Cosmic Structures in Brans-Dicke-like Theories

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Overview

1. Part I: No-hair theorem in BDT with $\Lambda > 0$
   - General Relativity
   - Brans-Dicke Theory
   - No-Hair conjecture
2. Part II: Spherical Star Solutions
   - Turnaround radius in ΛCDM
   - Perturbations in BD theory
   - Astrophysically Interesting Solutions
3. Conclusions

S.Bhattacharya, KFD, A.E.Romano, T.N.Tomaras: PRL 115 181104 (2015)

S.Bhattacharya, KFD, A.E.Romano, C.Skordis, T.N.Tomaras: in preparation

GR: Action and EoM

The action of general relativity is given by:

$$\mathcal{S}_{GR} = \mathcal{S}_{EH} + \mathcal{S}_m = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R + \mathcal{L}_m\right) ,$$

and the equations of motion are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} .$$

An alternative gravity theory: Brans-Dicke

Mach's principle

inertial are the frames that do not accelerate relative to the "fixed stars".

Dicke's form of Mach's principle

the gravitational constant κ should be a function of the mass distribution of the universe.

Brans and Dicke

replaced $1/\kappa \to \phi$ and added a kinetic Lagrangian for ϕ so that

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\phi R - 2\Lambda - \frac{\omega}{\phi} (\nabla \phi)^2 + \mathcal{L}_m \right) .$$

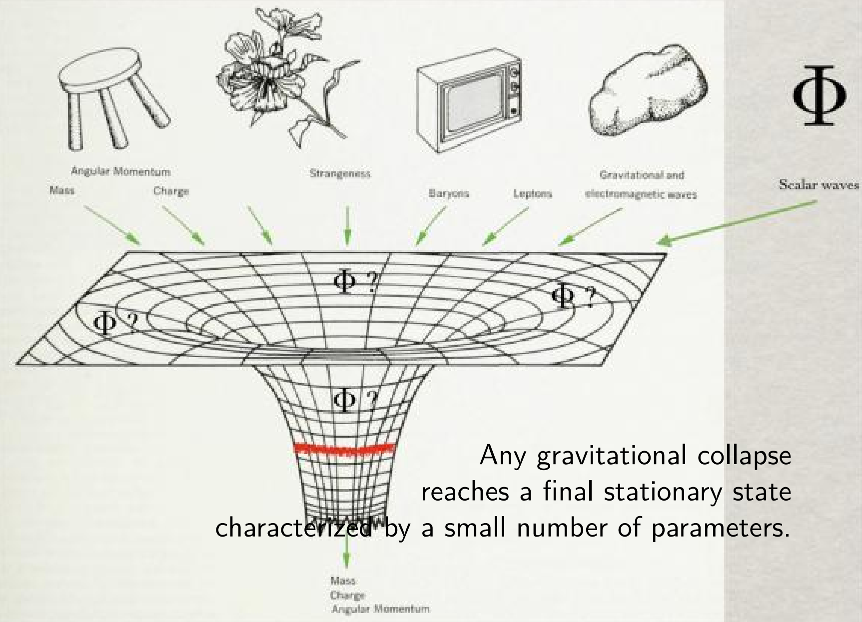
An alternative gravity theory: Brans-Dicke

Equations of Motion

$$\begin{aligned}\phi G_{\mu\nu} + \Lambda g_{\mu\nu} &= 8\pi T^{M}_{\mu\nu} + \frac{\omega}{\phi}\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right) \\ &\quad + \nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\,, \\ \Box\phi &= \frac{8\pi T - 4\Lambda}{2\omega+3}\,,\end{aligned}$$

and the ricci scalar is

$$R = \frac{4\Lambda - 8\pi T}{\phi}\frac{2\omega}{2\omega+3} + \frac{\omega}{\phi^2}(\nabla\phi)^2\,.$$



Any gravitational collapse
reaches a final stationary state
characterized by a small number of parameters.

No-hair in BDT with $\Lambda > 0$: set up

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- spacetime: stationary & axisymmetric

$$g_{\alpha\beta} = -\beta^{-2}\chi_\alpha\chi_\beta + f^{-2}\psi_\alpha\psi_\beta + \gamma_{\alpha\beta}\,,$$

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where $\chi_\alpha\psi^\alpha = 0$.

No-hair in BDT with $\Lambda > 0$

By using $\sqrt{-g} = \beta\sqrt{h}$ we obtain

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}\partial^\alpha\phi) = \frac{1}{\beta}D_a[\beta D^a\phi]$$

$$\Rightarrow D_a(\beta D^a\phi) = \frac{\beta(8\pi T - 4\Lambda)}{2\omega + 3}$$

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Multiply by $e^{-\phi}$ and integrate over Σ:

$$\int_{\partial\Sigma} e^{-\phi}\beta n^a D_a\phi = \int_\Sigma \beta e^{-\phi}\left[-(D^a\phi)(D_a\phi) + \frac{8\pi T - 4\Lambda}{2\omega + 3}\right].$$

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Conclusion

For finite ω, the Brans-Dicke field and consequently the curvature scalar diverge on the horizon.

Monotonicity of the BD field

$$\int_{\partial\mathcal{S}} e^{\epsilon\phi}\beta n^a D_a\phi = \int_{\mathcal{S}} \beta e^{\epsilon\phi}\left[\epsilon(D^a\phi)(D_a\phi) + \frac{8\pi T - 4\Lambda}{2\omega+3}\right].$$

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Conclusion

For finite ω and in the presence of a positive cosmological constant, the field ϕ of a stationary non-black hole spacetime must be monotonic.

No-hair generalization in scalar-tensor theories

We write the action as

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - 2\Lambda - \omega(\phi)(\nabla\phi)^2 - V(\phi)\right] + S_M\left[g_{\mu\nu}\right]$$

and the scalar equation becomes

$$(2\omega(\phi)+3)\,\Box\phi + \frac{d\omega(\phi)}{d\phi}(\nabla\phi)^2 + \phi\frac{dV}{d\phi} - 2V = 8\pi T - 4\Lambda$$

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or after a conformal transformation

$$\Box_{\mathcal{G}}\psi = U'(\psi) \equiv \frac{8\pi T - 4\Lambda}{\phi^2\sqrt{2\omega(\phi)+3}} + V'(\psi)$$

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- For $\Lambda = 0 = T \Rightarrow V'' \geq 0$,
- For $\Lambda \neq 0$ and/or $T \neq 0 \Rightarrow U'' \geq 0$ + restrictions on $\omega(\phi)$.

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- Consider the Schwarzschild-de Sitter metric

$$ds^2 = -(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})dt^2 + (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})^{-1} dr^2 + r^2 d\Omega^2 \,,$$

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$$r_{ta} = \left(\frac{3M}{\Lambda}\right)^{1/3} \,.$$

Turnaroun radius in ΛCDM

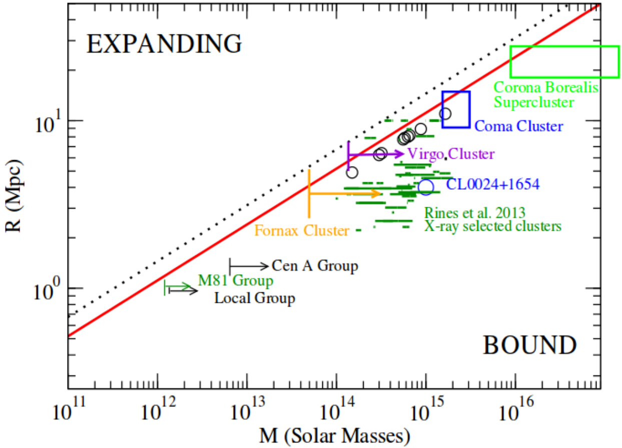

Figure : Data masses and turnaround radii for several local-Universe structures compared to the ΛCDM turnaround radius bound (red line).
V.Pavlidou, T.N.Tomaras; JCAP 1409 (2014) 020

Perturbations

Consider Schwarzschild-de Sitter background and large ω so that

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{\omega}\delta g_{\mu\nu} \ , \ \phi(r) = \phi_0 + \frac{1}{\omega}\delta\phi(r)$$

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and its solution up to a constant is

$$\delta\phi = \frac{C_1}{r_H}\ln\left(1 - \frac{r_H}{r}\right) + \left(1 + \frac{C_1}{2r_C}\right)\ln\left(1 + \frac{r}{r_C}\right) + \left(1 - \frac{C_1}{2r_C}\right)\ln\left(1 - \frac{r}{r_C}\right)$$

where $r_H \simeq 2M$ and $r_c \simeq \sqrt{3/\Lambda}$.

Astrophysically interesting solutions

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(i) Assume absence of the cosmological horizon, and thus the solution is valid only for $r \ll r_c$. To avoid the divergence at r_H we set $C_1 = 0$ and

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Conclusion

For $r_H \leq r \ll r_c$ and "hidden" cosmological horizon black-holes have scalar-hair.

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(ii) Assume absence of the black-hole horizon.
To avoid the divergence at r_c we set $C_1 = 2r_c$ and

$$\delta\phi \sim \frac{2r_c}{r_H}\ln\left(1-\frac{r_H}{r}\right)+2\ln\left(1+\frac{r}{r_c}\right) .$$

which is regular, monotonic, unique and reliable for $r_c/\omega \ll r \leq r_c$.

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It is proven that:

- at the surface of the star the slope is decreasing, while
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Conclusion

Smooth solution regular at both $r = 0$ and $r = r_c$ can exist only for $\omega = \infty$ and $\phi = const.$ everywhere, with the exterior being just SdS.

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Thus, to describe a star in BDT with $\Lambda > 0$ and finite ω, one has to "hide" the cosmological horizon as well.

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(iii) In the region $r_H \ll r \ll r_c$ and assuming $C_1 \ll r_c$ we obtain

$$\delta\phi(r) = -\frac{C_1}{r} - \frac{\Lambda r^2}{3}\,.$$

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The metric equations are

$$R^{(1)}_{\mu\nu} = \nabla_\mu\delta\phi\nabla_\nu\delta\phi + \nabla_\mu\nabla_\nu\delta\phi + \Lambda\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta\phi - \bar{g}_{\mu\nu}\right)\,.$$

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and the first order correction to the metric is

$$\begin{aligned} \delta f(r) &= \frac{2M}{r} \left(\frac{C_1}{M} - \frac{C_2}{18M} + 3C_3 \right) + \frac{\Lambda r^2}{3} (3C_3 + 1) - 3C_3 \,, \\ \delta h(r) &= \frac{C_2}{9r} - \frac{2\Lambda r^2}{3} \,. \end{aligned}$$

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Background + $\frac{1}{\omega}$ Corrections

$$\begin{aligned} f(r) &= 1 - \frac{2M}{r} \left(1 + \frac{1}{\omega} \left(\frac{C_2}{18M} - \frac{C_1}{M} \right) \right) - \frac{\Lambda r^2}{3} \left(1 - \frac{1}{\omega} \right) \\ h(r) &\simeq \left(1 - \frac{2M}{r} \left(1 + \frac{C_2}{18M\omega} \right) - \frac{\Lambda r^2}{3} \left(1 - \frac{2}{\omega} \right) \right)^{-1} \\ \phi(r) &= 1 - \frac{1}{\omega} \left(\frac{C_1}{r} + \frac{\Lambda r^2}{3} \right) \end{aligned}$$

Astrophysically interesting solutions

$$C_1 = -M \ , \ \ C_2 = -9M \, .$$

and the solutions become

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Thus the maximum turnaround radius is given by

$$r_{ta} = \left(\frac{3M}{\Lambda}\right)^{1/3}\left(1 + \frac{1}{2\omega}\right) \, .$$

Numerical Verification

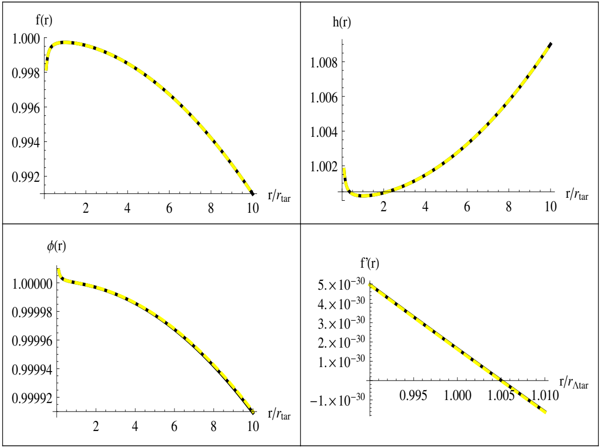

Figure : $M = 10^{17} M_{\odot}, R_0 = 10^3 M, \omega = 100$

Numerical Verification

Mass	Maximum Turnaround Radius (m)				
($M_\odot$)	ΛCDM	an.sol. $\omega_{BD} = 10^2$	an.sol. $\omega_{BD} = 10^4$	num. sol. $\omega_{BD} = 10^2$	num. sol. $\omega_{BD} = 10^4$
10^{11}	1.6436×10^{22}	1.6518×10^{22}	1.6437×10^{22}	1.6518×10^{22}	1.6437×10^{22}
10^{13}	7.6289×10^{22}	7.667×10^{22}	7.6293×10^{22}	7.6668×10^{22}	7.6293×10^{22}
10^{15}	3.541×10^{23}	3.5587×10^{23}	3.5412×10^{23}	3.5586×10^{23}	3.5412×10^{23}
10^{17}	1.6436×10^{24}	1.6518×10^{24}	1.6437×10^{24}	1.6518×10^{24}	1.6437×10^{24}

Table : The maximum turnaround radius is given, for different values of the mass and of ω. In the second column are the values of the ΛCDM model. The other columns are for the BD theory, calculated with the analytical and the numerical solutions respectively.

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- No-hair theorem: We constrain not only the field configuration but also the theory itself.
- For more general theories with $\omega = \omega(\phi)$, the usual no-hair theorem with a convex potential need not **necessarily** hold for $\Lambda > 0$, which **might** lead to hairy black holes.
- There is no regular star solution in presence of a stationary cosmological event horizon with the BD field.

Conclusions

- No-hair theorem: We constrain not only the field configuration but also the theory itself.
- For more general theories with $\omega = \omega(\phi)$, the usual no-hair theorem with a convex potential need not **necessarily** hold for $\Lambda > 0$, which **might** lead to hairy black holes.
- There is no regular star solution in presence of a stationary cosmological event horizon with the BD field. This means that there should be a screening effect for the BD theory at very large length scales.


Danke schön!