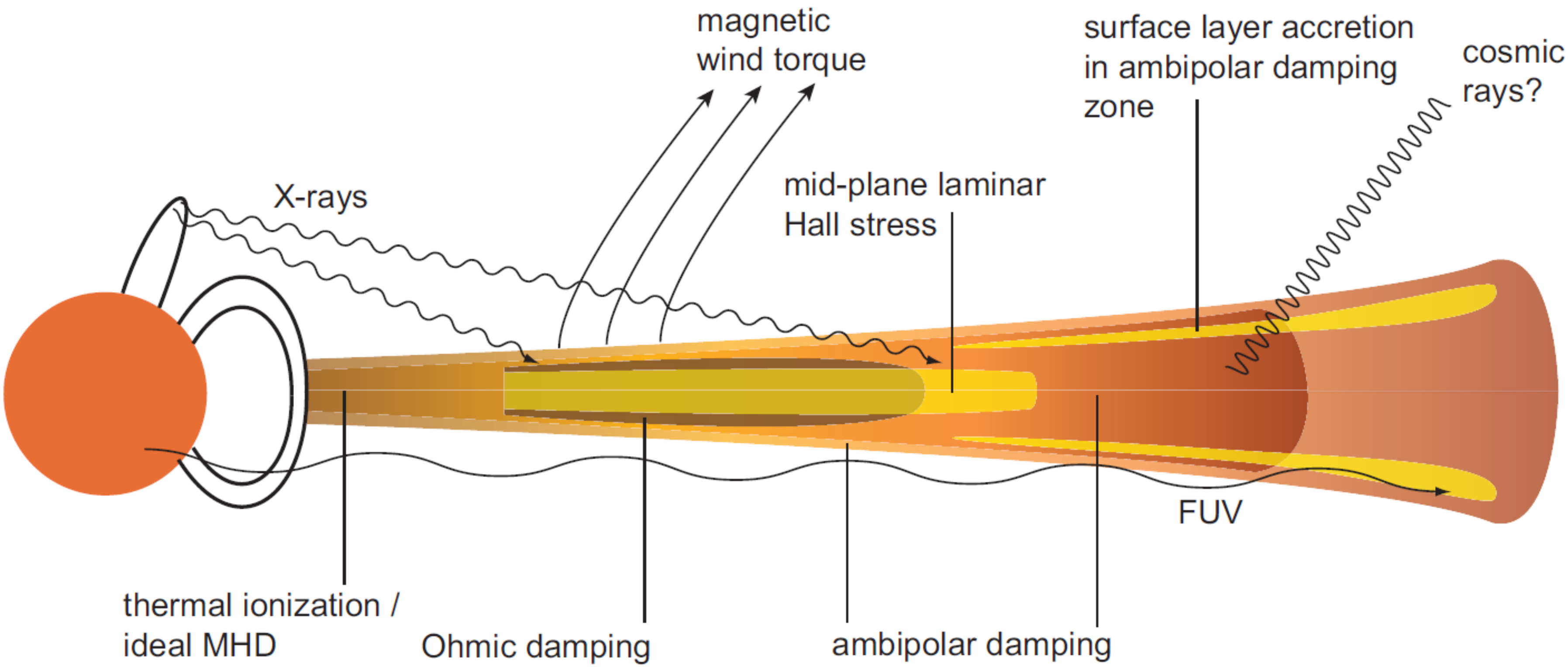
Magnetorotational Instability

- Chandrasekhar 1961, Balbus & Hawley 1991
- Modified stability condition: $(kv_A)^2 + \frac{d\Omega^2}{d\ln r} > 0$



[1]

Dead zones



magnetic
wind torque
surface layer accretion
in ambipolar damping
zone
cosmic
rays?
mid-plane laminar
Hall stress
X-rays
FUV
thermal ionization /
ideal MHD
Ohmic damping
ambipolar damping

[1]

Hydrodynamical Instabilities

Gravitational Instability

Streaming Instability?

Critical Layer Instability

Subcritical Baroclinic Instability

Convective Overstability

Vertical Shear Instability

Goldreich-Schubert-Fricke Instability

Convective Overstability

- neglect vertical structure:

$$\rho_0 = \bar{\rho}_0 \cdot \left(\frac{R}{R_0}\right)^{\beta_q}, c_s^2 = \bar{c}_s^2 \cdot \left(\frac{R}{R_0}\right)^{\beta_p}, p = c_s^2 \rho$$

- Consider thermal relaxation: $S = \frac{p}{T}\frac{(T - T_0)}{\tau}$

$\tau \rightarrow 0$: locally isothermal limit

$\tau \rightarrow \infty$: adiabatic limit

1D $\tau \to \infty$

Worst slide in this talk...

$$\partial_t\,(\rho_0+\rho') + (\rho_0+\rho')\partial_x\,u' + u'\,\partial_x\,(\rho_0+\rho') \;=\; 0$$

$$\partial_t\,u' + u'\,\partial_x\,u' + \frac{1}{\rho_0+\rho'}\,\partial_x\,(p_0+p') \;+ \partial_x\,\Phi = \;0$$

$$\partial_t\,(p_0+p') + u'\,\partial_x\,(p_0+p') + \gamma\,(p_0+p')\partial_x\,u' \;=\; 0$$

$$\partial_t\,\rho' \;+ \rho_0\,\partial_x\,u' \;+\; u'\,\partial_x\,\rho_0 \;=\; 0$$

$$\partial_t\,u' + \frac{\partial_x p'}{\rho_0} - \frac{\rho'}{\rho_0^2}\partial_x\,p_0 \;=\; 0$$

$$\partial_t\,p' + u'\,\partial_x\,p_0 \;+ \gamma\,p_0\,\partial_x\,u' = \;0$$

1 D
$\tau \to \infty$

Dispersion relation

$$\mathbf{Q}'(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk e^{ikx} \sum_l \alpha_l \hat{\mathbf{Q}}_l(k) e^{-i\omega_l(k)t}$$

$$M = \begin{pmatrix} -i\omega_l & \rho_0 ik + \partial_x \rho_0 & 0 \\ -\frac{\partial_x p_0}{\rho_0^2} & -i\omega_l & \frac{ik}{\rho_0} \\ 0 & \gamma p_0 ik \partial_x p_0 & -i\omega_l \end{pmatrix}$$

$$\Rightarrow \omega_0 = 0 \quad \vee \quad \omega_{1,2} = \pm\sqrt{c_s^2 k^2 + \frac{\partial_x p_0 \partial_x \rho_0}{\rho_0^2}}$$

3 D
finite τ

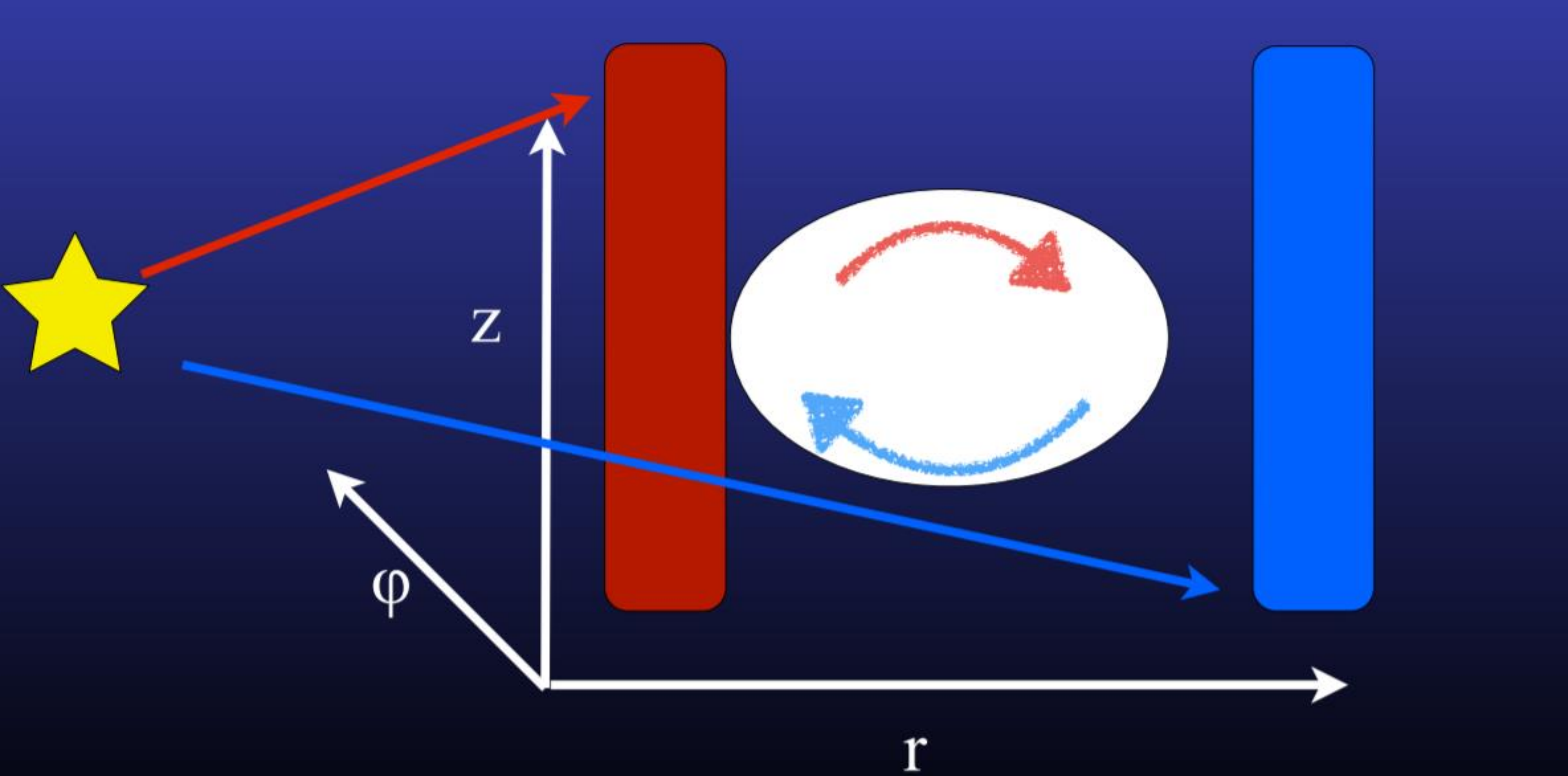
Eigenvectors

$$\boldsymbol{v} = \left[\rho', u_r', u_\phi', u_z', p'\right]^T \qquad\qquad M(\omega)\cdot\boldsymbol{v} = 0$$

growth rate

$$\Rightarrow\ \boldsymbol{v}_{\max} = \left[0, \underbrace{1, -\frac{1}{2}\left(\frac{\sigma}{\Omega} + i\right)}_{\textit{overstable epicycle}}, 0, 0\right]^T$$

[3]

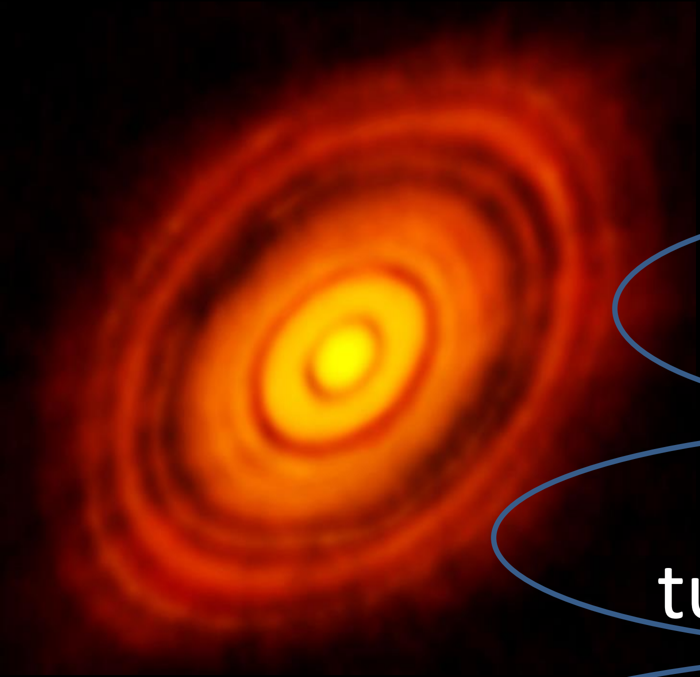


Linear Convective Overstability: Klahr and Hubbard 2014, Lyra 2014

Numerical Simulations

- test non-linear instabilities
- PLUTO code (Mignone et al.):
  - discretization, static or adaptive mesh
  - finite volume or finite difference approach
  - 1D, 2D or 3D simulations
- shows vortex formation and growth

Summary



Credit: ALMA (ESO/NAOJ/NRAO)

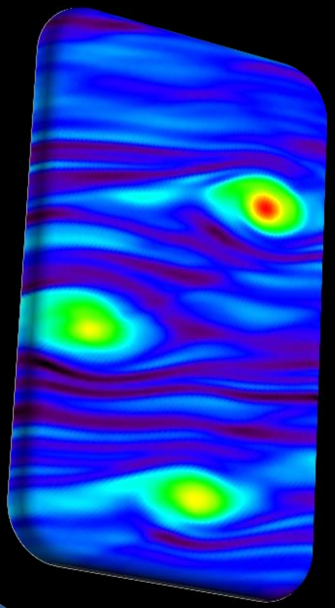
Pp-disks are turbulent.

MRI creates turbulence, but…

… there are dead zones, where…

…hydrodynamic instabilities operate ...

… under baroclinic conditions and finite cooling times to create vortices.



Credit: N. Raettig

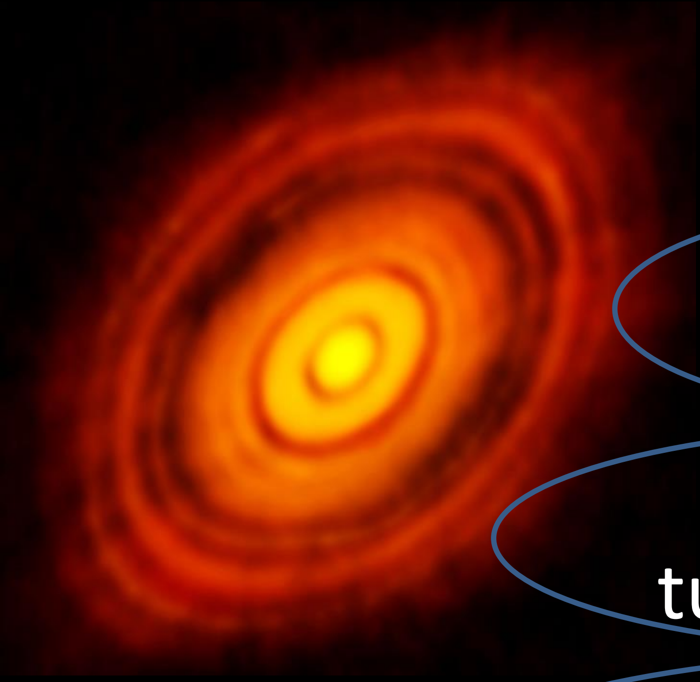
Next talk in about 6 months…

Thank you

and

stay tuned!

Summary



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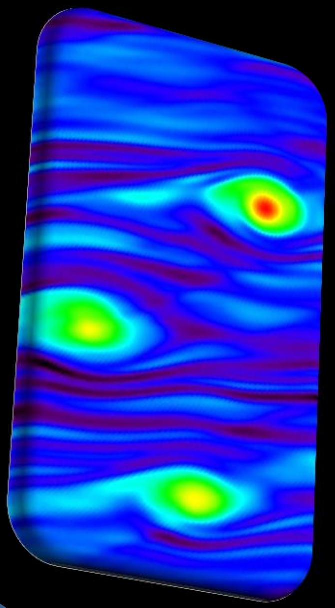


Credit: N. Raettig

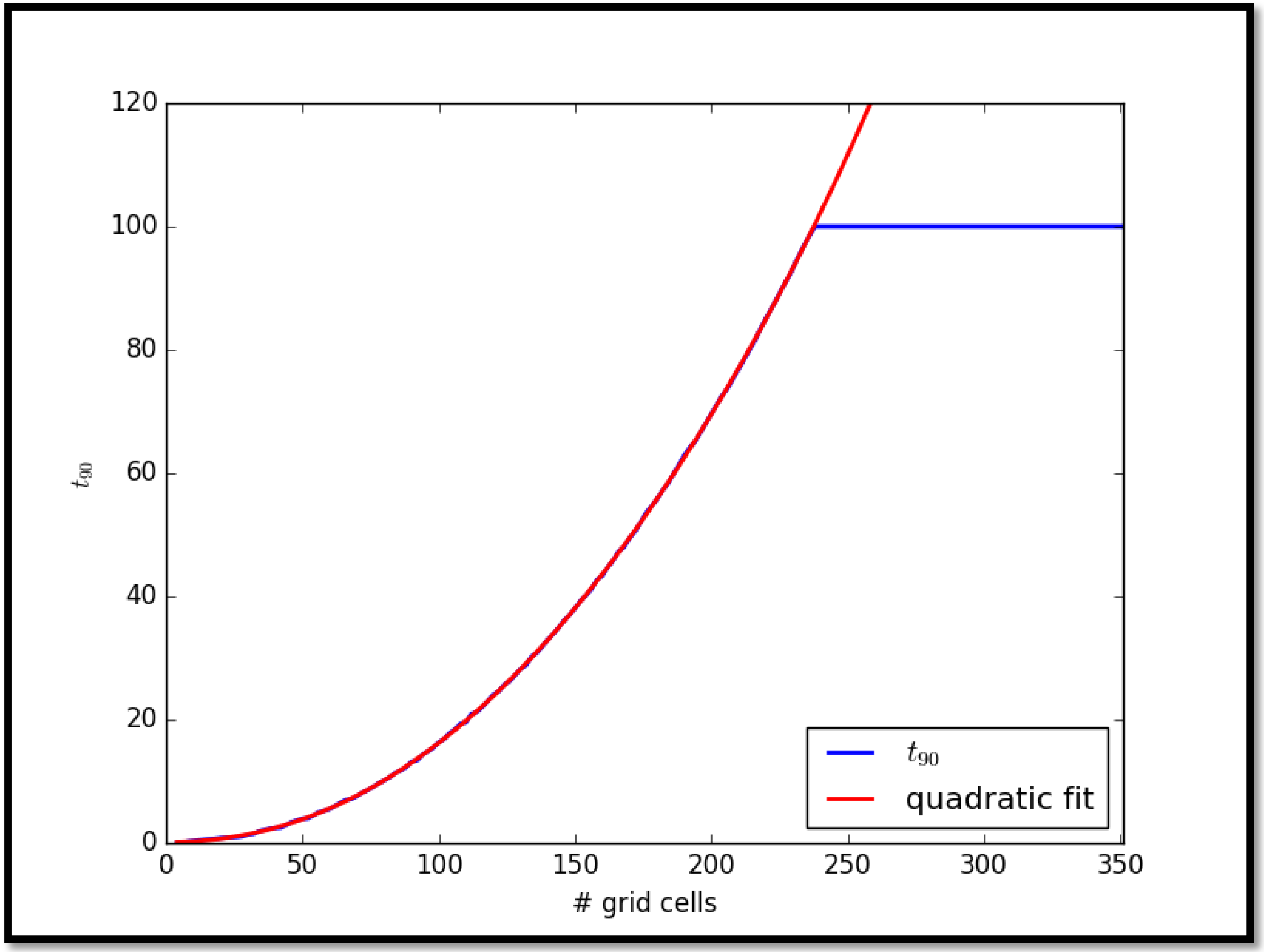
Sources

[1] Philip J. Armitage: „Astrophysics of Planet Formation“, Cambridge University Press, New York, USA, 2013

[2] https://upload.wikimedia.org/wikipedia/commons/d/d4/Johannes_Kepler_1610.jpg (January 6, 2018, 19:05 h)

[3] Lyra W., 2014, ApJ, 789, 77

Backup



120
100
80
60
40
20
0
t_{90}
0
50
100
150
200
250
300
350
grid cells
t_{90}
quadratic fit